

## Chapter 5 : FILTERS

Parallel and series tuned circuits filter electronic signals. Their filtering performance is only determined by the quality factor of the circuit. Quite often it is necessary to have filters with improved performance compared to what tuned circuits can offer.

The theory underlying design of electronic filters is formidable. We confine our discussion of filters to the extent necessary to implement the filters used in TRC-10, in this chapter.

### 5.1. Motivation

Any electronic filter can be visualized as a block between a source (input) and a load (output). This is depicted in Figure 5.1. What is expected from this is to maintain the signal components at *wanted* frequencies, and eliminate the ones at *unwanted* frequencies, as much as possible.

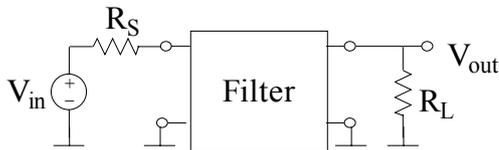


Figure 5.1 Electronic filter

Consider for example the RC LPF given in Figure 5.2(a), which is similar to the ones we used in Chapter 2.

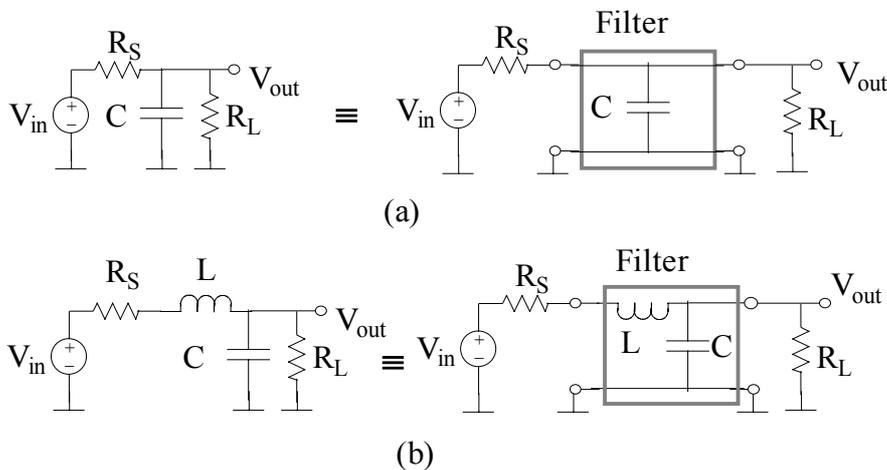


Figure 5.2 LPF (a ) one element, (b) two elements

The 3-dB cut-off frequency of such a filter is,

$$\omega_c = 1/C(R_S // R_L)$$

and its transfer function is

$$H_1(\omega) = [R_L / (R_S + R_L)] / (1 + j\omega / \omega_c).$$

We can get a similar filtering effect, if we use a series inductance instead of a parallel capacitance. If we use both a series inductance and a parallel capacitance in the filter block, we expect a better filtering function. Indeed the filter in Figure 5.2(b) has a transfer function

$$H_2(\omega) = 1 / [(1+R_S/R_L) - \omega^2 CL + j\omega(L/R_L + C R_S)].$$

This expression looks like a tuned circuit transfer function and it is not easy to immediately recognize it as a LPF. However for the special case of  $R_S = R_L$ , and with a choice of  $C$  and  $L$  to yield *maximally flat* behavior in the pass-band (we discuss this later in this chapter), i.e.  $\omega_c CR_S = \sqrt{2}$  and  $\omega_c L/R_S = \sqrt{2}$ ,  $H_2(\omega)$  becomes

$$H_2(\omega) = 1 / \{2[1 - (\omega/\omega_c)^2 + j\sqrt{2} \omega/\omega_c]\},$$

where  $\omega_c$  is  $\sqrt{2/LC}$ , for this circuit. Under the same condition  $R_S = R_L$ ,  $H_1(\omega)$  is

$$H_1(\omega) = 1 / [2(1 + j\omega/\omega_c)].$$

The magnitudes of both transfer functions are plotted with respect to frequency in Figure 5.3.

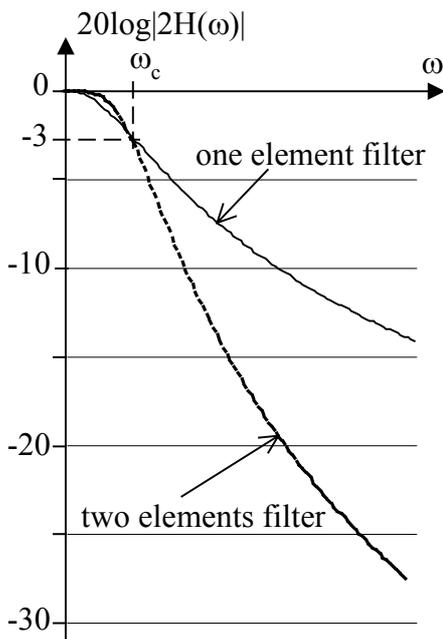


Figure 5.3 Transfer function of two LPFs, (a) one element, (b) two elements

Note that the filter with two elements is superior to the single element one in two respects:

1. Suppression of signal components at frequencies higher than  $\omega_c$  is significantly improved;
2. The signal components with frequencies less than  $\omega_c$  are better preserved, or less distorted.

How many filter elements must be used? Which kind of elements must be used? Or, what must be the values of the elements?

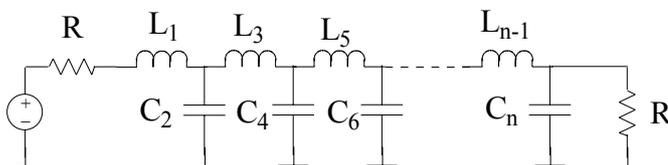
Modern filter theory addresses these questions by a systematic filter design technique.

## 5.2. Polynomial filters

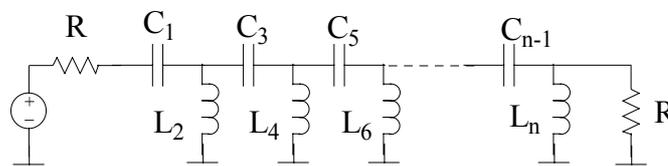
Modern filter theory maps the desired transfer function of a filter to the properties of a class of polynomials like Chebyshev polynomials, elliptic polynomials, etc. We use maximally flat, or *Butterworth filters* in TRC-10, which are also polynomial filters.

The basic building blocks in polynomial filters are LPF and HPF. BPF and band-stop filters are derived from these blocks.

The circuit morphology on which polynomial filters are based is of *ladder* type, as shown in Figure 5.4.



(a)



(b)

Figure 5.4 Ladder filters (a) LPF, (b) HPF

The parallel elements in the LPF configuration of Figure 5.4(a) are capacitors, and series elements are inductors. At low frequencies, inductors provide a low impedance path to the input signal to the output, while capacitors maintain high impedance to ground, and hence low loss. At high frequencies on the other hand capacitor impedance is low, and therefore there is a loss in the signal at every node. Inductors, on the other hand, have high impedance, and the division effect at each node on the signal is increased. In the HPF ladder in Figure 5.4(b), series and parallel elements are interchanged compared to LPF, thus yielding exactly an opposite function.

### 5.2.1. Butterworth filters

Butterworth filter seeks to have a power transfer function of

$$P_L/P_A = 1 / [1+(\omega/\omega_c)^{2n}]$$

where  $\omega_c$  is the 3-dB cut-off frequency,  $n$  is the number of elements (capacitors and inductors),  $P_L$  is the power delivered to load and  $P_A$  is the available power at the source.  $P_L/P_A$  is plotted versus frequency for different number of elements,  $n$ , in Figure 5.5. As  $n$  increases the power transfer function approaches to an ideal LPF.

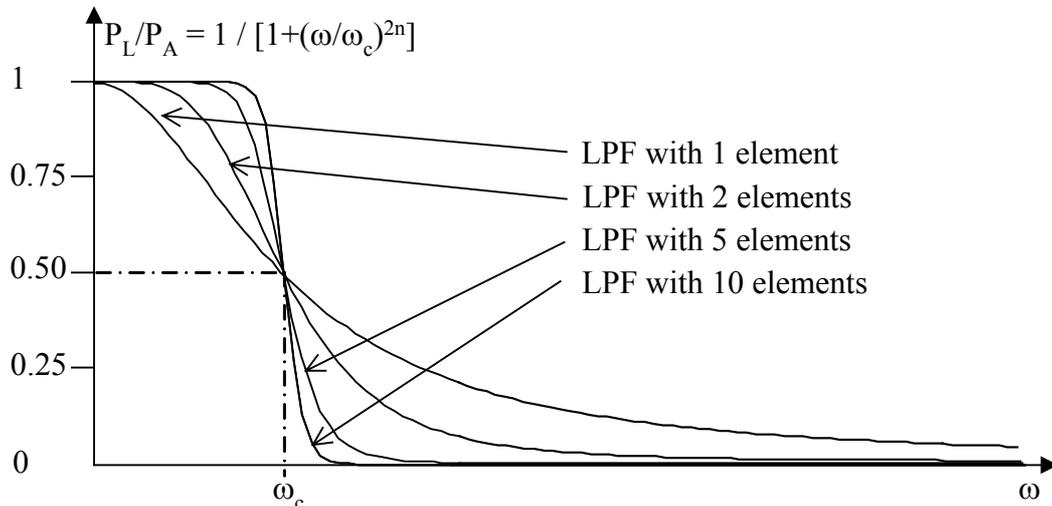


Figure 5.5 Butterworth LPF power transfer function

The number of elements in a filter can be odd or even and the ladder filter can start with a series or a parallel component. Usually configurations with fewer inductors are preferred, for constructional simplicity.

The component values for Butterworth filters, *normalized* with respect to the termination impedance and cut-off frequency, are provided in tables. Normalized coefficients are correct reactance and susceptance values in  $\Omega$ , for  $1 \Omega$  source and load resistance and for a cut-off frequency of  $\omega_c = 1$  rad/sec. A table of these coefficients, up to eight elements, is given in Table 5.1.

Using this table for designing a filter is straightforward. Since the coefficients are normalized component values, we must de-normalize them for given termination resistance and cut-off frequency.

$n$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
1	2.0000							
2	1.4142	1.4142						
3	1.0000	2.0000	1.0000					
4	0.7654	1.8478	1.8478	0.7654				
5	0.6180	1.6180	2.0000	1.6180	0.6180			
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176		
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902

Table 5.1 Normalized Butterworth filter coefficients

For example, consider a LPF with three elements depicted in Figure 5.6(a). The filter has two parallel capacitors and one series inductor. The source and load resistances of this filter are  $50\ \Omega$ . Assume that we want to have a cut-off frequency of 20 MHz.

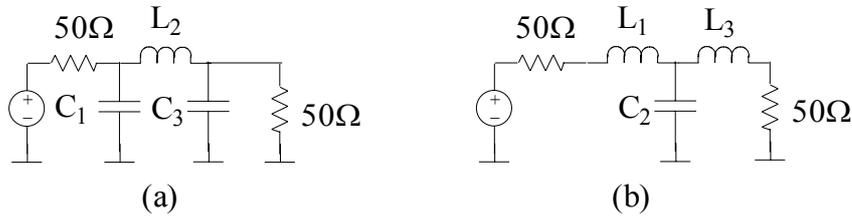


Figure 5.6 Three element LPFs

The filter coefficients are  $b_1=1$ ,  $b_2=2$  and  $b_3=1$  from Table 5.1 and  $\omega_c$  is  $1.257\ \text{E}8\ \text{rad/sec}$ , or 20 MHz. The first element is a parallel element and its susceptance is  $B_1 = \omega_c C_1$ . The normalized value of  $B_1$ ,  $B_1/(1/50\ \Omega)$ , is equal to the first coefficient  $b_1$ . Hence

$$\begin{aligned} B_1 &= b_1/50\ \Omega \\ &= 20\ \text{mS}, \end{aligned}$$

and the capacitance  $C_1$  becomes

$$\begin{aligned} C_1 &= B_1/\omega_c \\ &= 159\ \text{E-12}\ \text{F} \\ &\text{or } 159\ \text{pF}. \end{aligned}$$

The second element is a series element and therefore its normalized reactance  $X_2 = \omega_c L_2$ , is related to the second coefficient as

$$X_2/50\ \Omega = b_2.$$

Hence  $X_2$  becomes

$$X_2 = 100\ \Omega$$

and  $L_2$  at 20 MHz becomes

$$L_2 = X_2/\omega_c = 796\ \text{nH}.$$

The last element is again a parallel capacitance with a normalized value equal to that of  $C_1$ , hence  $C_3 = C_1 = 159\ \text{pF}$ . With these component values, this filter becomes a *third order Butterworth LPF* with a 3-dB cut-off at 20 MHz.

