# Bat-like Mobile Robot for Tracking a Moving Obstacle

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## ABSTRACT

The problem of tracking and capturing a moving obstacle in two dimensions by a mobile robot equipped with a wide-beam sonar system is discussed. In a practical system the range and azimuth measurements contain random errors and are available over a limited region. This pursuer/prey problem is treated as a feedback control system for which the pursuer action is dependent of the observed measurements. Two measures of performance are considered: the capture probability and the mean capture time when capture occurs. The lower bound for the mean capture time is determined from game theory, which assumes complete information about the prey. Strategies employing either qualitative (prey is to the left or right) or quantitative (range and azimuth to prey) information are implemented and compared. It is found that qualitative information is sufficient for prey capture, although quantitative information allows more efficient prey capture.

## I. INTRODUCTION

A bat-like sonar system employing a wide-beam transmitter (the mouth) and two receivers (the ears) was implemented to simulate and explore the capabilites of biological sensing systems [1]. In this paper, the investigation is extended to characterizing the performance of a mobile robot equipped with the sonar system whose task is to catch a second mobile robot moving along a linear path in an uncluttered environment. The sonar system estimates the range $r$ and azimuth $\theta$ of an object by extracting the time-of-flight (TOF) information from the detected echoes. Reliable prey localization is achieved over a limited region, the *active region*, defined by the intersection of the sensor sensitivity patterns. Both ears detect echoes only when the prey is present in the active region. Because of the symmetry of the transmitter/detector system, localization is most accurate when the prey is situated along the center of the active region at close range, where the signal-to-noise ratio in the echoes is large. Because of the similarity of our system to the biological bat/moth system, we refer to the mobile robot with sonar as $\mathcal{R}$, for ROBAT, and to the other as $\mathcal{M}$, for MOTH.

To treat the problem analytically and provide experimental verification, the capture of prey having *linear motion* is considered. Two measures of success are employed: probability of capture and mean capture time after initial detection (if capture occurs). In Section II a lower bound for the mean capture time is derived under the condition of perfect knowledge of prey dynamics. Section III describes the phases of prey capture for our robotic system. Section IV discusses qualitative and quantitative information. Simulation studies are described in Section V. The results are presented and interpreted in Section VI. A description of the robotic system in our laboratory is given in Section VII.

## II. PREY CAPTURE WITH LINEAR MOTION

A lower bound on the time to capture linearly moving prey is derived assuming that the pursuer has complete knowledge about the prey at all times and at all points in space. Consider the case in which $\mathcal{M}$ moves along a linear trajectory at speed $V_{\mathcal{M}}$. Let $\mathcal{R}$ first detect $\mathcal{M}$ at range $r$ and azimuth $\theta$, as shown in Fig. 1. The

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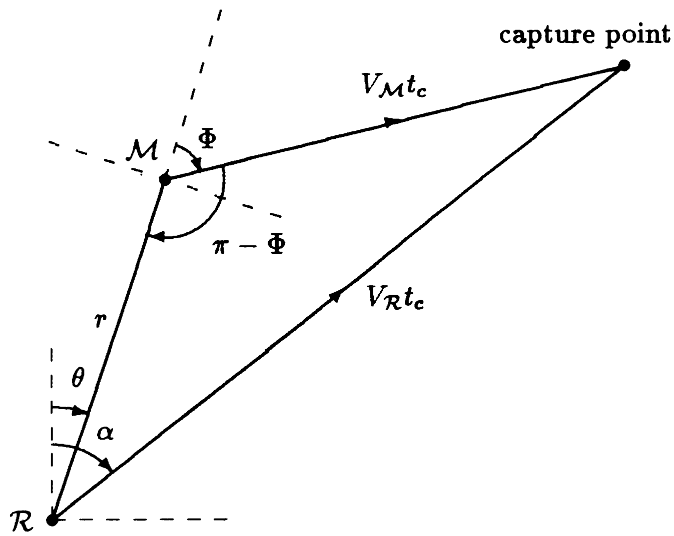


Fig. 1. Complete knowledge assumption for linearly moving prey.

