QR-RLS Algorithm for Error Diffusion of Color Images

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ABSTRACT

Printing color images on color printers, and displaying them on computer monitors requires a significant reduction of physically distinct colors, which causes degradation in image quality. An efficient method to improve the display quality of a quantized image is error diffusion which works by distributing the previous quantization errors to neighboring pixels exploiting the eye's averaging of colors in the neighborhood of the point of interest. This creates the illusion of more colors. In this paper, a new error diffusion method is presented in which the adaptive Recursive Least Squares (RLS) algorithm is used rather than the deterministic approaches in literature. To improve the performance, a diagonal scan is used in processing the image.

Keywords: halftones, color, displays, printing, quantization, image processing

This paper is presented in IEEE International Conf. Image Processing (ICIP'98) and it was supported in part by the NATO Grand CRG-971117.

1. INTRODUCTION

Color output devices such as halftone color printers and palette-based displays are capable of producing only a limited number of colors, whereas the human eye can distinguish around ten million colors under optimal viewing conditions.[1] The eye perceives only a local spatial average of the color spots produced by a printing device, and is relatively insensitive to errors made in high frequencies in an image.[1] Halftoning algorithms, therefore aim to preserve these local averages while forcing the errors between the continuous tone image and the halftone image to high frequency regions. The existing halftoning techniques can be broadly classified as dithering, error diffusion, and optimization-based halftoning techniques.

The basic idea in dithering methods is thresholding each pixel value after adding noise. The ordered dithering techniques are attractive in the sense that they are very simple to implement, and computationally inexpensive because they require pixelwise operations. However, dithering results into regular and periodic error patterns, which lowers the quality of the output image.

The second class of halftoning methods are error diffusion techniques first introduced by Floyd and Steinberg.[2] They proposed an algorithm which works by distributing the quantization error of the current pixel to neighboring pixels. Typically, at each pixel, the weighted sum of previous quantization errors is added to the current value, and then quantized to produce the output pixel value. These weights form an error diffusion filter. The error diffusion aims to preserve the local average value of the image, therefore a unity gain lowpass finite impulse response (FIR) filter is used for distributing the error.

Error diffusion was first developed for grayscale images. For color images, error diffusion can be applied to each color component independently, which is called scalar error diffusion, or as in,[3] a color pixel can be error diffused in a vectorized manner.

Some directional artifacts seen in error diffusion are due largely to the traditional raster of processing.[4] Previous approaches for improving error diffusion employed various choices of space filling curves to define the order of processing, such as serpentine curves,[4] Peano curves,[5] random space filling curves.[6]

In contrast to deterministic error filter kernels, some recent research employed dynamically adjusting the error filter kernel using adaptive signal processing techniques. Akarun *et* al[3] have used a vectorized error diffusion approach, and updated the error diffusion filter coefficients adaptively. Wong[7] minimizes a local frequency-weighted error criterion to adjust the error diffusion kernel dynamically using the well known least mean square(LMS) algorithm.[8]

In optimization-based halftoning techniques, the problem of halftoning is formulated as an optimization problem that minimizes an error metric between the continuous tone original image and its halftone version. Disadvantages of optimization-based methods for halftoning is that there are many local optima, the methods are iterative, and they require substantially high computational power. For color images, processing requirements further increase.

Some hybrid schemes that combine different aspects of halftoning methods are proposed in the literature such as the blue noise halftoning,[9] and green-noise halftoning.[10]

2. DIAGONAL ERROR DIFFUSION

Block diagram of the standard error diffusion technique is given in Figure 1. Usually, the image is processed in a raster scan fashion, and each input color pixel $\boldsymbol{x}(s)$ is a 3×1 vector, where the index $s = s_1 M + s_2$ and M is the number of horizontal pixels in the image. The current pixel $\boldsymbol{x}(s)$ together with the diffused error is quantized. The resultant image $\boldsymbol{y}(s)$ is the dithered image.

Here, $\boldsymbol{Q}$ is the quantizer, and $\boldsymbol{h}$ is the error diffusion filter. Some well-known error diffusion filter masks[2,11] are shown in Figure 2 where $\bullet$ denotes the origin. These masks determine the support of the error diffusion filter. A common characteristic of these filters is that they are causal, i.e. their region of support is wedge to ensure that these filters can be applied in a sequential manner.[12] The filter coefficients are deterministic, lowpass in nature, and adds up to 1 so that errors are neither amplified nor reduced.