We can have the same characteristics with the filter shown in Figure 5.6(b), with same normalized values. In this configuration, the first component is a series element. In this case  $X_1$  is  $\omega_c L_1$  and

$$X_1/50\ \Omega = b_1 = 1,$$

yielding

$$L_1 = X_1/\omega_c = 398 \text{ nH.}$$

Again, since  $b_3=b_1=1$ ,  $L_3=L_1=398 \text{ nH}$ . The second element is a parallel capacitance. Normalized susceptance  $B_2$ ,  $B_2/(1/50\Omega)$ , is equal to  $b_2$  and  $C_2$  is

$$\begin{aligned} C_2 &= B_2/\omega_c = 2(20 \text{ mS})/(1.257 \text{ E8 rad/sec}) \\ &= 318 \text{ pF.} \end{aligned}$$

### Generalization

Assume that resistances equal to  $R_o$  terminates the ladder filter on both sides. Let  $G_o=1/R_o$ . The algorithm of filter design using tables is as follows:

- If the  $i^{\text{th}}$  element in the ladder is a *series* element, then its normalized absolute reactance is equal to the  $i^{\text{th}}$  coefficient in the table, i.e.  $|X_i|/R_o = b_i$ .
  - ◆ If the *series* element is an *inductance*, then  $L_i = X_i/\omega_c = b_i R_o/\omega_c$ ;
  - ◆ If the *series* element is a *capacitance*, then  $C_i = 1/\omega_c |X_i| = 1/\omega_c b_i R_o$ ;
- If the  $i^{\text{th}}$  element in the ladder is a *parallel* element, then its normalized absolute susceptance is equal to the  $i^{\text{th}}$  coefficient in the table, i.e.  $|B_i|/G_o = b_i$ , or  $|B_i| = b_i/R_o$ ,
  - ◆ If the *parallel* element is a *capacitance*, then  $C_i = B_i/\omega_c = b_i/\omega_c R_o$ ;
  - ◆ If the *parallel* element is an *inductance*, then  $L_i = 1/\omega_c |B_i| = R_o/\omega_c b_i$ .

The coefficients for Butterworth filters up to 8 elements are given in Table 5.1. The coefficients for filters with more elements can be obtained from the following relation:

$$b_i = 2\sin[(2i-1)\pi/2n]$$

where  $i$  is the position of the element and  $n$  is the total number of elements in the filter, i.e.  $i = 1, 2, \dots, n$ .

#### 5.2.2. Harmonic filter

In case of TRC-10, we have a TX output amplifier which is expected to deliver about 10 mW to the antenna, at 29 MHz. Although the amplifier is a linear amplifier, there is distortion on the RF signal when the OPAMP is driven at its limits. Distortion means that there are signal components clustered around harmonics of 30 MHz.

Radio-communication regulations are very specific about such harmonic emission. Regulations specify the limitation above 30 MHz as follows:

*“In transmitters with 25 W or less mean output power, spurious emissions must be at least 40 dB below the mean power of fundamental emission and never greater than 25  $\mu$ W, but need not be reduced further than 10  $\mu$ W.”*

Signals produced by transmitters at frequencies other than the transmission frequency are called *spurious emissions* (like harmonics of the carrier), which must be suppressed. 30 dB below 10 mW is 10  $\mu$ W. The harmonic emissions of TRC-10 must not exceed 10  $\mu$ W.

The TX amplifier is a linear amplifier and the carrier waveform at the output is predominantly sinusoidal. The data sheets of LM7171 reveal that we must be able to get an output signal with about 3% distortion at full power. If we allow a safe margin by assuming 10% distortion, then the spurious signals are at least 20 dB below the mean output power to start with. Part of this spurious power (actually a small part in TRC-10) is at second harmonic, at about 58 MHz. If a filter to suppresses the second harmonic by about 10 dB, the higher harmonics will be suppressed more. To be on the safe side, let us design a filter, which suppresses the second harmonic by 20 dB.

Any design starts with a requirement specification. The harmonic filter specification for TRC-10 can be derived from above discussion. We need a LPF just before the antenna, which must have 20 dB attenuation at 58 MHz, it must not attenuate 29.7 MHz at any significant level (for example at most 1 dB) and it must have a minimum number of elements.

Let us design a Butterworth filter and write the first two requirements quantitatively to see what we have for third requirement:

$$10\log(P_L/P_A)|_{f=29.7\text{ MHz}} = -10\log[1+(29.7E6/f_c)^{2n}] = -1\text{ dB}$$

$$\Rightarrow (29.7E6/f_c)^{2n} = 0.259$$

$$\Rightarrow n(7.473)-n \log f_c = -0.293$$

and

$$10\log(P_L/P_A)|_{f=58\text{ MHz}} = -20\text{ dB} \quad \Rightarrow n(7.763)-n \log f_c = 0.998.$$

Solving the two equations simultaneously for n and  $f_c$ , we obtain n = 4.45 and  $f_c = 34.6$  MHz.

Number of elements in the filter must be larger than 4. We take n = 5. In order to keep the number of inductors low, we shall choose the ladder configuration with three parallel capacitors and two series inductors as in Figure 5.7.

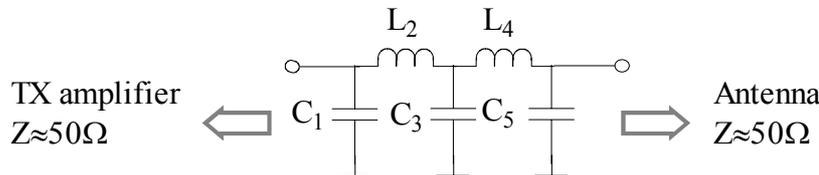


Figure 5.7 Harmonic filter

The filter coefficients are  $b_1 = b_5 = 0.618$ ,  $b_2 = b_4 = 1.618$  and  $b_3 = 2$ , from Table 5.1. Then,

$$C_1 = C_5 = 0.618/\omega_c(50\ \Omega)$$

and

$$C_3 = 2/\omega_c(50 \Omega).$$

At 34.6 MHz,  $C_1$  is 56.9 pF. The capacitor values must be standard values. We must choose the cut-off frequency to yield standard values. Hence we choose  $C_1 = C_5 = 56$  pF.

$C_3$  becomes 181.2 pF  $\Rightarrow C_3 = 180$  pF. With these capacitor values  $f_c$  becomes approximately 35.2 MHz. We cannot be very precise about matching the capacitor values, the cut-off frequency or the termination resistance. The capacitors have tolerances, which exceed this mismatch easily.

The inductors are,

$$L_2 = L_4 = 1.618 (50 \Omega)/\omega_c \approx 370 \text{ nH}.$$

Let us check the attenuation at few frequencies before we finish the design:

$$10\log(P_L/P_A)|_{f=29.7 \text{ MHz}} = -0.72 \text{ dB},$$

$$10\log(P_L/P_A)|_{f=29 \text{ MHz}} = -0.58 \text{ dB},$$

$$10\log(P_L/P_A)|_{f=58 \text{ MHz}} = -21.7 \text{ dB},$$

$$10\log(P_L/P_A)|_{f=87 \text{ MHz}} = -39.3 \text{ dB}.$$

To calculate attenuation at harmonics, we can use the approximation,

$$10\log(P_L/P_A) = -10\log[1+(\omega/\omega_c)^{2n}] \approx -10\log(\omega/\omega_c)^{2n} = -20n \log(\omega/\omega_c).$$

The attenuation provided by the filter is  $-20n$  dB per decade increase of frequency.

We use the filter designed above in TRC-10 as output harmonic filter. The components of this filter are C77, C78, C79, L6 and L7.

### 5.2.3. Butterworth HPF

HPF ladder uses inductors as parallel elements and capacitors as series elements. The power transfer function for Butterworth HPF is

$$P_L/P_A = 1 - 1 / [1+(\omega/\omega_c)^{2n}] = (\omega/\omega_c)^{2n} / [1+(\omega/\omega_c)^{2n}].$$

Consider the fourth order filter given in Figure 5.8. The normalized values are  $b_1 = b_4 = 0.7654$  and  $b_2 = b_3 = 1.8478$ .

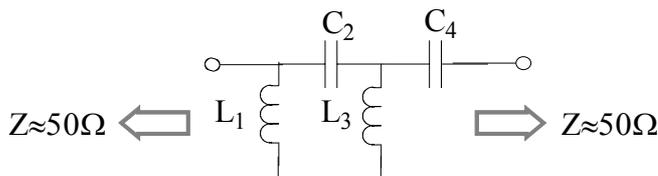


Figure 5.8 4-element HPF

If for example  $f_c$  is 20 MHz, the component values of the circuit in Figure 5.8 are

$$\begin{aligned} L_1 &= (50 \Omega)/\omega_c b_1 = 520 \text{ nH}, \\ C_2 &= 1/\omega_c b_2(50 \Omega) = 86 \text{ pF}, \\ L_3 &= (50 \Omega)/\omega_c b_3 = 215 \text{ nH}, \\ C_4 &= 1/\omega_c b_4(50 \Omega) = 208 \text{ pF}. \end{aligned}$$

Note that we could have chosen the 4-element ladder starting with a series capacitor instead of a parallel inductor. In that case the first element value is equal to  $C_4$  above and the resulting filter is the same the one in Figure 5.8 flipped horizontally. This shows that we can interchange the input and output of these filters. This property is called *reciprocity*.

#### 5.2.4. Butterworth BPF

BPF design starts with the design of a LPF with a 3 dB frequency,  $\omega_c$ , equal to the 3 dB bandwidth,  $\Delta\omega$ , of the BPF. Once LPF is designed, every series element (and every parallel element) is series (parallel) tuned out at an appropriate frequency, to obtain the BPF. In other words, a parallel resonant LC circuit replaces every parallel element and a series resonant LC circuit replaces every series element. The center frequency,  $\omega_{\text{center}}$ , of a BPF is placed at the arithmetic mean of the 3 dB frequencies,  $\omega_1$  and  $\omega_2$ , such that

$$\omega_{1,2} = \omega_{\text{center}} \pm \Delta\omega/2.$$

The resonance frequency  $\omega_0$  is, however, the geometrical mean of the two 3 dB frequencies. Design specifications of a BPF usually include a center frequency and BW.  $\omega_1$  and  $\omega_2$  are calculated first, and then the resonance frequency  $\omega_0$  is determined.

