EEE 424
Midterm 1

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March 17, 2016

Duration: 90 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do NOT use CALCULATOR.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NO CREDIT will be given for ANSWERS written out of place.

NAME: ____

ID NUMBER: ______________________________

SIGNATURE: ______________________________________

You may or may not need the following formulas:

$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$ $\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$ $\sum_{k=M}^{N} \alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\sin^2\theta + \cos^2\theta = 1$

Continuous-Time Fourier Series	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0 = \frac{2\pi}{T_0}, a_k = \frac{1}{T_0}\int_{<T_0>} x(t)e^{-jkw_0 t}dt$ $X(w) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Continuous-Time Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}dw$ $X(w) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$
Discrete-Time Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$ $a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j\frac{2\pi}{N}kn}$ $X(\theta) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta - \frac{2\pi}{N}k); a_{k+N} = a_k$
Discrete-Time Fourier Transform	$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(\theta)e^{j\theta n}d\theta$ $X(\theta) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\theta n}$

$$x(t) = \frac{\sin(\omega_0 t)}{\pi t} \quad \leftrightarrow \quad X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\Omega) = \frac{\sin(\Omega(N_1 + 1/2))}{\sin(\Omega/2)}$$

$$x[n] = a^n u[n],\ |a|<1, \quad \leftrightarrow \quad X(\Omega) = \frac{1}{1 - ae^{-jw}}$$

$$x[n] = \frac{\sin(a\pi n)}{\pi n} \quad \leftrightarrow \quad X(\Omega) = \begin{cases} 1 & |\Omega| < a\pi \\ 0 & a\pi < |\Omega| < \pi \end{cases}$$

PROBLEM 1: (40 points) No credit will be given to answers without proper justification.

a) (15 Points) An LTI system is given by the following difference equation:

$$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n-1]$$

i. What is the frequency response $H(e^{j\Omega})$? (4 Points)
ii. What is the DTFT of $z[n]=n\,h[n]$? (4 Points)
iii. What is the DTFT of $w[n]=\sin(\pi n/2)h[n]$ (4 Points)
iv. What is the DTFT of $r[n]=(-1)^n h[n]$ (3 Points)

b) (10 Points) Suppose x[n] is a length 8 sequence and X[k] is the length 8 DFT of x[n]. We define a length 24 sequence y[n]

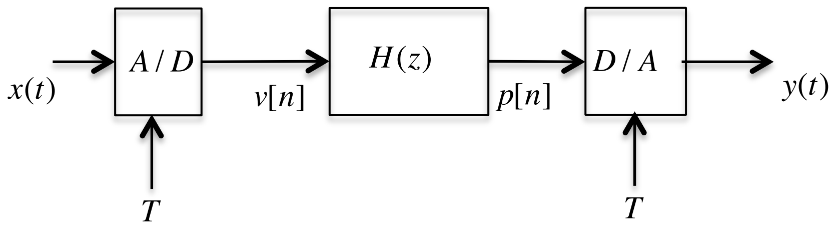
$$y[n]=\begin{cases} x\left[\frac{n}{3}\right], n=0,\pm 3,\pm 6,... \\ 0, \text{ otherwise} \end{cases}$$

What is the relationship between the 8 point DFT X[k] and 24 point DFT Y[k]?

c) (10 Points) Given a length N sequence x[n], where x[n] is non-zero for n=0,…,N-1, we define $y[n]=x[n]+x[N-n]$. What is the N point DFT of y[n] in terms of the N point DFT of x[n], i.e., X[k]?

d) (5 Points) A person starts to walk on train a track from minus infinity towards a particular track T. At each time, he/she throws a fair dice and takes exactly n steps where n is the output the dice throw. What is the probability that he/she will hit the particular track T?

PROBLEM 2: (30 points) No credit will be given to answers without proper justification.



The digital filter is casual and described by

$y[n] = (1/2)y[n-1] + x[n].$

a) What is the kind of the filter, i.e., FIR or IIR? LPF or HPF? (10 Points)

b) What is p[n] if $v[n] = 2\cos(\pi n/3 + \pi/4)$? (5 Points)

c) Given that A/D and D/A converters are ideal and use anti-alising filters with cut off π/T.

Derive frequency response of the analog filter from x(t) to y(t). (15 Points)

PROBLEM 3: (30 points) No credit will be given to answers without proper justification.

a) For an LTI system h[n], the output is given by

$$y[n] = 2\delta[n-1],$$

given that

$$x[n] = \delta[n]-2\delta[n-1]+ 2\delta[n-2].$$

i) (5 Points) Find the transfer function $H\left(e^{jw}\right)=\dfrac{Y\left(e^{jw}\right)}{X\left(e^{jw}\right)}$.

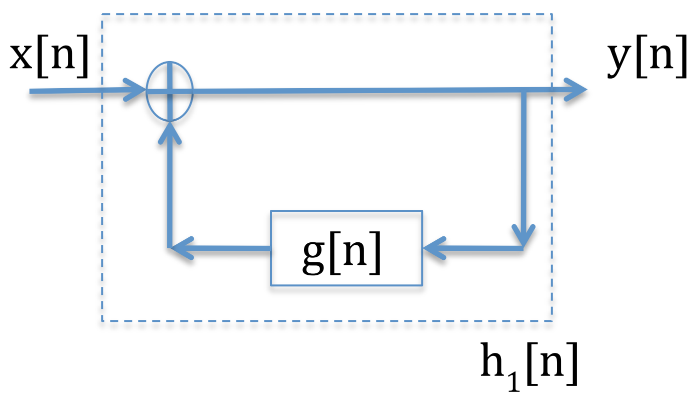
ii) (5 Points) Find the difference equation of the overall system.

b) (10 Points) Plot the magnitude of the DTFT of the Hanning window:

$$w[n]=\begin{cases} 0.5-0.5\cos(2\pi n/M), 0 \le n \le M, \\ 0, \text{ otherwise} \end{cases}$$

c) (10 Points) The block diagram of a causal LTI system, $h_1[n]$, is given below (input x[n], output y[n]). In this block diagram, the impulse response of the particular block is equal to

$$g[n] = \alpha\delta[n-1].$$



Determine $h_1[n]$ (assume $\alpha>1$ and α is real). Check for the stability of $h_1[n]$, while explaining its reason clearly.