Elec 201
FINAL

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Duration: 2 hours 30 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do not use any reference material other than that provided with this set.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NAME: ____

SIGNATURE: ____________________________________

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2	
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4	
5	
TOTAL	

You may or may not need the following formulas:

$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$ $\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$ $\sum_{k=M}^{N}\alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\sin^2\theta + \cos^2\theta = 1$

Continuous-Time Fourier Series	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0 = \frac{2\pi}{T_0}$ $a_k = \frac{1}{T_0}\int_{<T_0>} x(t)e^{-jkw_0 t}dt$ $X(w) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Continuous-Time Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}dw$ $X(w) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$
Discrete-Time Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$ $a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j\frac{2\pi}{N}kn}$ $X(\theta) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta - \frac{2\pi}{N}k); a_{k+N} = a_k$
Discrete-Time Fourier Transform	$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(\theta)e^{j\theta n}d\theta$ $X(\theta) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\theta n}$

$$x(t) = \frac{\sin(\omega_0 t)}{\pi t} \quad \leftrightarrow \quad X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\Omega) = \frac{\sin(\Omega(N_1 + 1/2))}{\sin(\Omega/2)}$$

$$x[n] = \frac{\sin(a\pi n)}{\pi n} \quad \leftrightarrow \quad X(\Omega) = \begin{cases} 1 & |\Omega| < a\pi \\ 0 & a\pi < |\Omega| < \pi \end{cases}$$

PROBLEM 1: (20 points) You must show all steps to receive credit.

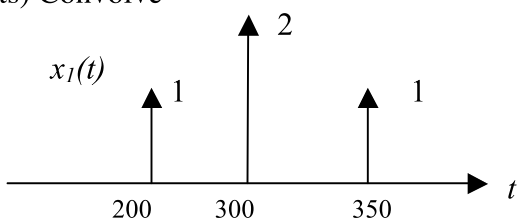
a) (2 pts) Simplify as much as possible:

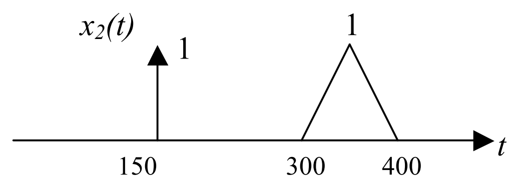
$$f(t) = \int_{-\infty}^{\infty} e^{-200t}\delta(3t-6)dt$$

b) (2 pts) Evaluate the integral

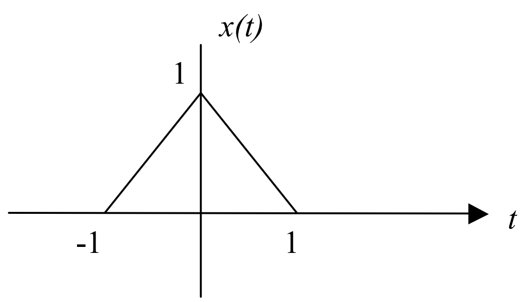
$$I = \int_{-\infty}^{\infty} \cos(\pi t)\left[\delta(t/2+4)+\delta(t/2-1)\right]dt$$

c) (6 pts) Convolve





d) (10 pts) Convolve x(t) and h(t), where



and

$$h(t) = u(-t) - u(-t-1) - u(t) + u(t-1)$$

PROBLEM 2: (20 points)

a) (15 pts) A signal x(t) has the CT Fourier transform

$$X(\omega) = \frac{1}{\omega^2 - 5j\omega - 2}$$

Write the CTFT of the following:

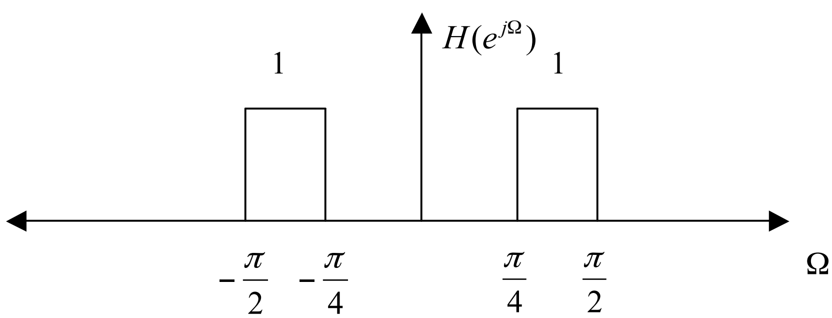
i) $x(t)\sin(200\pi t)$

ii) $\frac{d^2 x(t)}{d^2 t}$

iii) $jtx(t)$

iv) $e^{j800\pi t} x(t)$

a) Write the impulse response h[n] of the following bandpass digital filter, given between $[-\pi, \pi]$: (5 Points)

