Elec 303
Midterm 2

ASST. PROF. SERDAR KOZAT

May 24, 2012

Duration: 90 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do NOT use CALCULATOR.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NAME: ____

ID NUMBER: ___________________________

SIGNATURE: __________________________________

You may or may not need the following formulas:

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \qquad \sum_{k=M}^{N}\alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$$

$$\cos^2\theta = \tfrac{1}{2}\left(1+\cos 2\theta\right) \qquad \sin^2\theta + \cos^2\theta = 1$$

Continuous-Time Fourier Series	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0 = \frac{2\pi}{T_0}$ $a_k = \frac{1}{T_0}\int_{<T_0>} x(t)e^{-jkw_0 t}dt$ $X(w) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Continuous-Time Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}dw$ $X(w) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$
Discrete-Time Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$ $a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j\frac{2\pi}{N}kn}$ $X(\theta) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta - \frac{2\pi}{N}k); a_{k+N} = a_k$
Discrete-Time Fourier Transform	$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(\theta)e^{j\theta n}d\theta$ $X(\theta) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\theta n}$

$$x(t) = \frac{\sin(\omega_0 t)}{\pi t} \quad \leftrightarrow \quad X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\Omega) = \frac{\sin(\Omega(N_1 + 1/2))}{\sin(\Omega/2)}$$

$$x[n] = \frac{\sin(a\pi n)}{\pi n} \quad \leftrightarrow \quad X(\Omega) = \begin{cases} 1 & |\Omega| < a\pi \\ 0 & a\pi < |\Omega| < \pi \end{cases}$$

PROBLEM 1: (30 points) No credit will be given to answers without proper justification.

a) (5 Points) Express the Z-Transform of

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

in terms of X(z).

b) (10 Points) Let x[n] be a sequence with Z-Transform X(z). Find the Z-Transform of the following in terms of X(z).

$$x_1[n] = \begin{cases} x\left[\frac{n}{2}\right], & \text{if n is even} \\ 0, & \text{if n is odd.} \end{cases}$$

c)

(10 Points) Let x[n] be a sequence with Z-Transform X(z). Find the Z-Transform of the following in terms of X(z).

$x_2[n] = x[2n]$.

d) (5 Points) Find x[n] whose Z-Transform is given as

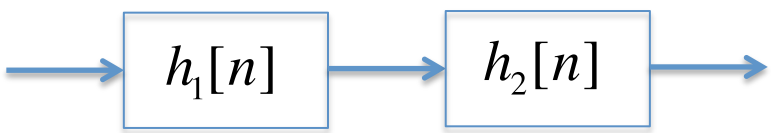
$X(z) = e^{z} + e^{1/z}$

PROBLEM 2: (35 points) No credit will be given to answers without proper justification.

A linear time invariant discrete time system has system function

$$H_1(z) = \frac{(1-2z^{-1}+2z^{-2})}{(1-z^{-1})(1-0.5z^{-1})(1-0.2z^{-1})}$$

a) Find and list all possible ROCs (10 Points)?
b) Find the impulse response of the system given that the system is causal (10 Points)
c) Find an $h_2[n]$ such that the overall system is stable and find the impulse response of the overall stable system. (15 Points)?



PROBLEM 3: (35 points) No credit will be given to answers without proper justification.

For an LTI system h[n], the output is given by

$$y[n] = 2\delta[n-1],$$

given that

$$x[n] = \delta[n]-2\delta[n-1]+ 2\delta[n-2].$$

a) Find the transfer function H(z) (7 Points).
b) Find the difference equation of the overall system (8 Points).
c) Given that the system is causal find h[n] (10 Points).
d) Given that the system does not have Fourier Transform, find h[n] (10 Points).