Conversion from LPF to BPF is depicted in Figure 5.9, where a BPF, with 3 dB BW of  $\Delta\omega = \omega_2 - \omega_1$  and a center frequency of  $(\omega_1 + \omega_2)/2$ , is derived from a LPF. In this filter, the resonance frequency is

$$\begin{aligned} \omega_0 &= \sqrt{\omega_1 \omega_2} \\ &= (L_1 C_1)^{-1/2} = (L_2 C_2)^{-1/2}. \end{aligned}$$

The out-of-band suppression of this filter follows the suppression properties of the LPF equivalent. In other words, if the low pass equivalent is of  $n^{\text{th}}$  order, the power transfer function of the BPF is,

$$P_L/P_A = 1 / \{1 + [(\omega - \omega_0)/\omega_c]^{2n}\},$$

or

$$P_L/P_A = 1 / \{1 + [(\omega - \omega_0)/\Delta\omega]^{2n}\}.$$

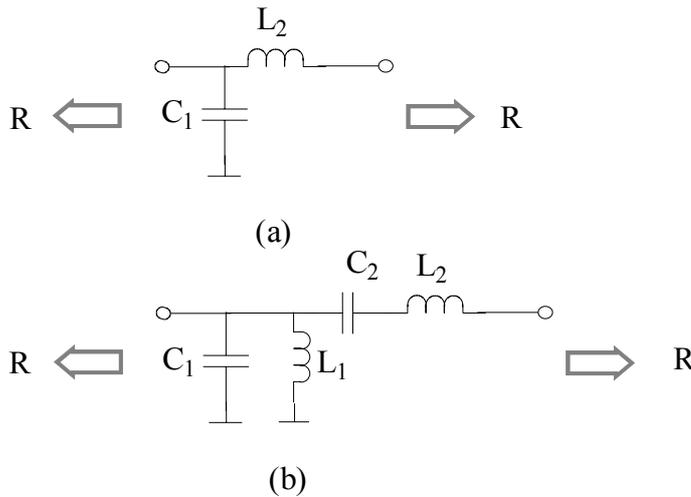


Figure 5.9 (a) LPF with  $\omega_c = \Delta\omega$ , (b) BPF obtained by tuning out  $C_1$  with  $L_1 = 1/\omega_0 C_1$  and  $L_2$  with  $C_2 = 1/\omega_0 L_2$

The wanted signal at the output of RX mixer is an AM signal with center frequency of 16 MHz and a BW of 6 KHz. This signal must be filtered out in order to avoid other products of the mixer to reach IF amplifiers and the detector. We need a 6 KHz BW BPF there. The frequency selectivity of this filter is very high, hence the center frequency and the resonance frequency can be considered to be equal (i.e. the arithmetic mean and the geometrical mean of  $\omega_1$  and  $\omega_2$  are almost equal).

The output resistance of SA602A (or SA612A) is 1.5 K. If we take R as 1.5 K, then  $C_1$  and  $L_2$  becomes

$$C_1 = b_1/\omega_c R = (1.4142)/[(2\pi \times 6E3)(1.5E3)] = 25 \text{ nF}$$

$$L_2 = b_2 R/\omega_c = [(1.4142)(1.5E3)]/[(2\pi \times 6E3)] = 56.3 \text{ mH.}$$

These are the values of the LC LPF with 6 KHz BW. The component values to tune these out at 16 MHz are

$$L_1 = 1/\omega_0^2 C_1 = 4 \text{ nH}$$

$$C_2 = 1/\omega_0^2 L_2 = 0.0018 \text{ pF.}$$

A BPF with above component values has the desired BW at 16 MHz, but such a filter cannot be realized. The required value of  $L_2$  is too high for a RF inductor to successfully implement at 16 MHz and values of  $L_1$  and  $C_2$  are too low.

We may expect an improvement if we decrease the termination resistance. We discuss methods of impedance transformation in the next section. Assume that output impedance of mixer IC is transformed to a low standard value like 50  $\Omega$ . The component values of BPF become

$$C_1 = 750 \text{ nF}$$

$$L_2 = 1.9 \text{ mH}$$

$$L_1 = 0.133 \text{ nH}$$

$$C_2 = 0.053 \text{ pF.}$$

All of the component values are out of range for good circuit design at 16 MHz, now (remember the parasitic lead inductance for capacitors; it is very effective on 750 nF at 16 MHz). Unrealistic component values are the result of very stringent requirement on the Q of the BPF. We require 0.04 % BW. Such filter problems are addressed by using special devices in electronic design. We use a *crystal ladder filter* in TRC-10 as IF filter, which we discuss later in this chapter.

### 5.3. Impedance matching

We often face the problem of converting impedance to another level, without any power loss or at least minimum power loss. One such case is the filtering problem we discussed in the previous section. Another one, and the most common one, is to have maximum power transfer to the load from a source. Impedance transformation is a fundamental topic in electronics and there is a wealth of information on it. The techniques that we use in TRC-10 are discussed below.

#### 5.3.1. Transformers

Transformers are most common impedance transformation devices. The ideal transformer of Section 2.6 is depicted in Figure 5.10 again, using phasor notation.

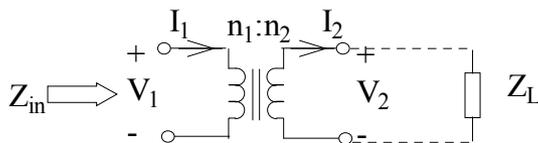


Figure 5.10 Ideal transformer

The primary and secondary voltage and current relations in an ideal transformer are

$$V_2/V_1 = n_2/n_1$$

and

$$I_2/I_1 = n_1/n_2,$$

where  $n_2/n_1$  is the ratio of number of turns in the secondary winding to the number of turns in the primary, which is commonly called the *transformer turns ratio*.

If a load impedance is connected across the secondary terminals, as shown in Figure 5.10, secondary terminal voltage and current must satisfy the relation

$$V_2/I_2 = Z_L.$$

On the primary side, we have

$$Z_{in} = V_1/I_1.$$

Substituting  $V_2$  and  $I_2$  we obtain,

$$Z_{in} = V_1/I_1 = (n_1/n_2)V_2/(n_2/n_1)I_2$$

$$= (n_1/n_2)^2 V_2/I_2$$

$$= (n_1/n_2)^2 Z_L.$$

The load impedance appears as  $(n_1/n_2)^2 Z_L$  across the primary terminals. By choosing appropriate turns ratio, we can convert  $Z_L$  virtually to any value.

### 5.3.2. Real transformers

Transformers are two (or more) coils wound on the same core. A transformer is shown in Figure 5.11.

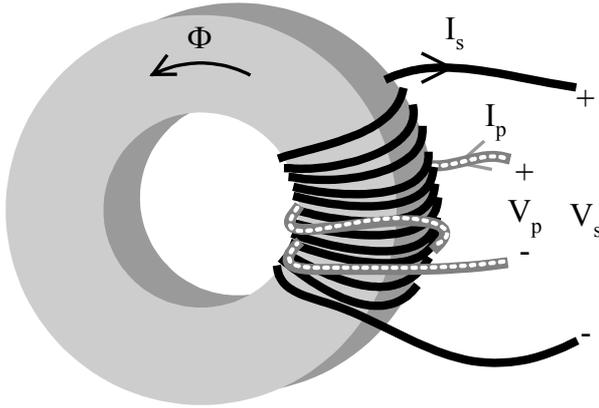


Figure 5.11 Real transformer

The flux generated in the core by two currents  $I_p$  and  $I_s$  is

$$\Phi = n_p A_L I_p - n_s A_L I_s$$

where  $n_p$  and  $n_s$  are the number of turns in primary and secondary windings respectively,  $A_L$  is the inductance constant of the core and  $I_p$  and  $I_s$  are primary and secondary currents, respectively. Both windings are wound in the same sense. If the currents in both windings have the same direction, they generate a flux in the same direction. By convention, we show the *primary current flowing into the transformer* and the *secondary current out of the transformer*. Therefore the total flux generated by two currents is the difference between primary and secondary. Using this equation, we can write  $I_p$  as

$$I_p = (n_s/n_p) I_s + \Phi/n_p A_L.$$

The first term is the relation between the primary and secondary currents in ideal transformer and is called the *transformer current*. The second term is called the *magnetizing current* and it is related to the finite value of the primary inductance (note that this inductance is implicitly assumed infinite in ideal transformer).

The same flux induces voltages across the primary and secondary windings. The induced primary and secondary voltages, on the other hand, are

$$V_p = j\omega n_p \Phi$$

and

$$V_s = j\omega n_s \Phi,$$

as in the case of inductance. Using these relations, we can relate two voltages as

$$V_p/V_s = n_p/n_s.$$

Also substituting  $\Phi = V_p/j\omega n_p$  into expression for  $I_p$ , we obtain

$$I_p = (n_s/n_p)I_s + V_p/j\omega n_p^2 A_L.$$

$n_p^2 A_L$  is the inductance of the primary winding,  $L_p$ . Hence we can write  $I_p$  as

$$I_p = (n_s/n_p)I_s + V_p/j\omega L_p.$$

The two equations relating the primary and secondary terminal voltages and currents can be shown in form of an equivalent circuit comprising an ideal transformer and a parallel inductance as in Figure 5.12. Note that in this model, if the primary inductance is indefinitely large, then its susceptance is zero and it can be removed from the equivalent circuit. The value of  $L_p$  depends on number of turns and the core, and can be calculated as discussed in Chapter 4.

There are two approaches to design RF transformers. If the requirements are such that we need the functions of a transformer in a relatively small bandwidth compared to a center frequency, we design *narrow-band* or *resonant transformers*. The idea is simple: if we tune out  $L_p$  by a parallel capacitor across the primary terminals (or equivalently the secondary terminals), we end up with only an ideal transformer left in the circuit. This is suitable only over a frequency range limited to the  $Q$  of the parallel tuned circuit. In this case the value of  $L_p$  need not be very large, but is chosen to provide the necessary bandwidth.

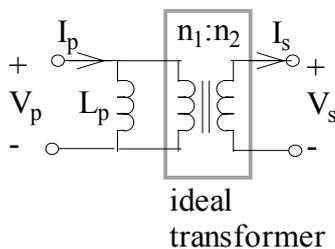


Figure 5.12 Equivalent circuit of a lossless real transformer

Second approach is *wide-band transformer* design. In this case we require that  $|j\omega L_p|$  is significantly larger than the effective impedance that appears across it over the frequency range of interest. Since the transformers are usually employed for converting and matching resistances,  $L_p$  is usually chosen such that its reactance at the lower end of the frequency band is more than *four* times the effective resistance across it. Both narrow-band and wide-band designs are given in Figure 5.13.

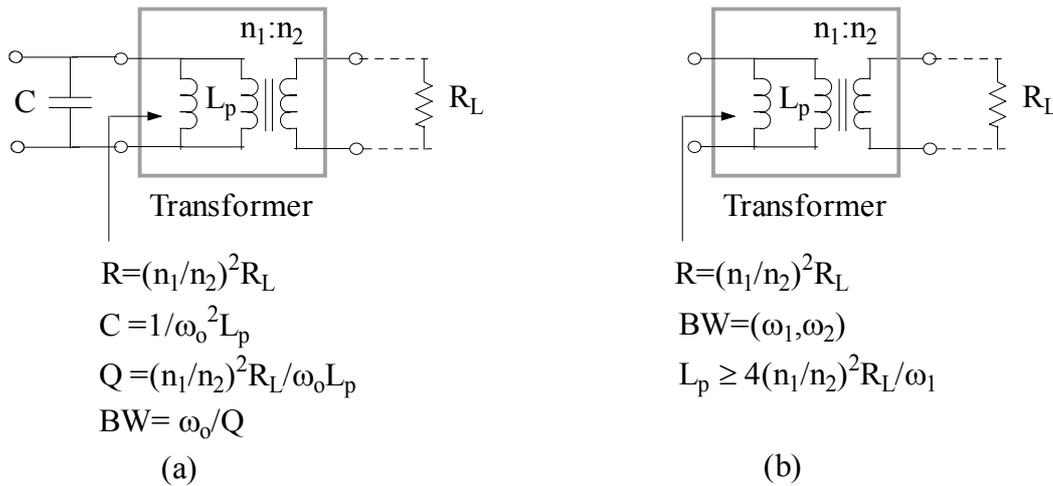


Figure 5.13 (a) Narrow-band transformer, (b) Wide-band transformer

There are two fundamental assumptions in the model of Figure 5.12:

1. There are no core or copper losses;
2. All the flux is confined within the core such that it intersects both coils.

We discussed how losses are handled in inductors in Section 4.4. We include losses related to  $L_p$  in the model similarly. Usually core losses are shown as a parallel equivalent resistance in transformer models and copper losses as series.

