EEE 424
Final

January 10, 2014

Duration: 150 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do NOT use CALCULATOR.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NAME: ____

ID NUMBER: ______________________________

SIGNATURE: ______________________________________

You may or may not need the following formulas:

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \qquad \sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$$

$$\cos^2\theta = \tfrac{1}{2}\left(1+\cos 2\theta\right) \qquad \sin^2\theta + \cos^2\theta = 1$$

Continuous-Time Fourier Series	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0 = \frac{2\pi}{T_0}$ $a_k = \frac{1}{T_0} \int_{<T_0>} x(t) e^{-jkw_0 t} dt$ $X(w) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Continuous-Time Fourier Transform	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{jwt} dw$ $X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$
Discrete-Time Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$ $a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-j\frac{2\pi}{N}kn}$ $X(\theta) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta - \frac{2\pi}{N}k); a_{k+N} = a_k$
Discrete-Time Fourier Transform	$x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(\theta) e^{j\theta n} d\theta$ $X(\theta) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\theta n}$

$$x(t) = \frac{\sin(\omega_0 t)}{\pi t} \leftrightarrow X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow X(\Omega) = \frac{\sin(\Omega(N_1 + 1/2))}{\sin(\Omega/2)}$$

$$x[n] = \frac{\sin(a\pi n)}{\pi n} \leftrightarrow X(\Omega) = \begin{cases} 1 & |\Omega| < a\pi \\ 0 & a\pi < |\Omega| < \pi \end{cases}$$

PROBLEM 1: (25 points) No credit will be given to answers without proper justification.

a) (5 Points) Given that

$$x[n] = \{4,3,2,1,0\} \xleftrightarrow{DFT,5} X[k]$$

$$s[n] = \{0,1,0,0,1\} \xleftrightarrow{DFT,5} S[k]$$

Determine y[n], which yields Y[k]=X[k]S[k] where Y[k] is 5 point DFT of y[n].

b) Evaluate the DTFT of the following in terms of the DTFT of x[n].(10 Points)

a. $x_1[n] = \begin{cases} x[n], \text{ if } n \text{ is odd,} \\ 0, \text{ otherwise.} \end{cases}$

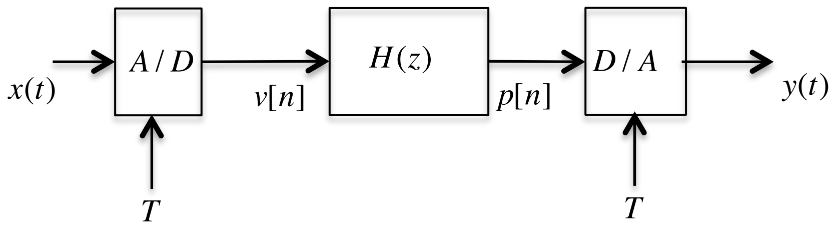
b. $x_2[n] = x[n]x[-n]$

c. $x_3[n] = \dfrac{x[n]}{(-1)^n}$

c) Given that $x[n] = 0.5^n u[n]$ and $X(e^{jw})$ is the DTFT of x[n].

Define $Y[k] = X\left(e^{j\frac{2\pi}{10}k}\right), k = 0,1,...,9$. Construct the sequence y[n] that is the length-10 IDFT of Y[k]. Provide y[n] in a closed form (if possible). (10 points)

PROBLEM 2: (25 points) No credit will be given to answers without proper justification.



The digital filter H(z) is casual and described by

$$y[n] = (1/2)y[n-1] + (1/2)x[n] + (1/2)x[n-1].$$

a) What is the kind of H(z), i.e., FIR or IIR? LPF or HPF? Linear phase? Minimum phase? (5 Points)

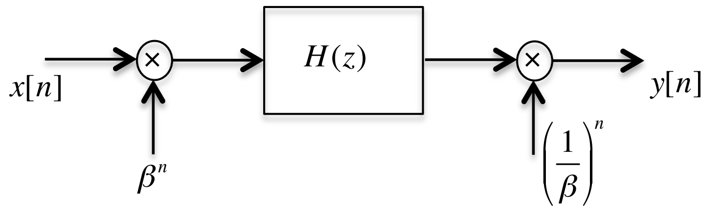
b) What is the 3dB bandwidth of H(z)? (5 Points)

c) What is p[n] if $v[n] = 2\sin(\pi n/3 + \pi/4)$? (5 Points)

d) Given that A/D and D/A converters are ideal and use anti-alising filters with cut off π/T.

What is the frequency response of the analog filter from x(t) to y(t)? (10 Points)

PROBLEM 3: (25 points) No credit will be given to answers without proper justification.

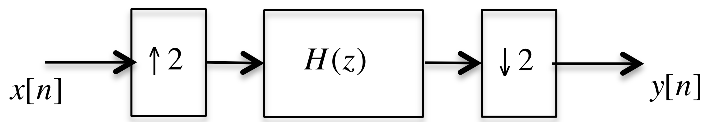


Given that x[n] is the input and y[n] is the output to the overall system, and

$H(z) = \dfrac{1}{1-(1/3)z^{-1}}$ is a casual LTI filter.

- **a)** Is the overall system linear? Is the overall system time invariant? (5 Points)
- **b)** What is Y(z) in terms of X(z)? What is the ROC of Y(z) in terms of X(z). (5 Points)
- **c)** What is y[n] given that $x[n] = \delta[n-1]$? (5 Points)
- **d)** Given that $x[n] = 2\sin(2\pi n/3 + \pi/4)$ and $\beta = e^{j(\pi/8)n}$, what is the output y[n]? (10 Points)

PROBLEM 4: (25 points) No credit will be given to answers without proper justification.



Given the input is x[n], the output is y[n] and H(.) is LTI.

a) Is the overall system linear? Is the overall system time invariant? (5 Points)
b) What is Y(z) in terms of X(z)? (5 Points)
c) Under what conditions x[n]=y[n]? (5 Points)
d) Given that $h[n]=\delta[n]+\delta[n-1]+\delta[n-2]$, can you derive the polyphase representation where the decimators are in the front and interpolators are at the end? (10 Points)