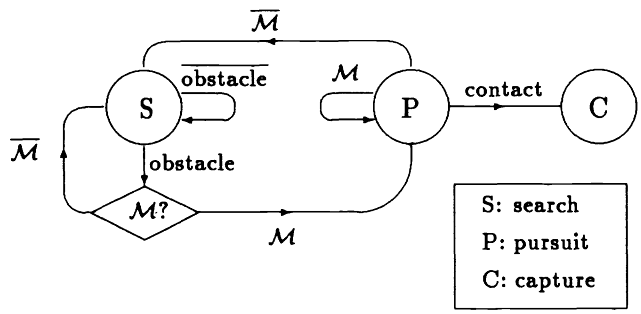


Fig. 2. State diagram for the motion of $R$.

orientation of $\mathcal{M}$'s linear path, measured with respect to $\theta$, is described by the random variable $\Phi$. With complete information about $\mathcal{M}$ (i.e. $r, \theta, \Phi$ and $V_{\mathcal{M}}$), it is ***natural*** to assume that $\mathcal{R}$ desires to intercept $\mathcal{M}$ as quickly as possible to minimize its energy expenditure [3]. This then defines the ***unique*** linear trajectory with orientation $\alpha$. From the geometry of Fig. 1, $\alpha$ and $t_c$ are found to equal

$$\alpha = \theta + \sin^{-1}\left[\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\sin\Phi\right] \tag{1}$$

$$t_c = \frac{r\left[\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\cos\Phi + \sqrt{1-\left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\right)^2\sin^2\Phi}\right]}{V_{\mathcal{R}}\left[1-\left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\right)^2\right]} \tag{2}$$

Note that Eq. (2) indicates that prey capture occurs in finite time only if $V_{\mathcal{M}} < V_{\mathcal{R}}$. With perfect knowledge, capture time is independent of $\theta$ and is maximum when $\Phi = 0$. Biologically, moths that flee directly away from the bat take the longest time to capture.

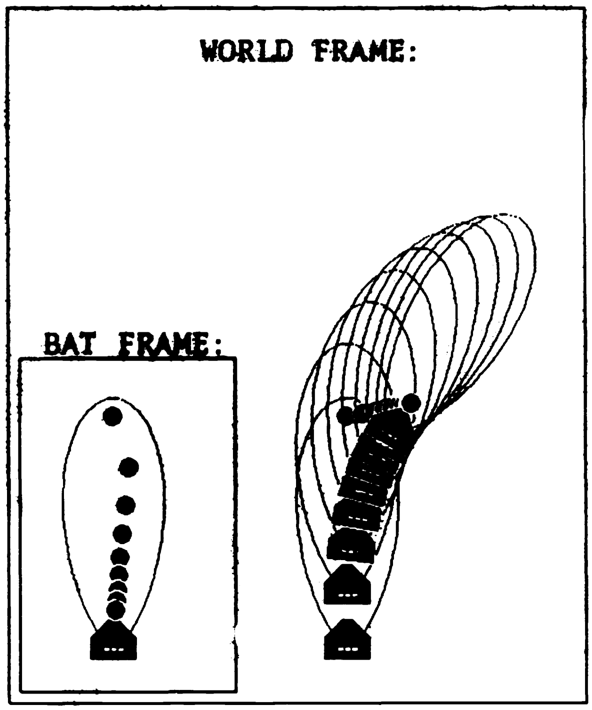


Fig. 3. A simulation example showing two frames of reference in the prey-capture problem.

For obtaining quantitative results, let us consider the case in which $\Phi$ is uniformly distributed over the interval $-\frac{\pi}{2} \leq \Phi \leq \frac{\pi}{2}$. This restriction of $\Phi$ limits the cases to $\mathcal{M}$ drifting *away* from $\mathcal{R}$, and eliminates the situation in which $\mathcal{M}$ has a velocity component towards $\mathcal{R}$ (suicidal moth). Taking the expected value of the minimum capture time over $\Phi$, we get

$$E_{\Phi}[t_c] = \frac{2r\left[\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} + \mathcal{E}\left(\frac{\pi}{2}, \frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\right)\right]}{\pi V_{\mathcal{R}}\left[1 - \left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\right)^2\right]} \tag{3}$$

where

$$\mathcal{E}\left(\frac{\pi}{2}, \frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\right) = \int_0^{\frac{\pi}{2}} \sqrt{1 - \left(\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}\right)^2 \sin^2 x}\, dx \tag{4}$$

is the complete elliptic integral of the second kind [4] with $\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}} < 1$. Eq. 3 indicates that the mean capture time increases with $V_{\mathcal{M}}$ and becomes infinite when $V_{\mathcal{M}}=V_{\mathcal{R}}$.

In this ideal case with complete information, capture probability $P_c=1$. This analytical result assuming perfect knowledge will be compared to our simulation and experimental results employing limited information due to measurement noise and a limited active region. We will find that in the limited information case, that occurs in nature, $P_c<1$.