Whether the flux is confined in the core or not is unimportant in making inductors. In transformers, however, having a large portion of the flux shared by both coils is critical. Assuming that all the flux is confined in the core is the equivalent to assuming that the permeability of the core material is infinitely large. However, the cores that can be used at high end of HF are usually of moderate permeability materials. Hence as a coil generates a flux, a large part of it remains in the core but some flux escapes to the surrounding medium. We call this part of flux *leakage*. Leaked flux contributes to the inductance of the generating coil, but it does not contribute to the induced voltage on the other coil. The other coil does not intersect the leaked flux. The part of flux intersected by both coils is called *linkage*. It is due to linkage that the transformer action exists.

Although the equivalent circuit which models flux leakage is more involved, it is possible to reduce it to the equivalent circuit given in Figure 5.14.  $L_p$  is divided into two parts,  $k^2 L_p$  and  $(1-k^2)L_p$ , where  $k^2 L_p$  represents the linkage and  $(1-k^2)L_p$  represents the leakage. The parameter  $k$  is called the *coupling coefficient*. In a transformer with low leakage,  $k$  is close to 1. Note that the coupling coefficient modifies the transformation ratio to  $kn_1/n_2:1$ , which is lower compared to turns ratio  $n_1/n_2:1$ .

$R_s$  and  $R_p$  represent the copper and core losses in Figure 5.14, respectively. All the current that flow into the primary winding experiences the copper losses. Hence  $R_s$ , representing this loss, is in series with the entire circuit. The leakage flux is part of flux, which leaks to the environment and not maintained in the core. The core losses are mainly associated with the linkage, hence  $R_p$  is connected in parallel with the inductance, which represents linkage.

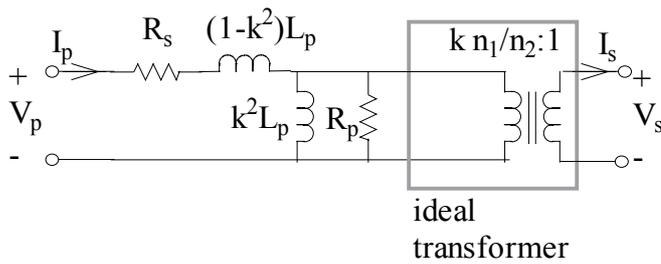


Figure 5.14 Equivalent circuit of a real transformer.

5.3.3. Matching by resonant circuits

The simplest way of narrow band impedance matching is using series RLC circuits. We discussed the equivalence of the series and parallel RLC circuits and amplification property of tuned circuits, in Section 4.3. Same property provides a means of matching. Figure 4.7 is repeated here as Figure 5.15, for convenience.

The resistance  $R_s$  in series with the inductance is transformed into a resistance  $(Q^2+1)R_s$  in parallel with the capacitance, as shown in Figure 5.15. There is a transformation ratio of  $Q^2+1$ . This transformation is valid within the 3 dB bandwidth of the tuned circuit. This impedance transforming LC circuit is called an *L-section*. The same circuit transforms a parallel resistance  $R_p$  into a series one with a value  $R_p/(Q^2+1)$ .

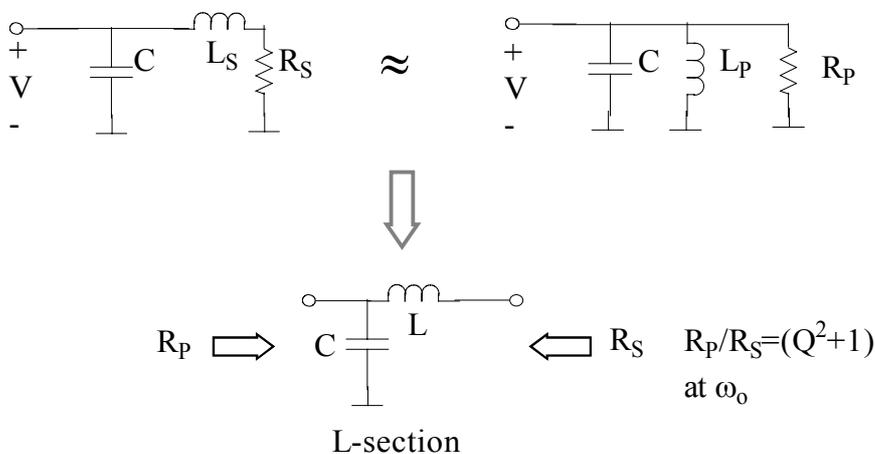


Figure 5.15 Narrow band impedance matching using L-section filters

5.3.4. Impedance inverters

Impedance inverters are narrow-band impedance transformers. An impedance inverter circuit is depicted in Figure 5.16. The circuit has a “T” form with two equal series reactances and a parallel reactance of same magnitude but opposite sign.

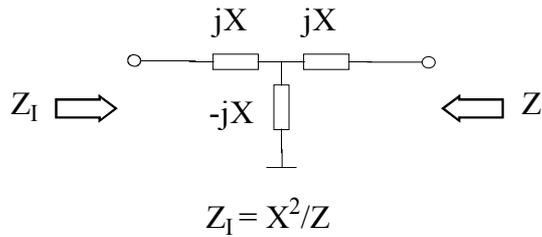


Figure 5.16 Impedance inverter

When an impedance  $Z$  is connected to one end of an inverter, the impedance seen at the other end becomes

$$Z_i = jX + 1/[-1/jX + 1/(jX + Z)] \\ = X^2/Z.$$

This useful circuit is used for many matching and filtering purposes. It can convert a series resonant circuit into a parallel resonant circuit. Assume that  $Z$  is the impedance of a series resonant RLC circuit, i.e.

$$Z = j\omega L + R + 1/j\omega C.$$

$Z_i$  becomes

$$Z_i = X^2/(j\omega L + R + 1/j\omega C),$$

and the admittance is

$$Y_i = 1/Z_i = (j\omega L + R + 1/j\omega C)/X^2 \\ = j\omega L/X^2 + R/X^2 + 1/j\omega CX^2.$$

This is the admittance of a parallel RLC circuit with a *capacitance* of value  $L/X^2$ , *conductance* of  $R/X^2$  and an *inductance* of  $CX^2$ .

#### 5.4. Crystal filters

Tuned circuits made of inductors and capacitors cannot meet very tight frequency selectivity requirements. The  $Q$  of such circuits is limited at about 100. Mechanical devices made of PZT ceramics and quartz crystals are used in such applications. PZT ceramics are *electrostrictive* and quartz crystals are *piezoelectric* materials, respectively. They can convert electric field applied on them into mechanical vibration within their body. Mechanical filter devices are usually made by evaporating electrodes on small plates of these materials. Applying a voltage across these electrodes produces an electric field in the material. Device converts this field into mechanical vibration at the same frequency of the applied voltage. The plate dimensions define various *modes of mechanical resonance*. When the frequency of the applied field matches to one of these resonance frequencies, the impedance that

appears across the electrodes makes a dip. The frequency of resonance is very stable and the losses are very small, limited to frictional activity during particle motion.

#### 5.4.1. Quartz crystals

Quartz crystals have the highest possible Q. A Q value of 100,000 is common in quartz crystals. The circuit symbol of a crystal (XTAL) is given in Figure 5.17(a). There are many modes of resonance in a XTAL and each mode has a fundamental resonance frequency and its *overtones*. We are interested in the fundamental resonance frequency of the only one mode of vibration. The equivalent circuit of the XTAL in the vicinity of this frequency is also given in Figure 5.17(b).

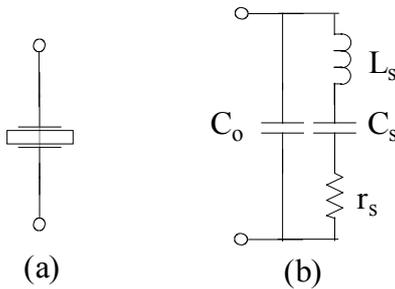


Figure 5.17 Quartz crystal (a) Circuit symbol and (b) Equivalent circuit

This equivalent circuit models the impedance at the (electrical) terminals of the XTAL. The mechanical properties of quartz and the dimensions of the plate, however, determine all three of the series circuit elements.  $L_s$  is proportional to the mass of the plate and  $C_s$  is determined by the *compliance* of quartz. Resistance  $r_s$  models the friction losses during vibration. The only inherently electrical component is  $C_o$ , which is the capacitance between the electrodes.

A *motional inductance*  $L_s$  of about 10 mH can be obtained easily with a small sized quartz plate.

The model in Figure 5.17(b) has two resonance frequencies:

$$f_s = 1/[2\pi(L_s C_s)^{1/2}]$$

and

$$f_p = 1/\{2\pi[L_s C_s C_o/(C_s + C_o)]^{1/2}\}.$$

$f_s$  is the resonance frequency of the series  $r_s L_s C_s$  branch and the impedance decreases down to  $r_s$  at this frequency (in parallel with  $j2\pi f_s C_o$ ).  $f_p$ , on the other hand, is the parallel resonance frequency, where the inductance  $L_s$  resonates with the series combination of  $C_s$  and  $C_o$ , i.e.,  $C_s C_o/(C_s + C_o)$ .

The motional element  $L_s$  is in the range of few mH to over 10 mH for XTALs at 16 MHz.  $C_s$  is in the order of fF (femto Farad,  $10^{-15}$  Farad) and  $r_s$  is in the range of few ohms to few tens of ohms.  $C_o$  is an electrical component, the *clamp capacitance*, and it is usually a few pF. We are interested in the frequencies in the vicinity of series

resonance. At  $f_s$ ,  $|j2\pi f_s C_o|$  is very small compared to  $1/r_s$ . Hence the total terminal impedance is  $Z_s \cong r_s$ . The only effect of  $C_o$  is to shift the series resonance slightly.

Quartz crystals are usually employed as they are supplied, i.e., with the highest possible  $Q$ . Timing circuits in watches, computers, or fixed frequency oscillators in communication equipment require very sharp filter characteristics at a precise resonant frequency.

We use quartz crystals for two different purposes in TRC-10. The 16 MHz clock module is a XTAL oscillator. It is an electronic circuit with an integrated quartz crystal in its case. We discuss this module in Chapter 7. We also use two quartz crystals to make a very narrow band IF filter with center frequency of 16 MHz.

#### 5.4.2. IF ladder filter

A 16 MHz XTAL has a typical bandwidth of about 100-200 Hz. We need 6 KHz BW at IF stage. A single XTAL can provide a limited attenuation of a single element filter (we view it as a BPF derived from a LP equivalent). We use two XTALs to make a Butterworth ladder filter.

Consider the circuit in Figure 5.18. Each XTAL provides a series resonance circuit. We need one series and one parallel resonant circuit, in order to employ techniques we use in designing Butterworth filters. We employ an impedance inverter to invert one of the series resonant circuits into a parallel resonant circuit, as in Section 5.3.4. We choose the external circuit parameters  $X$ ,  $R_1$  and  $R_2$  appropriately, to satisfy the bandwidth requirement.

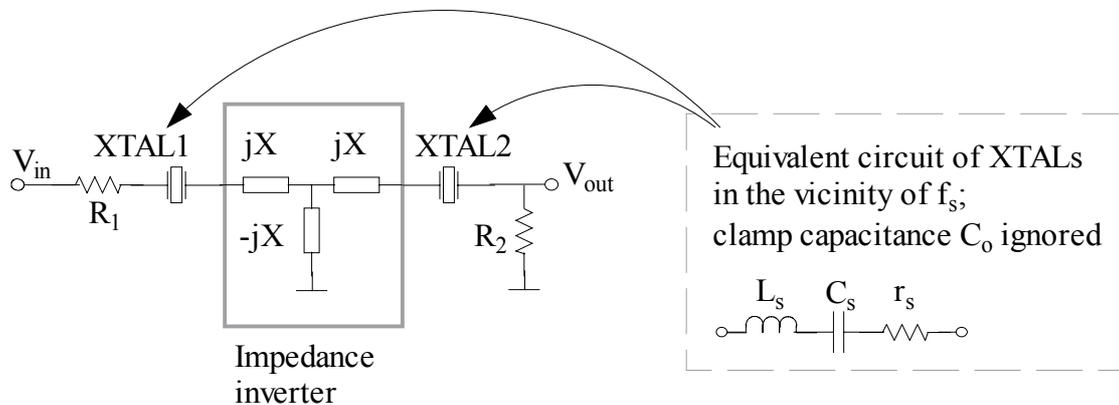


Figure 5.18 Two-crystal ladder filter

The inverter inverts the series circuit provided by XTAL2 and  $R_2$  into a parallel tuned circuit. The equivalent circuit of the resulting ladder filter is shown in Figure 5.19(a).

