EEE 424
Midterm 2

Assoc. Prof. S. SERDAR KOZAT

November 28, 2015

Duration: 90 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do NOT use CALCULATOR.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NO CREDIT will be given for ANSWERS written out of PLACE.

NAME: ____

ID NUMBER: ______________________________

SIGNATURE: ______________________________________

You may or may not need the following formulas:

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \qquad \sum_{k=M}^{N}\alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$$

$$\cos^2\theta = \tfrac{1}{2}\left(1+\cos 2\theta\right) \qquad \sin^2\theta + \cos^2\theta = 1$$

Continuous-Time Fourier Series	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0 = \frac{2\pi}{T_0}, a_k = \frac{1}{T_0}\int_{<T_0>} x(t) e^{-jkw_0 t} dt$ $X(w) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Continuous-Time Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{jwt} dw, X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$
Discrete-Time Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}, a_k = \frac{1}{N}\sum_{n=<N>} x[n] e^{-j\frac{2\pi}{N}kn}$ $X(\theta) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta - \frac{2\pi}{N}k); a_{k+N} = a_k$
Discrete-Time Fourier Transform	$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(\theta) e^{j\theta n} d\theta$ $X(\theta) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\theta n}$

$$x(t) = \frac{\sin(\omega_0 t)}{\pi t} \quad \leftrightarrow \quad X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\Omega) = \frac{\sin(\Omega(N_1 + 1/2))}{\sin(\Omega/2)}$$

$$x[n] = \frac{\sin(a\pi n)}{\pi n} \quad \leftrightarrow \quad X(\Omega) = \begin{cases} 1 & |\Omega| < a\pi \\ 0 & a\pi < |\Omega| < \pi \end{cases}$$

$$x[n] = a^n u[n],\ |a|<1, \quad \leftrightarrow \quad X(\Omega) = \frac{1}{1 - ae^{-jw}}$$

PROBLEM 1: (35 points) No credit will be given to answers without proper justification.

Given that the output of an LTI system is

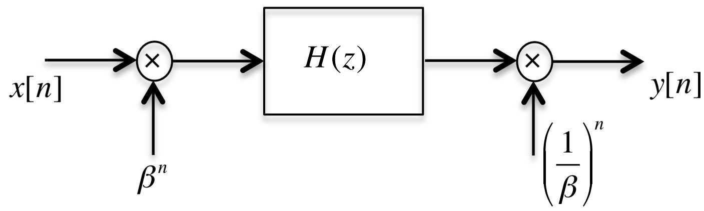
$$y[n] = \left(\frac{1}{5}\right)^{n-1} u[n-1],$$

and the input of the system is

$$x[n] = 3^n u[-n-1],$$

a) (5 Points) Write the corresponding difference equation.
b) (15 Points) Find H(z) and h[n] for all ROCs. Find the type of the filter, e.g., LPF, HPF, BPF, when the DTFT exits?
c) (15 Points) For each case, specify whether the system is stable, causal, zero phase or linear phase?

PROBLEM 2: (35 points) No credit will be given to answers without proper justification.



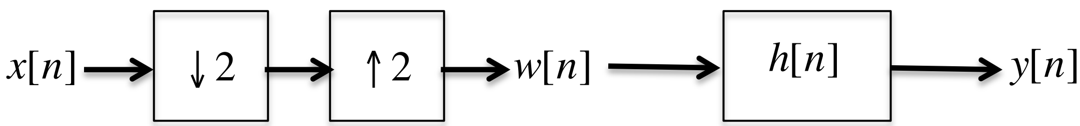
Given that x[n] is the input and y[n] is the output to the overall system, and

$H(z) = \dfrac{1}{1-(1/2)z^{-1}}$ is a casual LTI filter.

- **a)** Is the overall system linear? Is the overall system time invariant? (5 Points)
- **b)** What is Y(z) in terms of X(z)? What is the ROC of Y(z) in terms of X(z). (5 Points)
- **c)** What is y[n] given that $x[n] = \delta[n-1]$? (5 Points)
- **d)** Given that $x[n] = 2\sin(2\pi n/3 + \pi/4)$ and $\beta = e^{j(\pi/8)n}$, what is the output y[n]? (10 Points)

PROBLEM 3: (30 points) No credit will be given to answers without proper justification.

a) Given the system below, where h[n] is a perfect low pass filter with cutoff frequency $w_c = \pi / 3$



i) (5 Points) What is the DTFT of w[n] in terms of the DTFT of x[n]? (Hint: Just write w[n] in terms of x[n].)

ii) (10 Points) What is the output y[n] given that $x[n] = \sin(\pi n / 5)$?

b) (15 Points) Find the DTFT of y[n]=x[2n] in terms of DTFT of x[n].