## III. PHASES OF PREY CAPTURE

In our implementation, $\mathcal{R}$'s prey capture strategy consists of three modes, analogous to bat foraging behavior [5]. The state diagram is shown in Fig. 2. The initial state is the *search mode* S where $\mathcal{R}$ scans the environment at equal time intervals. This type of scan is observed frequently in nature and is called *saltatory*

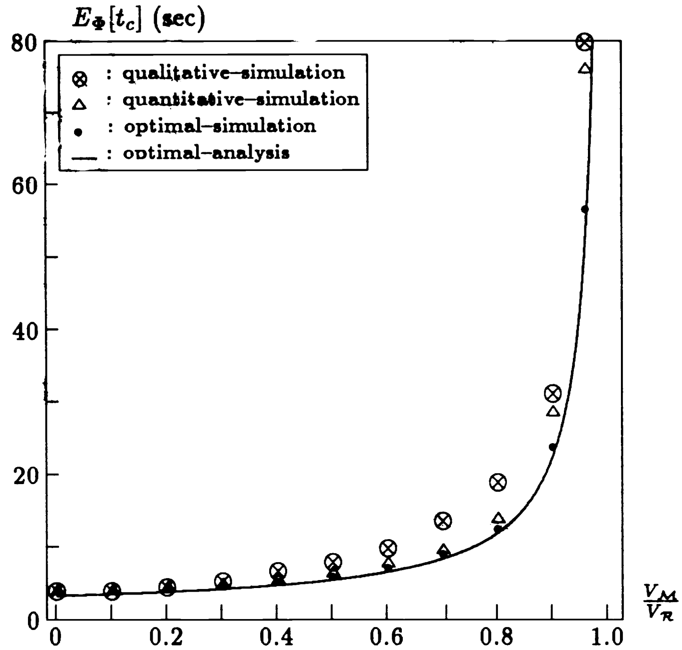


Fig. 4. Mean capture time vs. $\frac{V_\mathcal{M}}{V_\mathcal{R}}$ for $V_\mathcal{R}$=50 cm/s, $T_s$=1 s, $r$=200 cm, $\theta$=0°, and $\beta$=10°.

*search* [6]. In our current implementation, $\mathcal{R}$ scans by rotating the sensor system through a fixed angle while being stationary and then performs a translation in the direction bisecting the angle. The most efficient translation distance traveled between two scans is related to both the direction of the move and the shape of the region being searched: to maximize the efficiency of the search, $\mathcal{R}$ moves just far enough to avoid rescanning a previously searched region while minimizing the unsearched area [6].

When $\mathcal{M}$ is detected, $\mathcal{R}$ switches to the ***pursuit mode*** P. In P, $\mathcal{R}$ moves to reduce the range $r$ and azimuth $\theta$ to zero. When $r$ and $\theta$ are sufficiently small, $\mathcal{M}$ is considered captured and the process terminates in the capture mode C. However, it is not unusual for $\mathcal{M}$ to move out of the active region after it has been detected (not all moths are caught by bats). In this case, $\mathcal{R}$ returns to S by rotating in the direction where $\mathcal{M}$ was last detected.

## IV. RELATIONSHIP OF SENSING AND CONTROL

This section explores the relationship of sensing and control in robot systems that are to perform a task. Since the measurements of $\mathcal{M}$'s location influence the actions of $\mathcal{R}$, prey capture can be viewed in terms of a control problem [8]. To accomplish prey capture, there are a variety of options for controlling the action of the pursuer $\mathcal{R}$ (and possibly $\mathcal{M}$ for sensory equipped moths, although not considered here). Each option has its own trade-off in performance and complexity. Through analysis, simulation and experimental verification using our two mobile robots, an understanding of the sensing and control issues can be gained that allow prey capture to be accomplished.

In this paper, two different control strategies for the pursuit mode are described that use different levels of information. Both methods are memoryless, assume no knowledge of $\mathcal{M}$'s motion (nonparametric), and extract information sequentially from the environment. $\mathcal{R}$'s information about $\mathcal{M}$ is obtained at each scan instant and consists of noisy measurements of $r$ and $\theta$ whenever $\mathcal{M}$ is within the active region. Hence, the