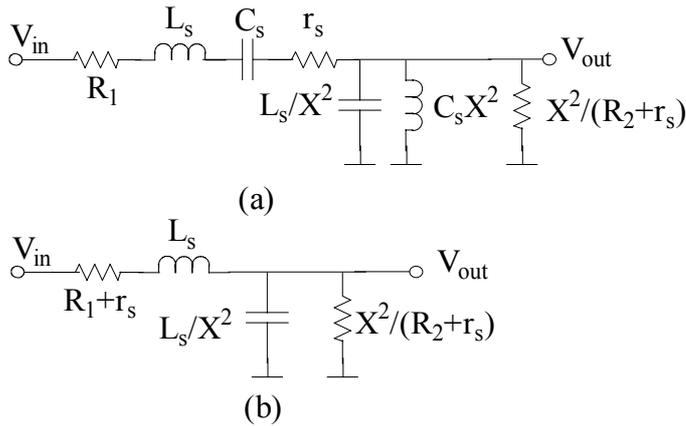


Figure 5.19 (a) Equivalent circuit of two-crystal ladder BPF and (b) LP equivalent of the filter

We must set all circuit parameters such that the filter becomes a second order Butterworth BPF. As we discussed in Section 5.2.4, we first consider the LP equivalent of this filter, which is given in Figure 5.19(b). Hence

$$R_o = R_1 + r_s = X^2 / (R_2 + r_s),$$

$$L_s = 1.4142 R_o / \omega_c,$$

and

$$L_s / X^2 = 1.4142 / \omega_c R_o$$

at the cut-off frequency  $\omega_c$  of the LPF for  $n=2$ , where  $R_o$  is the termination resistance. The cut-off frequency of the LP equivalent is 6 KHz.  $L_s$  and  $C_s$  are motional circuit elements and are fixed. Above relations specify  $X$  and  $R_o$  (hence  $R_1$  and  $R_2$ ) uniquely as

$$R_o = X = R_1 + r_s = R_2 + r_s = \omega_c L_s / 1.4142.$$

Once  $X$ ,  $R_1$  and  $R_2$  are specified as such, the parallel inductance  $C_s X^2$  and series capacitance  $C_s$  tunes out  $L_s / X^2$  and  $L_s$  respectively, producing a BPF with BW of  $\omega_c$  at the series resonance frequency of the XTAL.

A 16 MHz XTAL with  $L_s = 6$  mH requires  $X$ ,  $R_1$  and  $R_2$  to be

$$X = R_o = R_1 + r_s = R_2 + r_s = 160 \Omega,$$

for a BW of 6 KHz.

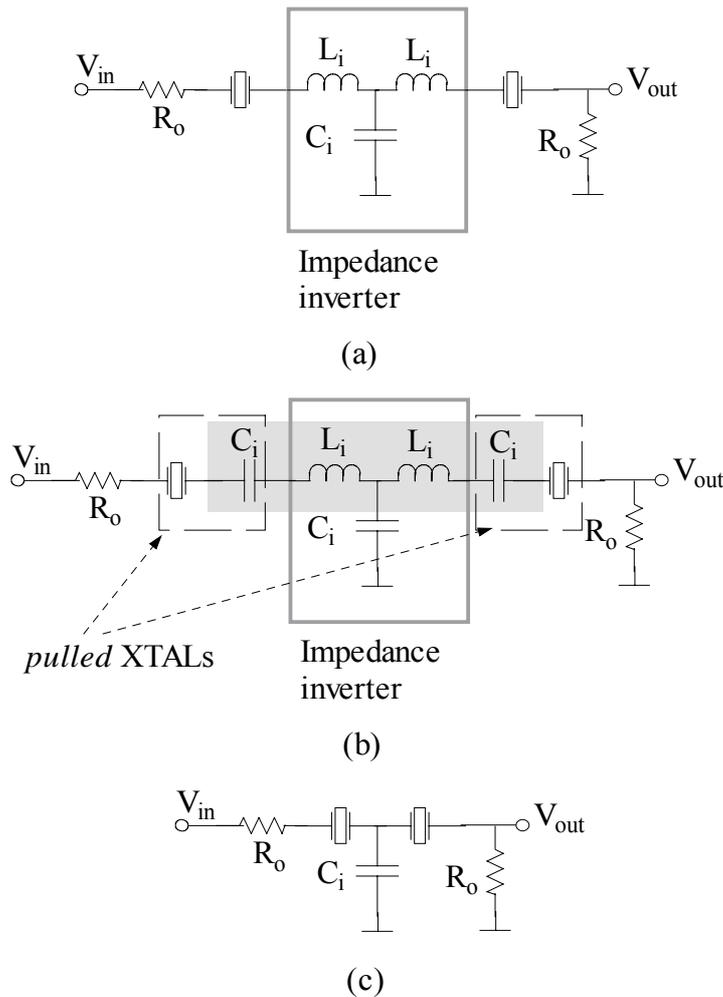


Figure 5.20 Impedance inverter for two-crystal ladder filter, (a) Proper inverter, (b) Inverter inductors tuned out by series capacitors, modifying the series XTAL branches, (c) Actual circuit with only one capacitor

Now let us examine how we shall realize the inverter. The inverter is expected to have a parallel capacitor ( $C_i$ ) and two series inductors ( $L_i$ ), providing  $-jX$  and  $jX$  at  $f_s$ , respectively. The circuit is depicted in Figure 5.20(a). Here the inverter circuit components are related as

$$X = 2\pi f_s L_i = 1/2\pi f_s C_i.$$

However we can tune the series inductors at  $f_s$  by including series capacitors  $C_i$ , into the circuit as shown in Figure 5.20(b). The series capacitor modifies the XTAL circuit only slightly, because the value of this capacitance is always few orders of magnitude larger than the motional capacitance of the XTAL. The effect is a slight increase in the series resonance frequency of the XTAL. This effect is called *pulling the frequency of the XTAL*. We are effectively left with an inverter and two *pulled* XTALs. The center frequency of the filter is now slightly higher than  $f_s$ .

Series connected  $C_i$  and  $L_i$  is simply a short circuit in the vicinity of  $f_s$ . If we replace all series  $C_i$  and  $L_i$  in the shaded area in Figure 5.20(b) by a short circuit, the circuit

behavior is left unchanged. The resulting circuit implementation is given in Figure 5.20(c).

The bandwidth requirements and the XTAL parameters determine the termination resistances. When we use XTAL filters, we must employ appropriate impedance transformation techniques to provide the proper termination to the filter.

### 5.5. Bibliography

There are excellent texts on filter theory and filter implementations. However the treatment of the topic at introductory electronics textbooks is usually very weak. In *The Electronics of Radio* filters are discussed excellently and in a detail, which is not available in other introductory electronics textbooks.

*The ARRL Handbook* has a very good chapter on filters (chapter 16).

There are very good application notes on HF transformer design in Philips Semiconductors and Ferroxcube web sites, [www-us6.semiconductors.com](http://www-us6.semiconductors.com) and [www.ferroxcube.com](http://www.ferroxcube.com), respectively.

A lot of information is available on every aspect of crystals. There is concise application information and data in PTI's web site, [www.piezotech.com](http://www.piezotech.com).

### 5.6. Laboratory Exercises

#### *Harmonic filter*

1. The harmonic filter of TRC-10 is given in Figure 5.21.

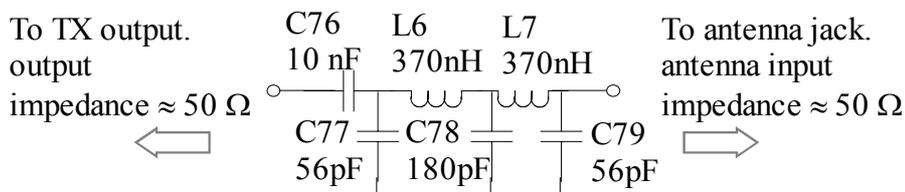


Figure 5.21 Harmonic filter

This filter is designed in Section 5.2.2. C76 is a 10 nF d.c. decoupling capacitor. The inductors of the filter, L6 and L7, are air-core inductors. Measure the diameter of the *coil former*, preferably using a compass. We shall use enameled wire of 0.5 mm diameter. Calculate the number of turns required for 370 nH using the formula given in Section 4.4.1. Make sure that the length of the coil does not exceed the spacing between the connection holes on the PCB. Check your calculations carefully.

Estimate the total wire length necessary for one coil. Add an extra 10 cm for easy manipulation and cut a piece of wire of the total length. Wind the coil of calculated number of turns on the coil former tightly. Stretch the coil to the calculated length, making sure that the coil is in good helical shape and wound

tightly on the coil former. Fix the winding on the coil former using a piece of tape (otherwise it springs out). Trim the leads and strip the enamel. Cover the leads by solder. (You can measure the inductance using LCR meter or a network analyzer. Ask the assistant or the lab technician to help you for measurement.) Using appropriate heat resistant glue, fix the winding, and remove the tape.

This is an air-core inductor and it does not have any core losses. Calculate the copper loss and the Q of the inductor at 29 MHz. Install this inductor as L6 and solder. Make L7 similarly, install and solder. Install C76, C77, C78 and C79 and solder them.

2. Use a very short piece of insulated cable as a jumper to connect the free end of C76 and the attenuator following R61 (better still connect free ends of C76 and 47 $\Omega$  resistor in the attenuator together- the attenuator circuit you constructed in Exercise 4.15), by-passing the TX/RX switching components.

The output section of the measurement circuit is given in Figure 5.22. This position of the cable from antenna jack is the correct position, but we must temporarily connect it using ground plane construction technique for this exercise. We have to remove the PCB several times to mount remaining components and we do not want this cable on our way.

Signal generator input is connected to C64 and the input part of the measurement set up is exactly as in Figure 4.16. Set the signal generator frequency to 28.85MHz and its voltage to the level you recorded in Exercise 7 in Chapter 4.

3. Measure second and third harmonic components. Compare the power of these components to the ones measured without harmonic filter, after making the correction for attenuator. Does the harmonic emission of TRC-10 satisfy regulatory requirements?

Estimate the attenuation provided by the harmonic filter at the second and third harmonic frequencies of 29 MHz. Compare this attenuation with the calculated ones.

4. Switch the power and the equipment off. Disconnect the spectrum analyzer and the signal generator. De-solder RG58/U you connected in Exercise 4.8. **Keep the 51  $\Omega$  and 1.5 K resistors (ground plane constructed) connected to C64.** You need them in TX/RX circuit experiments in Chapter 6.

Install R61 in its place. De-solder the attenuator and the antenna jack cable connection in gray brackets in Figure 5.22. De-solder the jumper. Remove and clean the remaining solder from the PCB, using de-soldering pump.

