Elec 201
Midterm 1

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APRIL 02, 2010

Duration: 90 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do NOT use CALCULATOR.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NAME: ____

ID NUMBER: ____________________________

SIGNATURE: ____________________________________

You may or may not need the following formulas:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad \text{AND} \quad c_k = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jk\omega_0 t} dt$$

$$H(\omega_0) = \int_{-\infty}^{\infty} h(t) e^{-j\omega_0 t} dt$$

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \quad \text{AND} \quad \sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta) \qquad \text{sinc}\left(\frac{x}{\pi}\right) = \frac{\sin x}{x}$$

PROBLEM 1: (30 points) No credit will be given to answers without proper justification.

a)

a. (4 points) $\int_{-\infty}^{\infty} [\sum_{k=-1}^{1} \delta\left(\frac{1}{2}t - k\pi\right)] \operatorname{Re}\{e^{jt}\} dt$ (Evaluate, i.e., simplify as simple as possible)

b. (3 points) $[u(t) - u(t-3)] * [\delta(t) - \delta(t-3)]$ (simplify and PLOT)

c. (3 points) $(\delta[n] - \delta[n-2]) * \left(\sum_{k=-1}^{1} \delta[n-2k]\right)$ (simplify and PLOT)

b) Compute and PLOT the convolution of the following two sequences (5 points)

$$x[n] = \begin{Bmatrix} 1 & n = 0 \\ 0 & n = 1 \\ 3 & n = 2 \end{Bmatrix} \text{ and } h[n] = \begin{Bmatrix} -1 & n = -1 \\ 2 & n = 0 \\ -1 & n = 1 \end{Bmatrix}$$

c) A discrete-time system is given by

$$y[n] = \left\{2 + \left(\sum_{i=-1}^{1} x[-n-i]\right)\right\}\left(\frac{j}{2}\right)^n$$

a. (4 points) Is this system linear? Prove your answer.

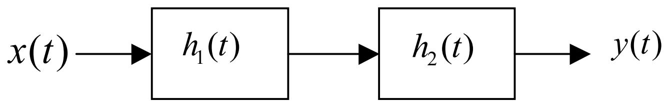
b. (4 points) Is this system time-invariant? Prove your answer.

c. (4 points) Is this system stable? Prove your answer.

d. (3 points) Is this system causal? Prove your answer.

PROBLEM 2: (35 Points)

a. (20 Points) Given the following LTI system and input x(t), WRITE y(t) in closed form and PLOT y(t).

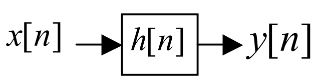


$h_1(t) = u(-t+1) - u(-t-1)$ $\quad$ $h_2(t) = u(t) - u(t-1)$

and x(t) is given as follows:

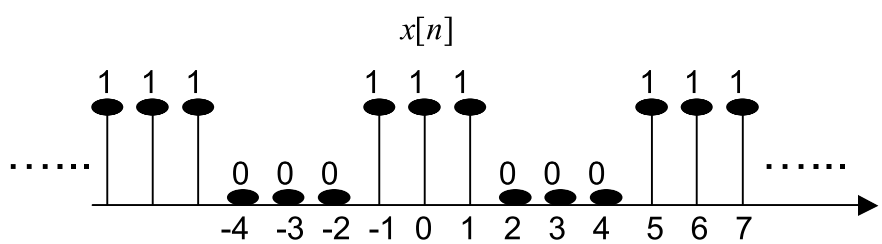
$x(t) = \delta(t-1) + \delta(t+1)$

b. (15 Points) Given the following LTI system and input x[n], calculate and PLOT y[n]



$h[n] = \delta[n+1] - \delta[n]$

Where x[n] is a periodic signal as follows:



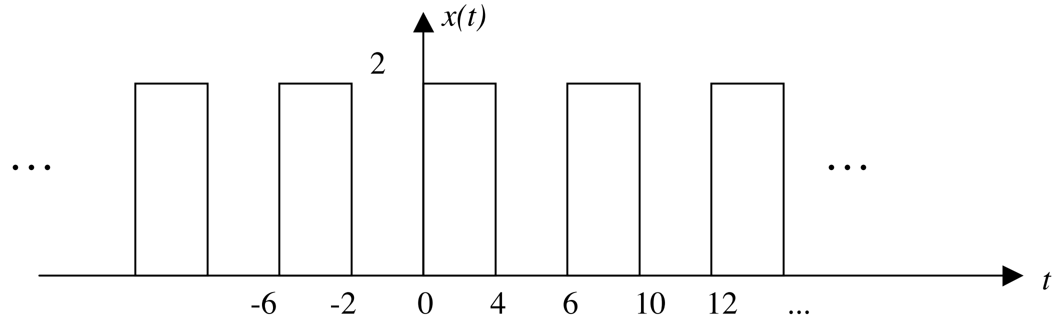
PROBLEM 3: (35 points)

Let an LTI system be defined by

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3\frac{dx(t)}{dt} - 2x(t)$$

a) Find the output *y(t)* when the input is $x(t) = e^{-j3\pi t}$ (5 points)

b) The input *x(t)* is given below. Find the Fourier series coefficients for *x(t)* (10 points)



c) For real signals, the trigonometric form of the Fourier series can be expressed as (15 points)

$$y(t) = c_0 + \sum_{k=1}^{\infty}\left[a_k \cos\left(\frac{2\pi}{T_0}kt\right) + b_k \sin\left(\frac{2\pi}{T_0}kt\right)\right].$$

Find c_0, a_k *and* b_k for the output signal *y(t)*, when the input *x(t)* is given above (as a shifted periodic square wave).

d) What is the average power of the input *x(t)* (as a shifted periodic square wave)? (5 Points)