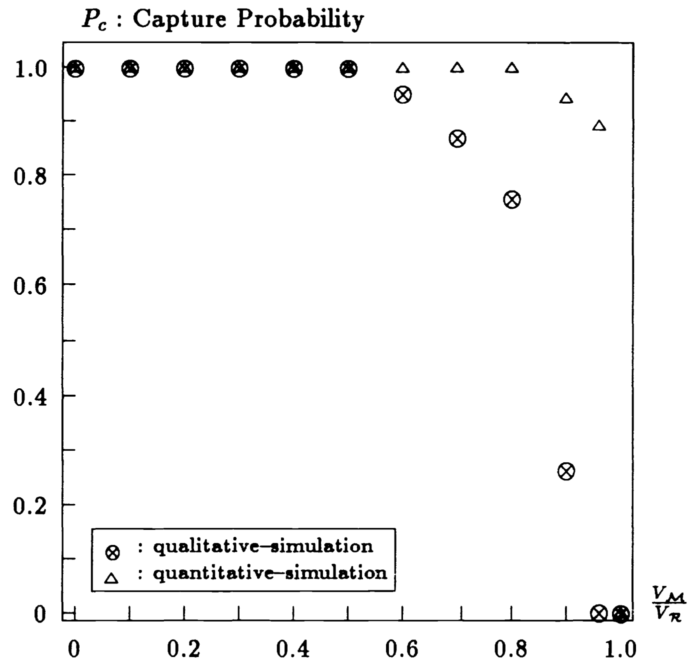


Fig. 5. Capture probability vs. $\frac{V_\mathcal{M}}{V_\mathcal{R}}$ for $V_\mathcal{R}$=50 cm/s, $T_s$=1 s, $r$=200 cm, $\theta$=0$^o$ and $\beta$=10$^o$.

complete knowledge assumption in Sec. II. used to derive the lower bound on capture time is an idealization.

### A. *Qualitative Information*

To address the so-called ***purposive imaging problem*** [9], the minimal information required to achieve prey capture was determined. This result is important for understanding how to implement a practical, economic and efficient robot system that is to accomplish a particular task. Previous results by others [2, 10] indicate that successful prey capture by $\mathcal{R}$ that is faster than $\mathcal{M}$ can be accomplished by employing only the direction $\theta$ of $\mathcal{M}$ relative to $\mathcal{R}$. Our results are less restrictive, indicating that only the binary information that $\mathcal{M}$ is either to the right or left of $\mathcal{R}$'s line-of-sight is sufficient for capture. Our system's corresponding *ad hoc* response to this information that the prey is on the right or left by $\mathcal{R}$ is a rotation by a fixed angle $\beta$ to the right or left. This minimum-information/minimum-response *bang-bang* strategy is the least sophisticated successful method of accomplishing capture that we have investigated.

### B. *Quantitative Information*

Having an estimate of range $\hat{r}$ and azimuth $\hat{\theta}$, determined from the most recent echoes, $\mathcal{R}$ moves toward the estimated location of $\mathcal{M}$ within the active region. Due to the echo travel time, delayed estimates of $\mathcal{M}$'s position are observed. Given the range and azimuth estimates, $\mathcal{R}$ makes a rotation that centers the beam on $\mathcal{M}$'s current location and moves forward. In this way, the accuracy characteristics of the sonar system are exploited: the prey is positioned in the most sensitive part of the beam and the signal-to-noise ratio of the next echo is improved by decreasing the range (reduced $1/r^2$ loss). In the capture process, this procedure is usually repeated several times, updating $\mathcal{M}$'s location after each iteration. This is a reasonable model given the information available about the bat brain [7].

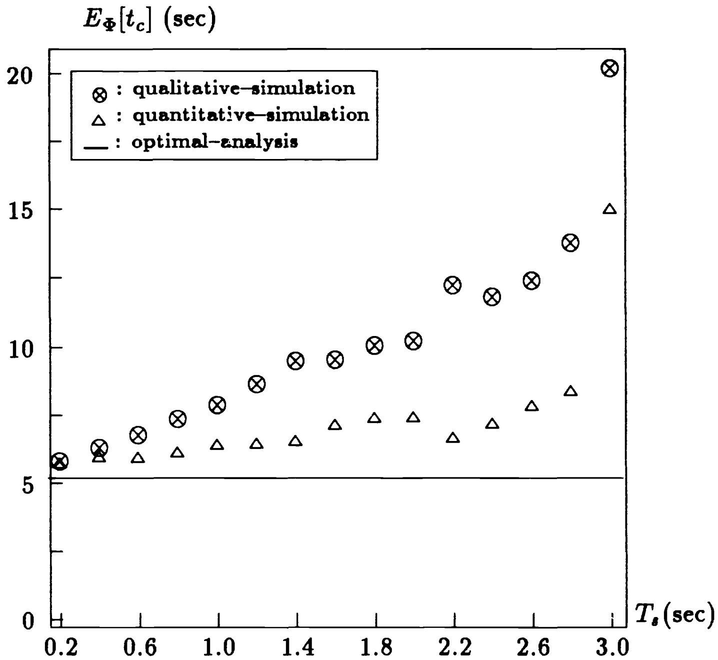


Fig. 6. Mean capture time vs. $T_s$ for $V_{\mathcal{R}}$=50 cm/s, $V_{\mathcal{M}}$=25 cm/s, $r$=200 cm, $\theta$=0° and $\beta$=10°.