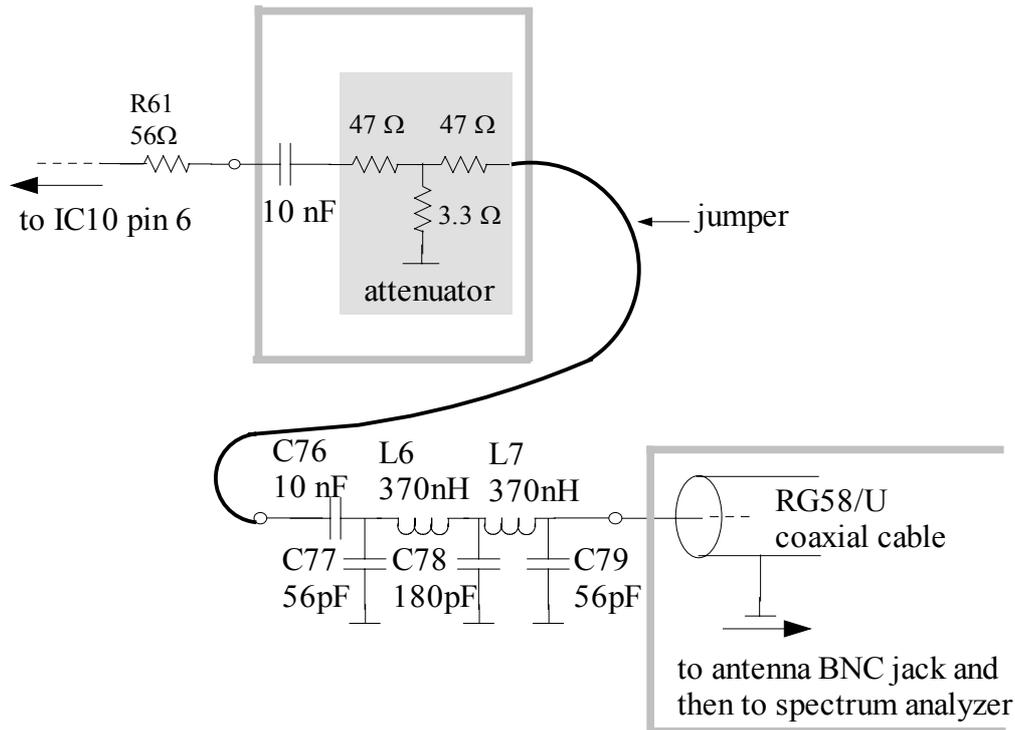


Figure 5.22 Harmonic measurement set up

### Crystals

- IF of TRC-10 is 16 MHz. XTALs at this frequency can be found abundantly in all component suppliers. The tricky thing about such XTALs is, on the other hand, in order to make best use of lower cost and for convenience, we acquire these crystals from local component stores, in this particular case from *Konya Sokak*. Usually the only thing that is known about these XTALs is their nominal series resonance frequency  $f_s$ , and the value of the pulling capacitance that must be used to obtain this frequency. The proper way of acquiring XTALs is to order them to the manufacturer with clearly delineated specifications, in order to get most reliable and repeatable results. This way of acquisition is the most expensive way also.

It is sufficient to have the  $f_s$  of two XTALs within few hundred Hz in TRC-10, since the bandwidth of the IF filter is 6 KHz. We must make sure that both crystals are of same make and come from the same production batch, in order to maintain this. Then their  $f_s$  are within at most few hundred Hz, and this is close enough for our purposes in this application.

We shall measure the  $f_s$  and  $Q$  of the XTAL in this exercise and then characterize the relevant XTAL parameters. The measurement set up is given in Figure 5.23.

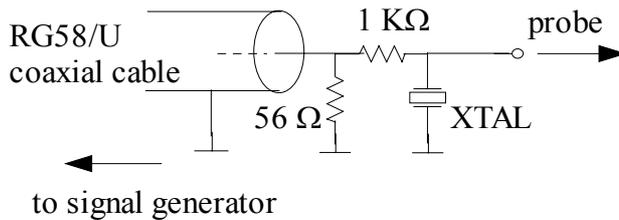


Figure 5.23 XTAL measurement set up

Set up the circuit in Figure 5.23. on a small piece of copper clad PCB, using ground plane construction technique. Connect the cables to the signal generator and the oscilloscope. Switch the power and the equipment on.

1 K $\Omega$  resistor is much larger than the expected series resistance of the crystal, which is less than 100  $\Omega$ . We use 1 K $\Omega$  resistor, to convert the signal source into a current source. We use 56  $\Omega$  resistor to terminate the coaxial cable connected to the signal generator. Adjust the signal generator to deliver 2 V pp sinewave at 16 MHz to 56  $\Omega$  resistor, and set it up so that you can change the frequency at 1 KHz intervals. Now the crystal is driven approximately by 2 mA pp, or 1 mA p, current at frequencies in the vicinity of  $f_s$ . Scan the frequency range between 15.990 MHz and 16.005 MHz. As we change the frequency we shall observe a minimum on the voltage across the crystal, because the crystal has a series resonance. At the frequency where you observe a dip on the voltage, decrease the scan step to 10 Hz. Find the frequency of minimum voltage. This frequency is  $f_s$ . Measure and record this frequency and this minimum voltage. Now calculate the series resistance  $r_s$ .

Shift the frequency and observe that the voltage increases. Record the frequency on both sides of  $f_s$  where the voltage is  $\sqrt{2}$  times the minimum you recorded. These two frequencies corresponds to the situation where reactance of the crystal is equal to  $r_s$ , hence the difference between them is  $-3$  dB BW. Calculate the series Q of the crystal.

Calculate  $L_s$  and  $C_s$  from  $f_s$ ,  $r_s$  and Q.

Most of the XTALs supplied in the kit for this course have  $L_s$  of about 5.5 mH. The design of the IF filter is based on this of value for  $L_s$ . Your measurement technique is likely to yield a less accurate result for  $r_s$ , which affect the calculated value of  $L_s$ . However, in case if your measurement results deviate from this value of  $L_s$  considerably, first check your measurements and calculations, and then contact the instructor for a discussion of your measurements.

Compare this value of  $L_s$  to other inductances in TRC-10. Calculate the number of turns necessary on T37-7 core to get this inductance value.

Can you think of a similar method to measure  $C_o$ ?

Desolder the cable and the XTAL. Desolder and remove the resistors.

**IF filter**

6. Following the procedure explained in Section 5.4.2, calculate  $R_o$  and  $X$  of the ladder filter, using the value of  $L_s$  you have found. Calculate the inverter capacitance value.
7. The IF filter of TRC-10 is given in Figure 5.24. Start building the filter by placing the crystals on its wide side on the marked rectangle on copper surface. Solder the case, at the top of the crystal, to the copper by using ample amount of heat and solder. This is necessary to keep the large metal surface of the crystal grounded. Otherwise it will be left floating, and will provide a stray path for capacitive coupling of various signals in the TRC-10 circuits to relatively high impedance nodes as unwanted signals.

The termination impedance for this filter is given as  $150 \Omega$ . This is the correct standard valued resistor for XTALs with 5.5 mH equivalent inductance. Also, when 68 pF is used for C100, it yields a reactance of  $150 \Omega$  at 16 MHz. Should the value of C100 be different than 68 pF, the only effect is to have a different termination resistance and hence a different BW and some mismatch.

Calculate the  $R_o$  and  $C_i$  for the XTAL at hand and determine the standard values for R90 and C100. connect and solder C100.

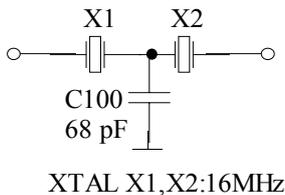


Figure 5.24 IF filter

**IF Transformer**

8. The RX mixer is a SA602A (or SA612A) IC. The output impedance of this chip is approximately 1.5 K. It has two  $180^\circ$  out-of-phase outputs. We use both outputs of this chip constructively to get four times more power.

Consider the equivalent circuit in Figure 5.25(a). In this circuit two out-of-phase outputs of the mixer chip are modeled as two voltage sources  $V_{IFo1}$  and  $V_{IFo2}$ , respectively, with equivalent 1.5 K source resistances.  $V_1$  becomes  $V_{IFo1} - V_{IFo2}$ , and the two out-of-phase outputs are constructively added at the transformer primary terminals.

The other function of the transformer is to convert the total source impedance of 3 K ( $1.5 \text{ K} + 1.5 \text{ K}$ ) to approximately  $150 \Omega$  at its secondary, which is the assumed termination impedance of the IF filter. This requirement sets the transformer ratio to

$$n_1^2/n_2^2 = 3000/150 \Rightarrow n_1/n_2 \approx 4.5.$$

If the termination impedance  $R_o$ , hence R90, of the IF filter is different than  $150\Omega$ , the turns ratio, and the number of turns, must be chosen accordingly.

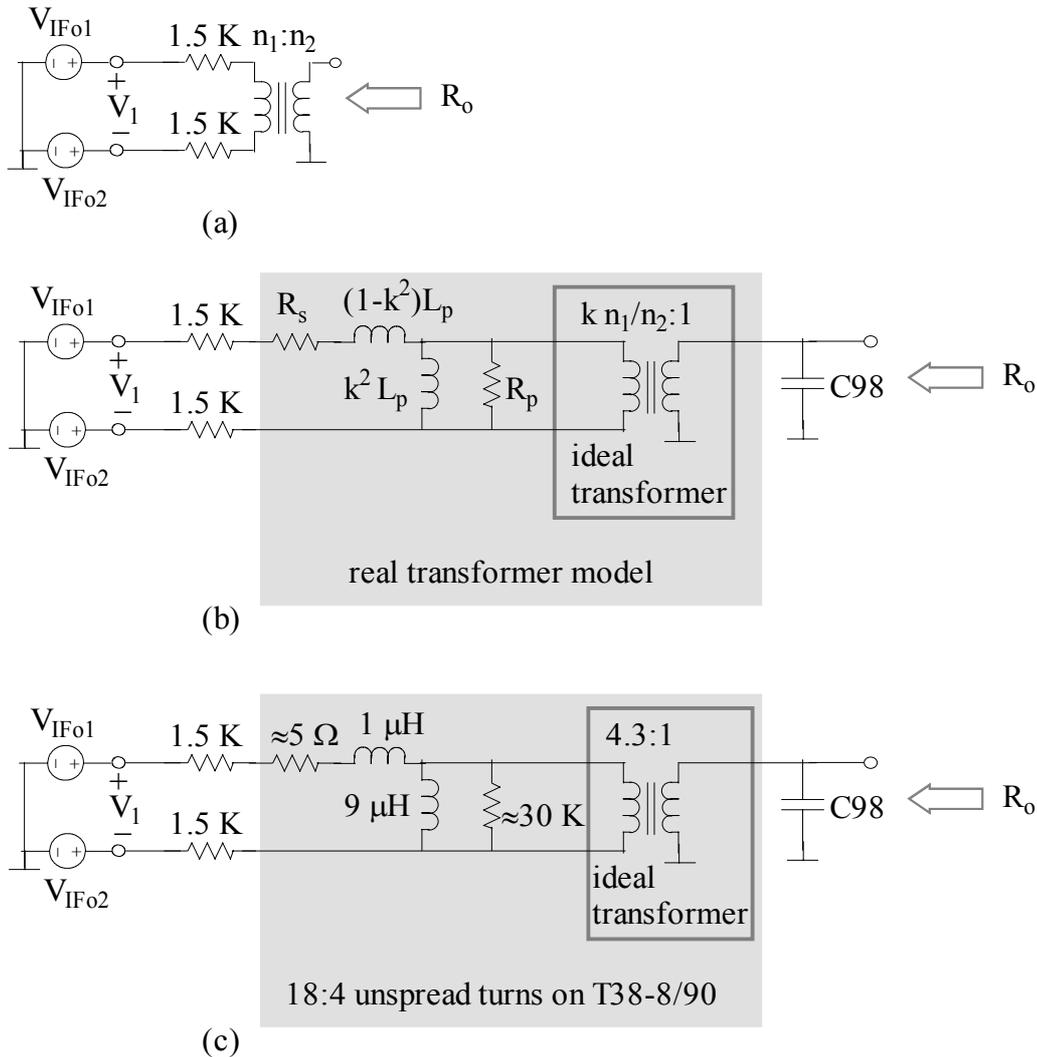


Figure 5.25 Impedance matching between RX mixer and IF filter (a) with an ideal transformer, (b) with a real tuned transformer model and (c) with the typical values of a transformer with 18:4 turns on a T38-8/90 toroid core.

The core for the IF transformer T3 is a T38-8/90 toroid, produced by Micrometals. T38 tells that the outer diameter of the core is 0.38 inch (or 9.53 mm) and 8/90 is the iron powder material type. The core materials for transformer applications must have a permeability as high as possible, in order to keep most of the flux within the core. This material has a relative permeability  $\mu_r$  of 35 and yields an  $A_L$  of  $20 \text{ nH/turn}^2$  with its given geometry. What follows is a discussion of how an IF filter is designed. Every design requires an output specification and it is again assumed that the termination is  $150\Omega$ . For other values all relevant component values must be modified accordingly.