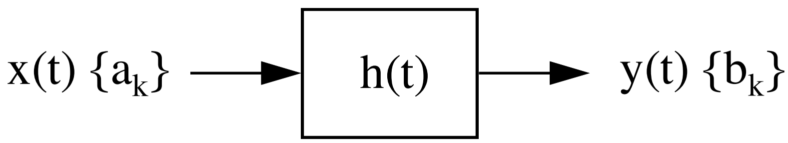


PROBLEM 3: (20 points)

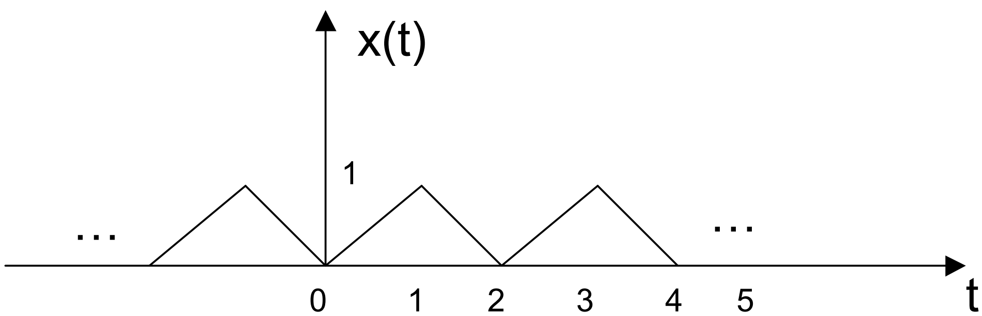
Let an LTI system be defined by

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} - 2x(t)$$

where the input *x(t)* and output *y(t)* are periodic with the Fourier series coefficients $\{a_k\}$ and $\{b_k\}$, respectively



a. Find the Fourier series representation of the following $x(t)$ in complex exponential form. Will the CTFS coefficients be real, purely imaginary, or neither? (5 Points)



b. Find the Fourier series representation of $x(t)$ in trigonometric form. (5 Points)

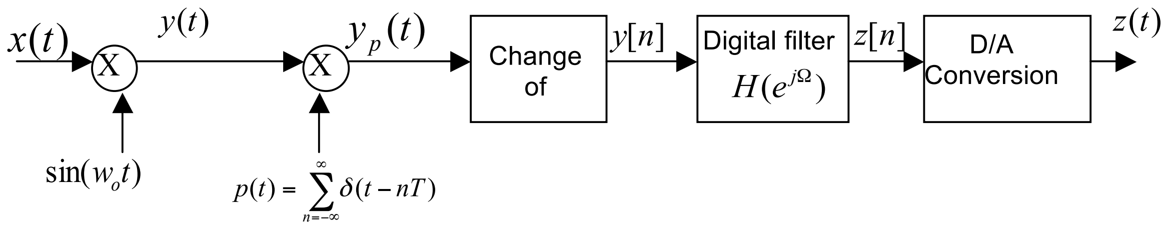
c. (10 pts) Suppose we input x(t) to an analog filter with the impulse response

$$h(t) = \frac{\sin\left(\frac{18\pi}{5}t\right)}{\pi t}$$

Write the Fourier series for the output signal $y(t)$. How many harmonics will there be in the output signal $y(t)$? (10 Points)

PROBLEM 4: (20 points)

Let $x(t) = \frac{\sin(Wt)}{\pi t}$



a) Suppose W=100 and $w_0 = 100$. We sample y(t).

i. What should be the minimum sampling rate T_0 of y(t) to avoid aliasing? (2 Points)

ii. Compute and plot the CTFT of $Y_p(w)$ if y(t) is sampled with the minimum sampling rate T_0. (3 Points)

iii. If y(t) is sampled with the minimum sampling rate T_0 and the digital filter has an impulse response $h[n] = \cos(\pi n / 2)$. What is the output z(t)? (5 Points)

b) Suppose W=100 and $w_0 = 150$. We sample y(t).

i. What should be the minimum sampling rate T_0 to avoid aliasing? (2 Points)

ii. Compute and plot the CTFT of $Y_p(w)$ if y(t) is sampled with the minimum sampling rate T_0. (3 Points)

iii. If y(t) is sampled with the minimum sampling rate T_0 and the digital filter has an impulse response $h[n] = \frac{\sin(\pi n / 2)}{\pi n}$. What is the output z(t)? (5 Points)

PROBLEM 5: (20 points)

An LTI system is described the differential equation:

$$\frac{d^3 y(t)}{dt^3} + \frac{d^2 y(t)}{dt^2} - 2y(t) = \frac{dx(t)}{dt}$$

a) Find the corresponding transfer function H(s) of this system (3 Points)
b) What is the corresponding ROC, if the system is known to be causal? Find the impulse response h(t) that corresponds to this ROC. (8 points)
c) Suppose for this part of the question, the system is again known to be a causal system. Find the corresponding output y(t) of the system if the input is $x(t) = \frac{d\delta(t)}{dt} - \delta(t)$. (9 Points)