The equations describing this system are written as follows:

$$\begin{aligned} \boldsymbol{u}(s) &= \boldsymbol{x}(s) + \sum_{k<s} \boldsymbol{h}(s-k)\boldsymbol{e}(k) && (1) \\ \boldsymbol{e}(s) &= \boldsymbol{u}(s) - \boldsymbol{y}(s) && (2) \\ \boldsymbol{y}(s) &= Q(\boldsymbol{u}(s)) && (3) \end{aligned}$$

where $k < s$ corresponds to a causal error diffusion mask, and $\boldsymbol{e}(s)$ is the quantization error. The error between the original input pixel and the output pixel is defined as the output error, $\boldsymbol{e}_{out}(s) = \boldsymbol{x}(s) - \boldsymbol{y}(s)$ which can be expressed as

$$\boldsymbol{e}_{out}(s) = \boldsymbol{e}(s) - \sum_{k<s} \boldsymbol{h}(s-k)\boldsymbol{e}(k). \qquad (4)$$

The normal raster used in error diffusion causes vertical or horizontal artifacts, and regular patterns that arise especially in uniform intensity regions. It is well-known that Human Visual System(HVS) is less sensitive to diagonal errors compared to the vertical or horizontal errors. To take advantage of this fact we scanned the image diagonally, hence the error is diagonally diffused.

Causal prediction windows shown in Figure 3 are used in the error diffusion algorithm. Here, we aim to break up the horizontal and vertical directionality of the possible error patterns, and force the accumulation of the error to be in diagonal orientation to which the human eye is less sensitive.

3. A NEW ADAPTIVE ERROR DIFFUSION

The error diffusion filter plays an important role in shaping the output error spectrum. In contrast to deterministic error diffusion filters, recent algorithms use the optimum filter coefficients for a given image, or update the coefficients adaptively using LMS type adaptive algorithms.[3,7]

As in standard dithering, in error diffusion, the aim is to decorrelate the quantization noise from the input signal. This results into a whiter error spectrum which is less disturbing. This requires the prediction of the quantization error from the previous quantization errors. The prediction aims to minimize the energy of the output error $\boldsymbol{e}_{out}(s)$:

$$\begin{aligned} E[||\boldsymbol{e}_{out}(s)||^2] &= E[||\boldsymbol{x}(s) - \boldsymbol{y}(s)||^2] \\ &= E[||\boldsymbol{e}(s) - \sum_{k<s} \boldsymbol{h}(s-k)\boldsymbol{e}(k)||^2]. \end{aligned} \tag{5}$$

Then the optimal filter coefficients are chosen as the minimizer of (5).

Since typical image characteristics are locally nonstationary, an adaptive algorithm is used in the minimization of the output error sequence. For this purpose, we propose to use an RLS type adaptation for better tracking of local characteristics than the slower converging LMS adaptation. To improve the robustness, in the implementation of the RLS adaptation, rotation based algorithms are utilized.[13]

In the implementation of the QR-RLS adaptation, the previous quantization errors in the causal so-called half plane window are used as inputs, and the current quantization error is used as the desired signal.

In the scalar implementation of the algorithm, the red, green and blue components of each pixel are processed separately by running three QR-RLS algorithms in parallel, each giving the output for each one of the color components red, green, blue, as shown in Figure 4.

In the vector implementation, all three color components of the previous quantization errors are used in the prediction of the each component of the current quantization error, as in Figure 5. Again, three parallel QR-RLS algorithms are run for each of the color components. Here, the aim is to use the correlation among the color components.