The presence of leakage inductance in a transformer reduces the coupling coefficient (see Section 5.3.2), and therefore reduces the effective transformer

ratio to a value below the winding ratio. The model of a real transformer of Figure 5.14 is repeated in Figure 5.25(b), for convenience.

We shall use a winding ratio of 4.5 as calculated above. Considerations on deciding the number of turns for the secondary (or equivalently the primary) of this transformer are as follows:

- The secondary inductance (hence the number of turns) must be as large as possible in order to keep the Q of the transformer secondary (with a tuning capacitor) low, consequently to avoid tuning problems;
- The inductance must be low enough to have 16 MHz sufficiently below the self-resonance frequency of the primary inductor  $L_p$  (the larger one).

We use 18 turns for primary and 4 turns for secondary windings. First wind the primary using 40 cm long 0.2 mm enameled wire. Wind 18 tight turns on T38-8/90 toroid. *Do not stretch the winding to cover the toroid*, keep it tight. This is useful to minimize the leakage flux since we have small number of turns in secondary winding. Fix the ends of the winding using a piece of tape. Wind 4 *tight* turns of primary *on* the secondary winding *centered on* it, as a second layer. Use 0.5 mm wire. Fix the ends using a tape. Trim all four leads, strip the enamel and cover with solder. Replace the pieces of tape by glue to fix the ends.

The inductance observed at the secondary terminals would be  $4^2 \times 20$  nH, or 320 nH, if the winding is stretched. With an unstretched winding this inductance can be up to twice as high. The value of the capacitor C98 to tune out this inductance must be about 160 pF to 200 pF. The resistance that appears across the secondary, on the other hand, is approximately  $150 \Omega / 150 \Omega = 75 \Omega$ , which yields a Q of 1.2 to 1.35. Hence the BW of the transformation is very large and the value of C98 is not critical at all. We choose it 180 pF standard capacitor value.

Transformers are characterized by a measurement technique called *short and open circuit tests*. This method is developed for low frequency transformers, but we can adopt a version of it here. We want to find  $k$ ,  $L_p$ ,  $R_s$  and  $R_p$  of our transformer. We first measure the impedance across the primary terminals using an RLC meter or a network analyzer at 16 MHz, *with secondary terminals left open circuit*. Then we short the secondary terminals and measure the impedance again. When we carry out this test on transformers with 18:4 turns on T38-8/90 toroid made as described above, the typical measurements are as follows:

open circuit test at 16 MHz:	10 $\mu$ H in series with about 45 $\Omega$ , as equivalent primary impedance, and
short circuit test at 16 MHz:	1 $\mu$ H in series with 5 $\Omega$ , as equivalent primary impedance.

Applying this test on the transformer model in Figure 5.14 or Figure 5.25(b), short circuit test immediately yields

$$R_s = 5 \Omega \text{ and } (1-k^2)L_p = 1 \mu\text{H}.$$

Solving for  $k^2L_p$  and  $R_p$  is somewhat more complicated. Since the  $Q$  of the primary winding is large (more than 20), the equivalent inductance of the primary impedance in open circuit test is  $L_p$ . In other words, the primary admittance in open circuit test is

$$\begin{aligned} Z_p &= R_s + j\omega_{IF}(1-k^2)L_p + 1/[1/R_p + 1/(j\omega_{IF} k^2L_p)] \\ &\approx R_s + R_p / [(R_p/\omega_{IF} k^2L_p)^2 + 1] + j\omega_{IF} L_p \\ &\Rightarrow L_p = 10 \mu\text{H} \end{aligned}$$

where  $\omega_{IF}$  is the IF frequency. The equivalent series resistance of  $45 \Omega$  in open circuit test cannot be measured accurately, since the series reactance to it is very large. However the above relation gives an indication of a quite large  $R_p$  of about  $30 \text{ K}$ .

Coupling coefficient can be found as  $k = 0.95$ . Hence the transformer ratio  $k n_1/n_2:1$  of the ideal transformer in the model becomes  $4.3:1$ . The equivalent circuit parameters of a typical IF transformer is given in Figure 5.25(c).

It is interesting to note that when a  $3 \text{ K}$  resistance is connected to the primary of such a transformer as in Figure 5.25(c), the impedance across the secondary terminals is measured as  $150 \Omega$  in parallel with  $510 \text{ nH}$ . This is very close to what the equivalent circuit predicts. The decrease in transformer ratio combines with the effect of the presence of  $R_p$  to keep the equivalent secondary resistance near  $150 \Omega$ .

Install the transformer T3, paying attention to the correct placement of primary and secondary pair of leads. Solder the leads. Install and solder the tuning capacitor C98, and two d.c. blocking capacitors C94 and C95.

9. The transformer T3 matches the output impedance of the mixer to  $R_o$  of the IF filter. The other terminal of the filter is terminated by R90. Install and solder R90.
10. Install IC12, an LM7171 for the first IF amplifier. Carefully check the pin configuration and the placement details on the PCB. Solder IC12. Install and solder the supply by-pass capacitors and feedback resistors, C102, C103, R91 and R92, respectively. Check all the connections, both visually and using a multimeter (as ohmmeter for connectivity), carefully.

Switch the power on. Using a multimeter check all d.c. voltages at the supplies, input and output terminals. Connecting R90 between the amplifier input and ground provides a d.c. path for the input bias current of the OPAMP. Carry out the d.c. analysis of this amplifier referring to the data sheet, and calculate the maximum d.c. off-set voltage that we may expect at the output of IC12.

Switch the power off.

11. The circuit part comprising IF transformer, IF filter, and IF1 amplifier (first IF amplifier) is given in Figure 5.26. In order to test this circuit, we simulate the balanced the mixer output by the circuit shown in gray bracket. Two  $1.5 \text{ K}$  resistors represent the output impedance of two output terminals of the mixer.

56 Ω terminates the signal generator cable and signal generator voltage simulates  $V_1$  in Figure 5.25.

- Mount and solder two 1.5 K resistors and the 56 Ω resistor, using ground plane construction technique. Solder the free end of coaxial cable with BNC connector for the signal generator connection. Solder a 56 Ω resistor between pin 6 of IC12 and ground plane using ground plane construction technique. Connect the oscilloscope probe to the pin 6 of IC12. Switch the equipment on.

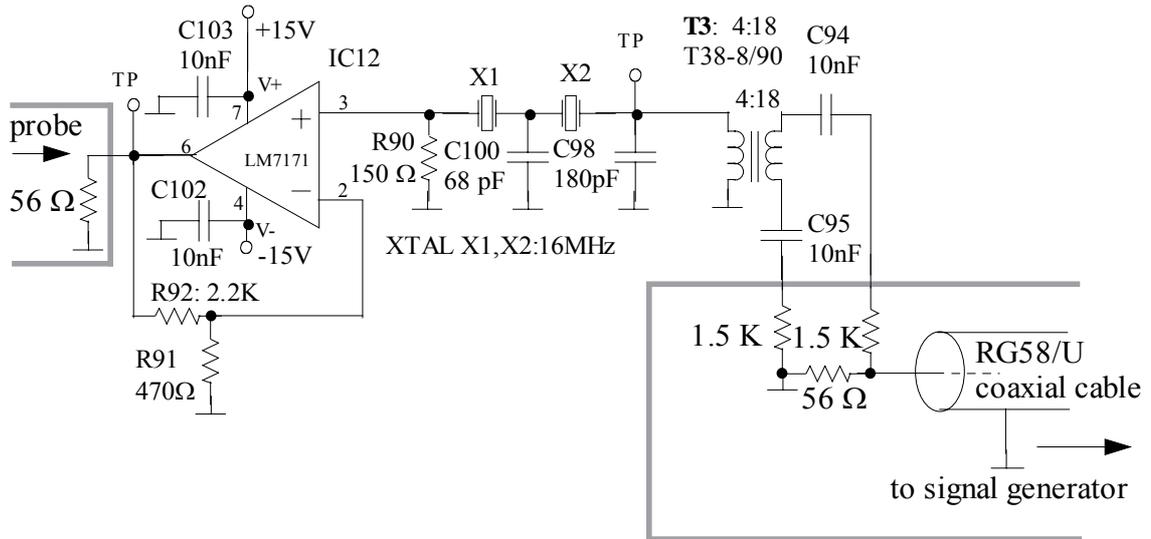


Figure 5.26 IF stage measurement set up

Set the signal generator output to -7 dBm or 0.1 V rms sine wave. Vary the frequency over the pass-band of IF filter around 16 MHz and, find and set it to the frequency where the amplitude of the amplifier output is maximized. This frequency is the center frequency of IF filter. Record it. Measure the magnitude of the peak output voltage amplitude  $V_{out1}$  and record it.

- The approximate equivalent circuit of this part at the center frequency of the IF filter is given in Figure 5.27. Following the method described in Section 4.5.1 and referring to the data sheet of LM7171, first determine the open loop gain of the OPAMP at 16 MHz and then calculate the voltage and power gain of the circuit in Figure 5.27.

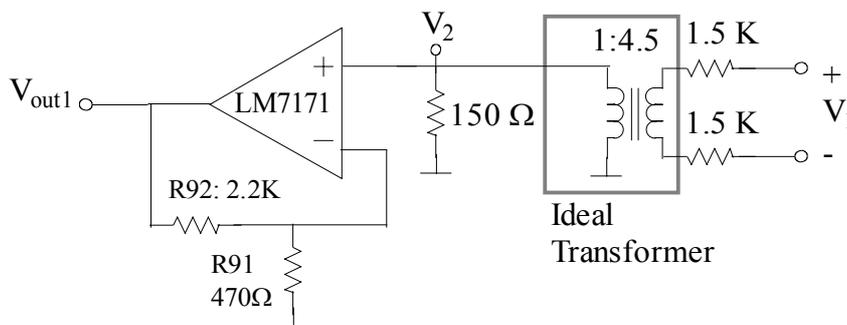


Figure 5.27 Equivalent circuit of IF stage at IF frequency

14. Set the signal generator output voltage amplitude to 1 V rms. Measure the peak to peak output voltage,  $V_{out1}$ , in a frequency range of 100KHz centered at passband. Calculate the loss

$$L = 20\log | V_{out1}/V_1 | \text{ dB}$$

at each frequency, where you can take  $V_1$  as the value displayed on the signal generator (take care to convert rms value to peak value). Plot this loss versus frequency. Vertical axis must be scaled so that it can accommodate 60 dB range. The number of measurements and the frequencies where they are taken must be such that the filter pass-band and stop-band are properly characterised. Find the -3 dB frequencies and the bandwidth of the IF filter.

Switch the power and equipment off. Desolder and remove the 56  $\Omega$  resistor connected to pin 6 of IC12. Remove the excess solder from the copper surface using your desoldering pump.

15. We need another inductor, L8, for the second IF amplifier, IF2. We use a T50-7 Micrometals toroid core to make L8. You already know what T50-7 means. Find the properties of this core from the data sheet. Calculate the number of turns necessary for an inductance of approximately 8.4  $\mu\text{H}$ . It must be about 38. We use 0.2 mm thick enameled wire to wind the inductor. Calculate the length of wire you must use. Cut that length of wire and make an excellent inductor. **Do not forget to spread the winding evenly around the core.** All the data provided assumes this. Install and solder L8.

Install and solder R93 and C104. C104 is a trimmer capacitor. Make sure that you place it such that ground pin corresponds to adjustment pin.

Install IC13, another LM7171. Carefully check the pin configuration and the placement details on the PCB. Solder IC13. Install and solder the supply by-pass capacitors and feedback resistors, C105, C106, R94 and R95, respectively. Check all the connections, both visually and using a multimeter (as ohmmeter for connectivity), carefully.

Install and solder a 100  $\Omega$  resistor between the pin 6 of IC13 and copper surface using ground plane construction technique.