## V. SIMULATION STUDIES

To test the importance of available information in a flexible manner, prey capture was simulated by a program that models the physical operation of the actual sonar system. The amplitude characteristics of the echoes as functions of range and azimuth and the system threshold setting allowed the active region to be defined. Additive random errors to the TOF measurements simulated noise effects. Dynamic equations for the motion of the physical robots $\mathcal{R}$ and $\mathcal{M}$ were implemented in the simulations. TOF readings were obtained from the relative location of $\mathcal{M}$ to $\mathcal{R}$ and employed to produce either the qualitative (right/left) or quantitative $(\hat{r}, \hat{\theta})$ information. Delays proportional to the range were added to simulate the time of flight.

A simulation example for the quantitative method for linearly moving prey is shown in Fig. 3. The sonar system is illustrated by a set of three small rectangle on $\mathcal{R}$ and its active region is shown in outline. The simulation starts by $\mathcal{R}$ sensing the location of $\mathcal{M}$, which is randomly positioned along the distant edge of the active region. $\mathcal{M}$ then moves away from $\mathcal{R}$ along a linear path having a random orientation. The sequence of $\mathcal{M}$'s locations is shown as solid dots, added with each measurement. In $\mathcal{R}$'s coordinate system, the relative positions of $\mathcal{M}$ are shown as $\mathcal{R}$ reacts to the sonar measurements and closes in for capture.

The simulation commences when $\mathcal{M}$ starts fleeing at random orientation $\Phi$. Usually, $\mathcal{R}$ manages to catch $\mathcal{M}$. If $\mathcal{M}$ escapes out of the active region $\mathcal{R}$ responds by a saltatory search with $\pm 45°$ rotations that cover the maximum possible area. If $\mathcal{M}$ escapes out of the active region and is not detected within 12 cycles of saltatory search, $\mathcal{M}$ is considered not captured, resulting in a decreased $P_c$. Using the qualitative (left-right) information, the bang-bang rotation angle $\beta$ was chosen to be 10° (close to its optimal value as shown below). One hundred realizations, each with a random $\theta$ and $\Phi$ were generated to evaluate the cost incurred on performance when different levels of information are extracted from the sensor system.

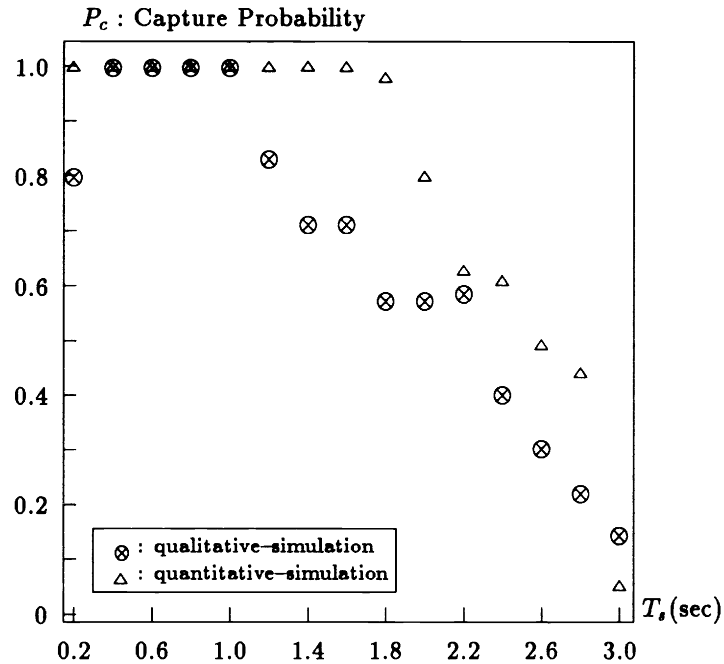


Fig. 7. Capture probability vs. $T_s$ for $V_{\mathcal{R}}$=50 cm/s, $V_{\mathcal{M}}$=25 cm/s, $r$=200 cm, $\theta=0^o$ and $\beta=10^o$.