The steps of the QR-RLS algorithm is given as[13]

$$\text{for} \quad s = 1, 2, \ldots \quad (\boldsymbol{R}(0) = \sqrt{\delta}\boldsymbol{I}_N, \quad \boldsymbol{\Gamma}(0) = \boldsymbol{0})$$

$$\boldsymbol{Q}(s) \begin{bmatrix} \sqrt{\lambda}\boldsymbol{R}(s-1) & \sqrt{\lambda}\boldsymbol{\Gamma}(s-1) & \boldsymbol{R}^{-T}(s-1)/\sqrt{\lambda} \\ \underline{\boldsymbol{e_p}}^T(s) & \boldsymbol{e}(s) & \boldsymbol{0}^T \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} \boldsymbol{R}(s) & \boldsymbol{\Gamma}(s) & \boldsymbol{R}^{-T}(s) \\ \boldsymbol{0}^T(s) & \tilde{f}(s) & \tilde{\boldsymbol{g}}^T(s) \end{bmatrix} \\ \tilde{\gamma}(s) &= \prod_{i=1}^{N} cos\theta_i(s) \\ f(s) &= \tilde{f}(s)\tilde{\gamma}(s) \\ \boldsymbol{h}(s) &= \boldsymbol{h}(s-1) - \tilde{\boldsymbol{g}}(s)\tilde{f}(s). \end{aligned}$$

Here $\underline{\boldsymbol{e_p}} = [\boldsymbol{e}_1(s), ..., \boldsymbol{e}_N(s)]^T$ denotes the $N \times 1$ data vector where $\boldsymbol{e}(s-i)$ are represented as $\boldsymbol{e}_i(s)$. The factor λ is an exponential forgetting factor which should be chosen in $[0, 1]$ interval.

4. SIMULATION RESULTS

To demonstrate the performance of our error diffusion algorithm, we carried out simulations with several images among which a representative one is presented here.[14] We compare the new method with Floyd-Steinberg's method, and the adaptive error diffusion with LMS algorithm both with normal raster and diagonal raster of the image. The results for the Peppers image is shown in Figure 7 in grayscale.* The image error diffused by Floyd-Steinberg's method in (b) contains color impulses, and the edges are smeared to each other. These artifacts, color impulses and false edges, are reduced in (c) and (d) which are obtained by the LMS-based error diffusion method. The image in (d) is obtained by diagonal processing which shows some improvement when compared to (c). However, in these images there are still false edges in slowly varying color regions. Images in (e) and (f) correspond to the vector adaptive error diffusion with QR-RLS algorithm. The resulting images with QR-RLS based algorithm are sharper and brighter giving the highest quality output. In (f) diagonal processing is used. It is observed that this method shows the best performance since the color impulses and false contours are greatly eliminated.

It is pointed out in[16] that whiter the quantization error power spectrum better the quality of the displayed image. In Figure 6, the spectrum of the quantization errors in a horizontal line of the Peppers image is shown. As can be observed from these plots, the power spectrum of the image error diffused by the QR-RLS type adaptive method has not only the lowest energy but also the flattest response whereas the error diffusion with Floyd-Steinberg has the highest energy. The LMS based method lies between the two curves. This experiment verifies the fact that QR-RLS based method produces the best results. Similar results are obtained for other lines of the image.

*The color image outputs can be viewed at.[15]

5. CONCLUSIONS

We proposed a new adaptive error diffusion method for color images. We used QR-RLS adaptive algorithm to update the error diffusion filter coefficients in the minimization of the output error so that it is least noticeable to the human eye. We exploited relative insensitivity of the HVS to diagonal orientations, and scanned the image diagonally. Our simulation studies show that the new adaptive error diffusion algorithm outperforms the deterministic and LMS type error diffusion algorithms. Also, diagonal scanning of the image produces better performance in removing the false contours.

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List of Figures

Biographies:

Gozde Bozkurt Unal received the B.Sc. degree in electrical engineering from Middle East Technical University, Ankara, Turkey, in 1996, and the M.Sc. degree in electrical engineering from the Bilkent University, Ankara, Turkey, in 1998. She is currently a research assistant in the Electrical and Computer Engineering Department at NCSU, Raleigh, pursuing her Ph.D. degree.

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Prof. Cetin is a member of the DSP technical committee of the IEEE Circuits and Systems Society. He founded the Turkish Chapter of the IEEE Signal Processing Society in 1991. He received the young scientist award of TUBITAK (Turkish Scientific and Technical Research Council) in 1993.

He is the chair of the IEEE-EURASIP Nonlinear Signal and Image Processing Workshop (NSIP'99) which will be held in June 1999 in Antalya, Turkey.