16. The series RLC circuit comprising R93, L8 and C104 is a reasonably high Q circuit. We must tune C104. First calculate the Q. Calculate the expected voltage and power gain between pin 6 of IC12 and pin 6 of IC13, in dB. Record your calculations and their results.
17. Switch the power on. Using a multimeter check all d.c. voltages at the supplies, input and output terminals of IF2 amplifier stage. What is the d.c. path for the bias currents of this OPAMP?

Connect the probe of Channel 1 of the oscilloscope to the pin 6 of IC12 and Channel 2 to pin 6 of IC13. **Make sure that the input circuit is exactly as the**

same as in Exercise 12 and the input voltage level is set to 0.1 V<sub>rms</sub>, as in Exercise 12, and the signal generator is connected. Switch the equipment on.

Observing the waveform on Channel 1, make sure that the signal generator frequency is set at the center frequency of the IF filter. Adjust the frequency, if necessary. Decrease the input signal amplitude, if you observe distortion.

Adjust C104 until the amplitude of the signal on Channel 2 is maximized, using the alignment tool. Decrease the input signal amplitude further, if you observe distortion. IF2 amplifier is tuned.

Measure the voltage gain between pin 6 of IC12 and pin 6 of IC13 and record it. Compare it with your calculation. Can you comment on the discrepancies on a quantitative basis?

Measure the overall voltage gain between the signal generator output and IF2 output. Convert it into dB and record it. Calculate the power gain in dB using this measurement and record it.

Switch the power and the equipment off.

Desolder and remove the coaxial cable at the input. Desolder and remove the 100 Ω resistor at the output of IF2. Remove the excess solder from the copper surface using your desoldering pump.

You need the two 1.5 K resistors and the 56 Ω resistor in the exercises of Chapter 6. Leave them on the PCB.

## 5.7. Problems

- Using a spread sheet and using the component values calculated in Section 5.2.1, calculate and plot the transfer response  $20\log|H(\omega)|$  of filters in Figure 5.6 from 1 rad/sec to  $10^8$  rad/sec.
- Find the amplitude and phase of the voltage at the output of the circuit given in Figure 5.28 when input voltage is  $2 \cos(2\pi \times 16 \times 10^6 \times t)$  volts. Find the amplitude of the current flowing through the capacitor.

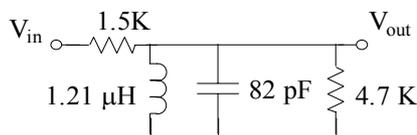


Figure 5.28

- Design a 3<sup>rd</sup> order Butterworth LP filter (i.e. find the values of all reactive components) with minimum number of inductors, whose cutoff frequency is 3 KHz and termination impedance are 1 K.

4. If  $V = V_p \angle \theta$  is the phasor of voltage across a resistor  $R$ , we know that the power delivered to that resistor is  $V_p^2/(2R)$ , or  $|V|^2/(2R)$ . It is given in Section 5.1 that the transfer function of the circuit in Figure 5.2(b) is

$$V_{out}/V_{in} = H_2(\omega) = 1/\{2[1-(\omega/\omega_c)^2 + j\sqrt{2}\omega/\omega_c]\},$$

For  $R_S=R_L=R$  and for maximally flat response. Now since the available power at the input of this circuit is  $|V_{in}|^2/(8R)$ , show that the ratio of the power delivered to the load  $P_L$  to the available power  $P_O$  is

$$P_L/P_O = 1 / [1+(\omega/\omega_c)^{2 \times 2}],$$

i.e. the power transfer function of a second order Butterworth filter

5. Find the amplitude and phase of the voltage at the output of the circuit given in Figure 5.29 when input current is  $10 \cos(1.82 \times 10^8 \times t)$  mA. Find the amplitude of the current flowing through the inductor.

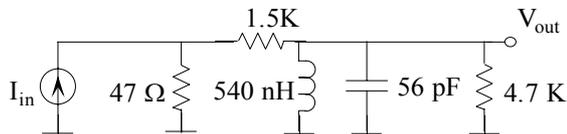


Figure 5.29

6. Find the 3-dB cut of frequency of the LPF given in Figure 5.30.

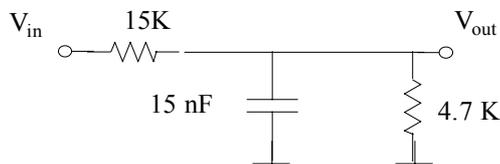


Figure 5.30

7. Design a 5<sup>th</sup> order Butterworth LP filter with minimum number of inductors, whose cutoff frequency is 3 KHz and termination impedance are 1 K.
8. Design a 3<sup>rd</sup> order Butterworth HP filter with minimum number of inductors, whose cutoff frequency is 3 MHz and termination impedance are 75 Ohm.
9. Design a Butterworth LP filter which has an attenuation of at most 1 dB at 16 MHz and an attenuation of at least 20 dB at 32 MHz (i.e. find the minimum number of components, find reactive element values for a filter with at minimum number of inductors, and finally check the attenuation at specified frequencies for your filter).
10. Design a Butterworth BPF with center frequency of 30 MHz and a 3-dB bandwidth of 3 MHz, such that its attenuation at 60 MHz is at least 20 dB.

11. Assume we have an *ideal* transformer with a primary/secondary winding ratio of 1:2. What is the impedance that appears across the primary if a  $300\ \Omega$  is connected across the secondary.
12. Assume we wind up a very good *real* transformer with no loss and no leakage with 1:2 primary/secondary winding ratio. The primary inductance is  $1\ \mu\text{H}$ . Find the impedance across the primary at 10 MHz when  $300\ \Omega$  is connected across the secondary. What is the phase shift between the primary voltage and primary current?
13. Calculate the *transformer current* and the *magnetizing current* for the transformer and load given in Problem 12 when the transformer is driven by a voltage of  $2\angle 0^\circ$  volts at 10 MHz.
14. A real transformer has a primary inductance of  $1\ \mu\text{H}$  and a primary/secondary turns ratio of 4:24. Assuming that the transformer has no loss and no leakage, calculate the impedance across the primary at 30 MHz when a 1.8 K load is connected across the secondary. What is the phase shift between the primary voltage and primary current?
15. Find the value of parallel capacitance required across the primary to tune out the primary inductance at 30 MHz, for the transformer and load given in Problem 14. Find the impedance across the primary at 30 MHz. Calculate the phase shift between the primary voltage and primary current when this capacitance is connected across the primary.
16. Find the impedance across the terminals of a XTAL at the parallel resonance frequency. The circuit in Figure 5.17(b) is redrawn below in Figure 5.31 for convenience. (Hint: First find the conductance and susceptance of the series rLC circuit, and then find parallel resonance frequency and effective resistance considering that total susceptance including  $C_o$  is zero at his frequency.)

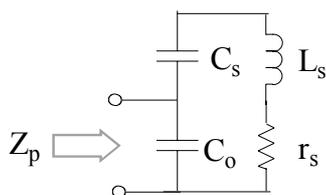


Figure 5.31 Quartz crystal equivalent circuit

17. A 16 MHz crystal is measured to have  $f_s = 15,992,875\ \text{Hz}$ , the parallel resonance frequency  $f_p = 16,024,600\ \text{Hz}$  and peak conductance at  $f_s$   $1/r_s = 135\ \text{mS}$ . The Q of the series resonance is found as 112,000. Calculate  $C_s$ ,  $L_s$ ,  $r_s$  and  $C_o$ .
18. Assume that a 16 pF *pulling* capacitor is connected to the XTAL in problem 17 in series. Calculate the *pulled* frequency.
19. A 16 MHz crystal is measured and the G and B vs frequency plots shown in

20. Figure 5.32 are obtained. Find the equivalent series inductance, capacitance and loss resistance for this XTAL. Calculate  $R_o$  and  $C_i$  of the IF filter if a pair of XTAL's with this characteristics are used. Determine the turns ratio for T3 and the values (standard) of R90, C100 and C98 for this filter. (Ans: 4:13; 270  $\Omega$ ; 27 pF; 180 pF;  $L_s = 10.3$  mH,  $X = 390 \Omega$ ,  $R_o = 275 \Omega$ )

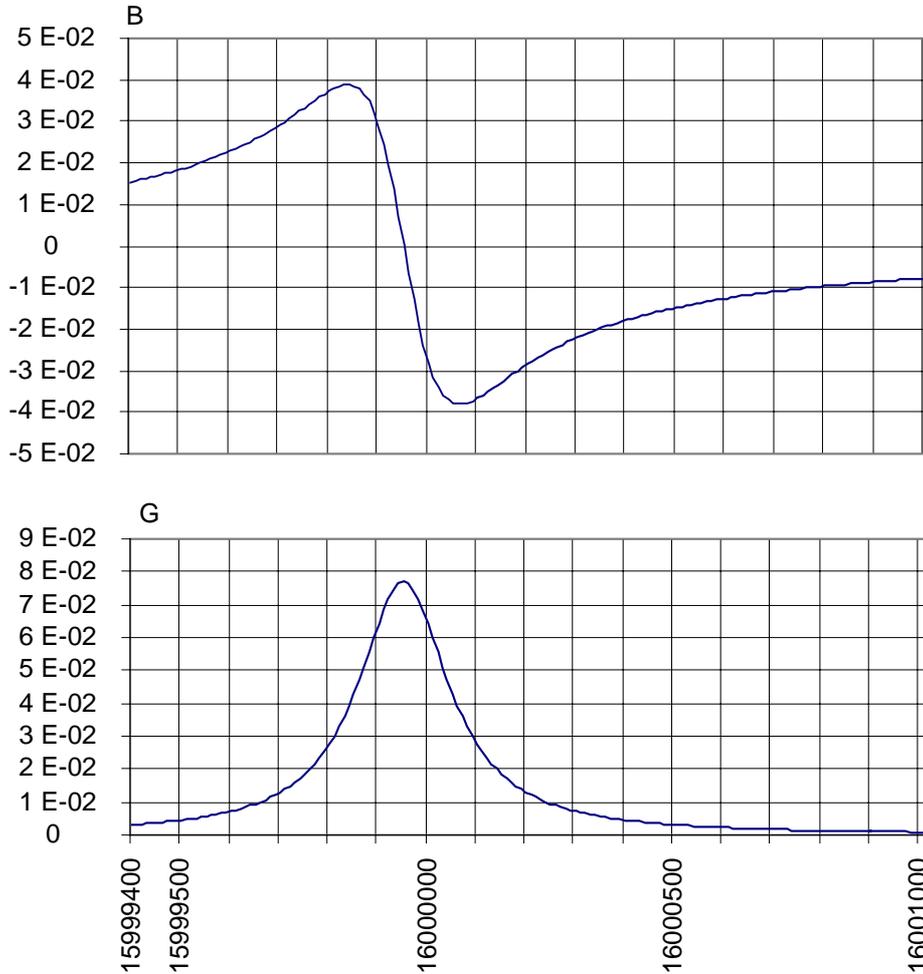


Figure 5.32 XTAL susceptance and conductance vs frequency: vertical axis- siemens; horizontal axis- Hz.

21. Consider the circuit given in Figure 5.28. The voltage across this circuit can be expressed as

$$v_{out}(t) = V_{dc} + V_1 \cos(\omega_0 t + \theta_1) + V_2 \cos(2\omega_0 t + \theta_2) + V_3 \cos(3\omega_0 t + \theta_3) + V_4 \cos(4\omega_0 t + \theta_4) + V_5 \cos(5\omega_0 t + \theta_5) + V_6 \cos(6\omega_0 t + \theta_6) + \dots$$

if  $v_{in}(t)$  is a square wave signal of 2 V pp amplitude, as shown in Figure 1.2, with period  $T = 0.0625 \mu\text{sec}$ . Calculate the amplitude and phase of the first six terms in  $v_{out}(t)$ . How much deviation is there in  $v_{out}(t)$  from a sinusoid?

22. Please match the  $200\ \Omega$  load resistor to the source side for maximum power transfer at 16 MHz, using T38-8/90 ( $A_L=20\ \text{nH/turns}^2$ ) in the following circuit. Determine  $H(\omega) = V_{\text{out}}(\omega)/V_s(\omega)$  at 16 MHz.