## VI. PERFORMANCE EVALUATION RESULTS

Quantitative measures of the importance of information are provided by determining the corresponding costs in capture probability and mean capture time for the qualitative and quantitative methods and comparing with the complete-knowledge case. The simulation results are shown in Figs. 4–9. As apparent from the results, the technique based on complete information, described in Sec. II. yields the highest capture probability and minimum capture time. We investigated the penalty incurring on performance when suboptimal, fast and robust techniques are used. From the analysis and verification of our bat-like sonar system, we have observed that the following parameters are important for prey capture.

### *A. Relative Speed $V_{\mathcal{M}}/V_{\mathcal{R}}$*

The speed of $\mathcal{M}$ compared to $\mathcal{R}$ is the most important factor in accomplishing prey capture. Mean capture time and capture probability are shown as a functions of $V_{\mathcal{M}}/V_{\mathcal{R}}$ in Figs. 4 and 5. As expected, quantitative information yields better performance than qualitative information, but there is a wide range of $V_{\mathcal{M}}/V_{\mathcal{R}}$ values over which the performance is comparable. The results indicate that moths with velocities below $0.5V_{\mathcal{R}}$ are always captured with both methods. With quantitative information, moths moving as fast as $0.8V_{\mathcal{R}}$ are successfully captured. For $V_{\mathcal{M}} > 0.8V_{\mathcal{R}}$, $\mathcal{M}$ escapes out of the active region more often and defies further detection. For the qualitative system, when $V_{\mathcal{M}} > 0.5V_{\mathcal{R}}$, the rotation limited by $\beta$ allows more moths to escape from the active region. Hence, the penalty of qualitative systems is not in a significant increase in capture time, but rather in a reduced capture probability.

### *B. Scan Interval $T_s$*

In biological bats, a new pulse is transmitted as soon as the echo from the previous pulse is detected and processed. Then, the scan interval $T_s$ is equal to the TOF measurement plus the processing time, if any, of the echoes. When $r$ is large, the TOF is sufficiently long so that $\mathcal{M}$ can move a significant distance from its measured location in the time between two scans. However, a given linear motion of $\mathcal{M}$ corresponds to

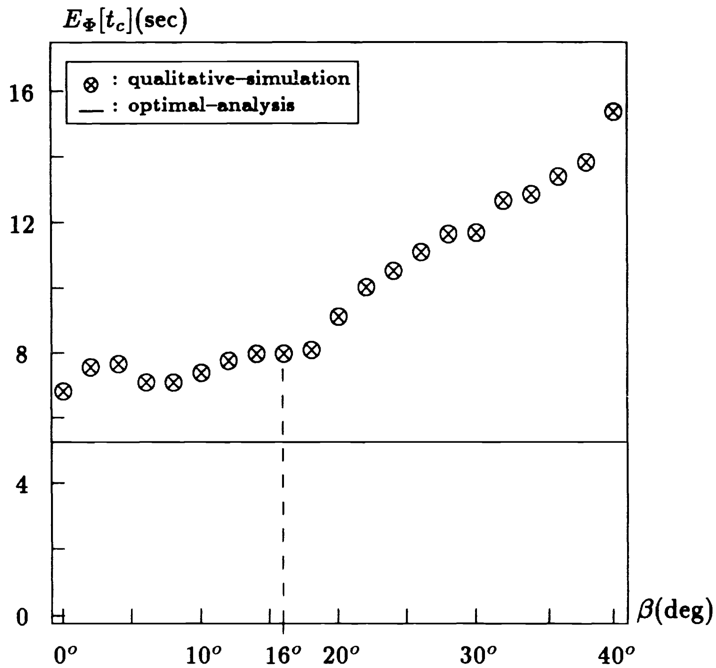


Fig. 8. Mean capture time vs. $\beta$ for $V_{\mathcal{R}}$=50 cm/s and $V_{\mathcal{M}}$=25 cm/s.

a larger change in $\theta$ at nearby ranges than when $\mathcal{M}$ is distant. Since the beam pattern is also narrower at small $r$, if the environment is not scanned frequently enough, $\mathcal{M}$ can easily escape out of the active region. To prevent this, bats set their scan interval $T_s$ proportional to range [7], by transmitting a new pulse as soon as the echo from the previous pulse is detected. Therefore, they scan much more often as they approach the prey. To investigate the effect of the scan interval, $T_s$ was varied by including a hypothetical processing delay in addition to the delay due to TOF. The effect of varying $T_s$ is shown in Figs. 6 and 7. From a control system point of view, the delay in receiving information produces an increase in the open-loop phase response of the system. With both methods, a reduction in $T_s$ reduces the phase lag of the control system. Therefore, the phase margin is increased which in turn allows more open-loop gain in the stable system and, hence, more maneuverability for the bat [11]. Feedback is especially important for the nonlinear bang-bang method which uses only qualitative information. In this case, a high open-loop gain is needed to reduce these nonlinear effects in the system when closing the feedback loop. This explains why the performance of this method deteriorates very quickly with increasing $T_s$.