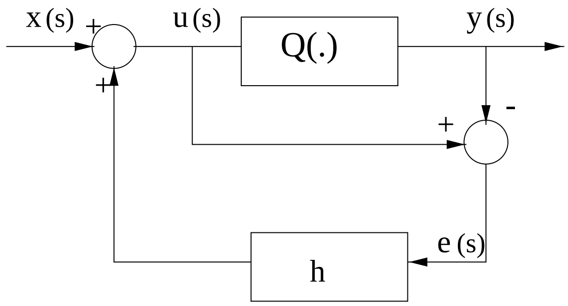

Figure 1. Block diagram of error diffusion method

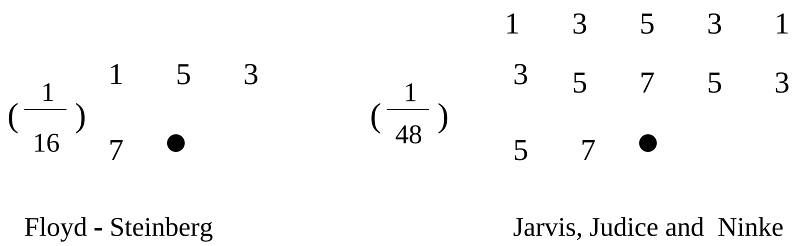

Figure 2. Error Diffusion Filter Masks

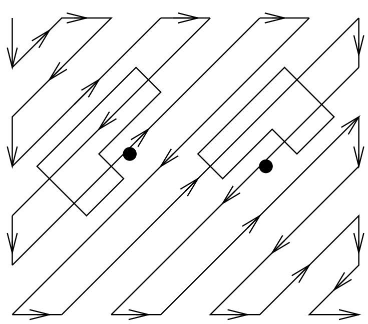

Figure 3. Diagonal Scanning: dots correspond to the current pixel, and the L-shaped window contains the previous pixels.

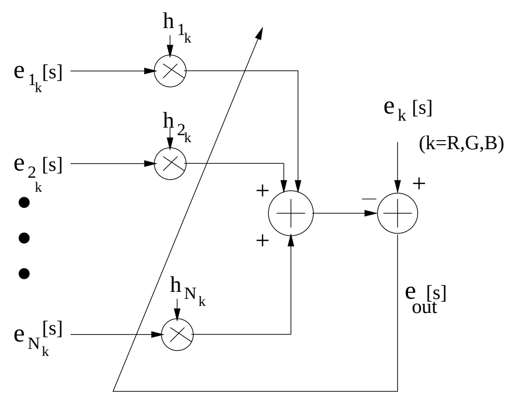

Figure 4. Linear Combiner, Scalar Implementation

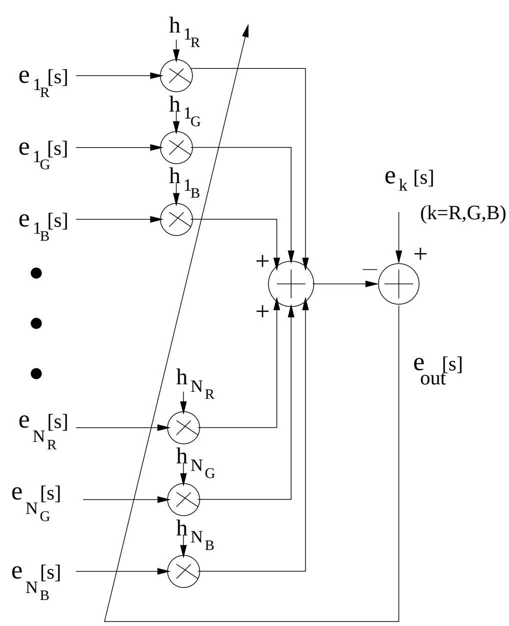

Figure 5. Linear Combiner, Vector Implementation

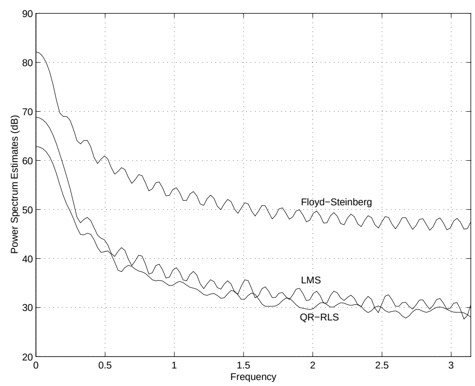

Figure 6. Comparison of the error spectra

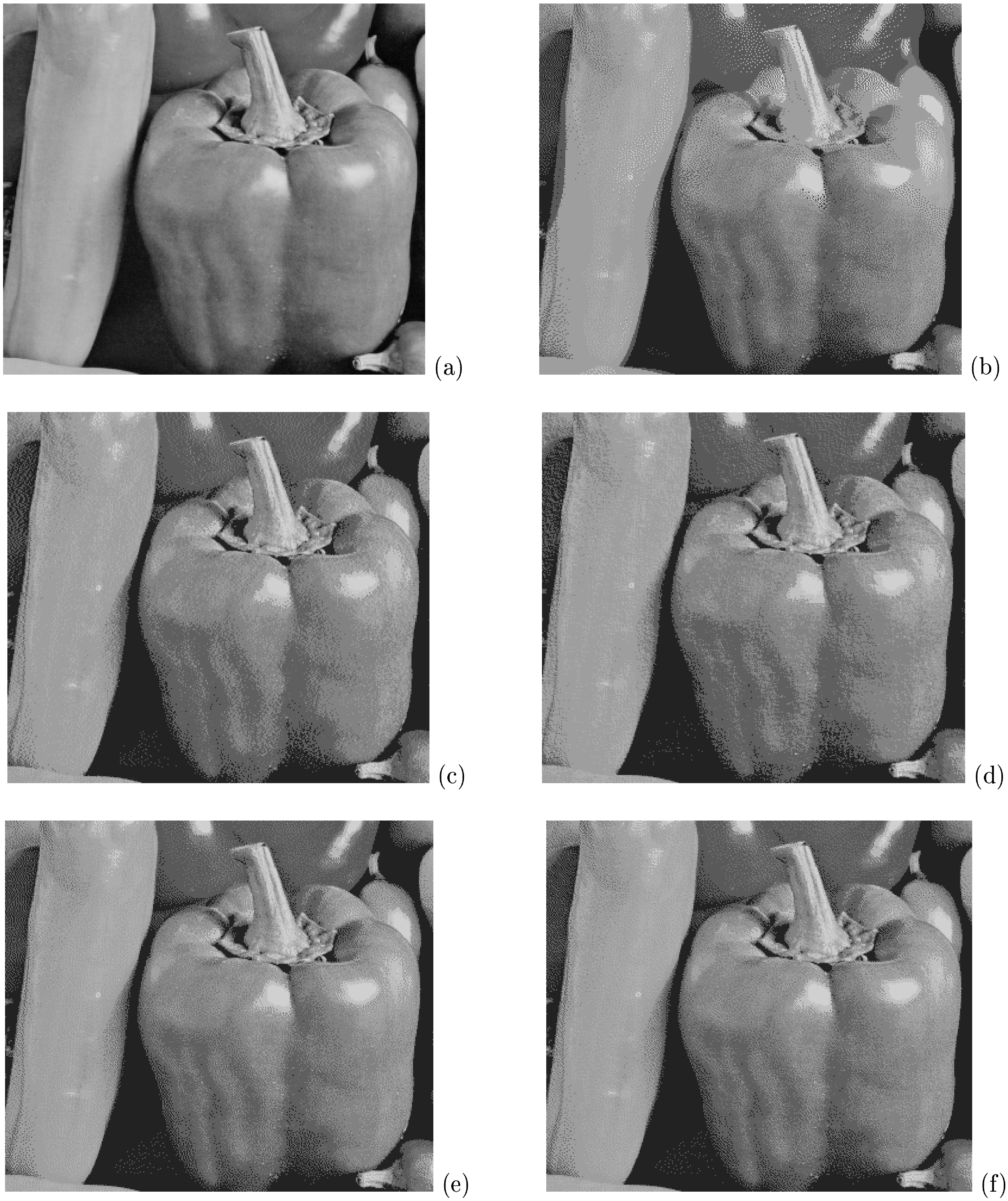

Figure 7. (a) Original Image, (b) Standard Error Diffusion, (c) Error Diffusion with LMS (raster scan), (d) Error diffusion with LMS (diagonal scan), (e) Error Diffusion with QR-RLS (raster scan), (f) Error Diffusion with QR-RLS (diagonal scan)