### C. *Effect of* $\beta$

The performance of the bang-bang algorithm for different values of $\beta$ is shown in Figs. 8 and 9. For these simulations, the saltatory search rotation angle was set equal to $\beta$, instead of the constant value of 45° used in the earlier simulation results. If $\beta$ is too small (or too large), the system is overdamped (or underdamped). An overdamped system does not allow $\mathcal{R}$ to follow $\mathcal{M}$, while an underdamped system causes $\mathcal{R}$ to overshoot $\mathcal{M}$. It is observed that the intermediate value of $\beta$=16° yields the highest capture probability and yet reasonably small capture time. The small values of capture time around $\beta$=0° are due to the observation that only the *easy* are captured.

## VII. EXPERIMENTAL VERIFICATION

Although the simulations are more flexible and efficient, real robots and sensor systems are essential to verify the assumptions. Experiments with the robots in our laboratory have indicated results similar to those

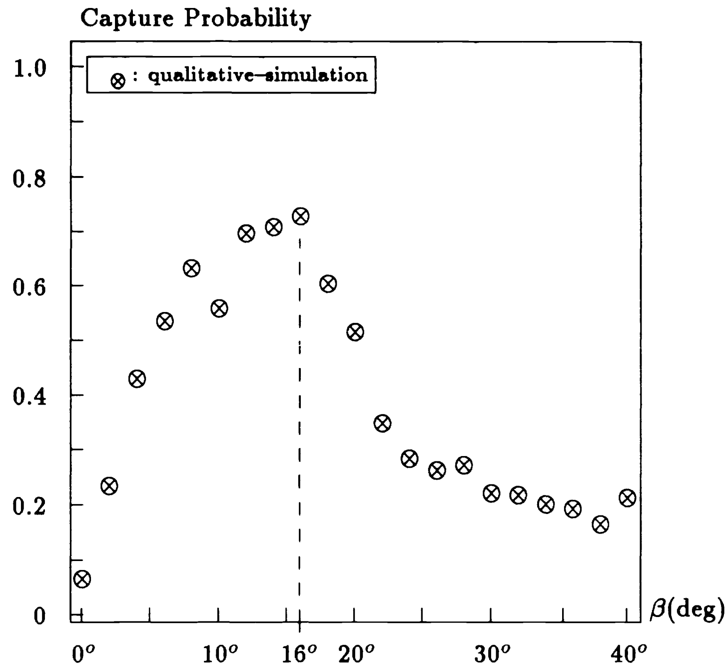


Fig. 9. Capture probability vs. $\beta$ for $V_{\mathcal{R}}$=50 cm/s and $V_{\mathcal{M}}$=25 cm/s.

observed in the simulations.

*A. Description of ROBAT*

A schematic illustration of our robot system is shown in Fig. 10. The mobile robot $\mathcal{R}$ is a position-controlled vehicle, driven by permanent-magnet stepper motors, that has a maximum speed of $V_{\mathcal{R}}$=50 cm/s. It consists of a triangular platform, placed on top of a passive front caster and two stepper-motor wheels. The sonar system is carried on-board with circuitry that provides excitations for pulse transmission and amplifier/filters for echo detection and envelope extraction. A cable carries the analog signal envelopes to an A/D converter having 16 bit resolution at a 50 KHz sampling rate. The transducers are all of one type, the Panasonic ultrasonic ceramic microphone part #EFR-OSB40K2. They are constructed using a piezoelectric ceramic, resonant at 40 KHz and are employed for both transmitting and receiving ultrasound signals. The transducers are located 40 cm above the platform (55 cm above the floor) to eliminate the reflections off the platform. This configuration also eliminates the troublesome secondary reflections that bounce off the floor to $\mathcal{M}$ and back to $\mathcal{R}$, by increasing the additional path length so that multipulse interference does not occur (the direct echo from $\mathcal{M}$ to $\mathcal{R}$ is the first observed signal). The control of the robot and the processing of the signals are accomplished by an IBM PC/XT-286 that extracts information from the sensor data, determines the action to be taken and sends commands to a PDP-11/23 for computing motor control signals.

*B. Description of MOTH*

Although smaller than $\mathcal{R}$, $\mathcal{M}$ is similar in that it is a platform driven by two stepper motors. $\mathcal{M}$ carries a vertically-mounted cylindrical reflector with diameter 16 cm and height 1 m. The cylinder provides an omni-directional reflecting surface that can be observed anywhere in the active region. Its height was dictated by that of the sonar system. Unlike $\mathcal{R}$, $\mathcal{M}$ has no sensory feedback, hence it acted as a *passive prey* unaware of the presence of $\mathcal{R}$. It is independently controlled through a cable by its own PDP-11/23 to move along a linear trajectory with maximum speed $V_{\mathcal{M}}$=50 cm/s. The environment is a 3 m$\times$3 m area free of

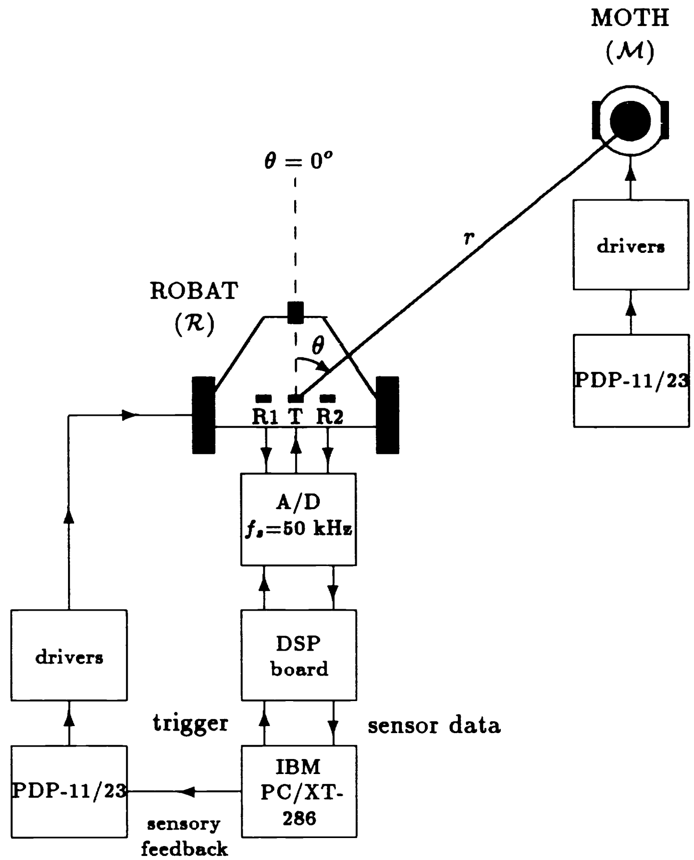


Fig. 10. Configuration of the robotic system.

obstacles other than $\mathcal{M}$.

*C. Experimental Setup*

To verify the analytical results and the comparison of the qualitative and quantitative algorithm performances, experiments were performed in real-time with ROBAT and MOTH. For the following experiments, an average scan interval of $T_s$=1 s. was used to correspond to the value in the simulations. The experiments are started when $\mathcal{M}$ is located at the center of $\mathcal{R}$'s active region along $\theta$=0$^o$ and starts fleeing along a linear path having the random orientation $\Phi$. Algorithms using both qualitative and quantitative information, described in Sec. IV., were implemented on the robotic system. For qualitative information, the bang-bang rotation angle $\beta$ was chosen to be 20$^o$. Five different trials were realized for each algorithm at angles $\Phi=0, \pm 30^o, \pm 60^o, \pm 90^o$ for different speeds of $\mathcal{M}$. For a given $V_{\mathcal{M}}$ and $\Phi$, the capture time was averaged over the five trials to reduce the effect of experimental errors.

*D. Experimental Results and Interpretation*

The results for $V_{\mathcal{M}}$= 8,6,4 cm/s are shown in Fig. 11 as a function of $\Phi$. As expected, the experimentally observed capture time decreases with increasing $|\Phi|$, exhibiting the same trend as the analytically-determined lower bound for capture time. The difference between using qualitative vs. quantitative information is observed not to be severe, but in certain cases the qualitative method may take as long as 8 s. longer to capture the prey. The differences between the experimental results and the analytically predicted minimum capture time are due mainly to measurement errors and the finite active area in the practical system. The

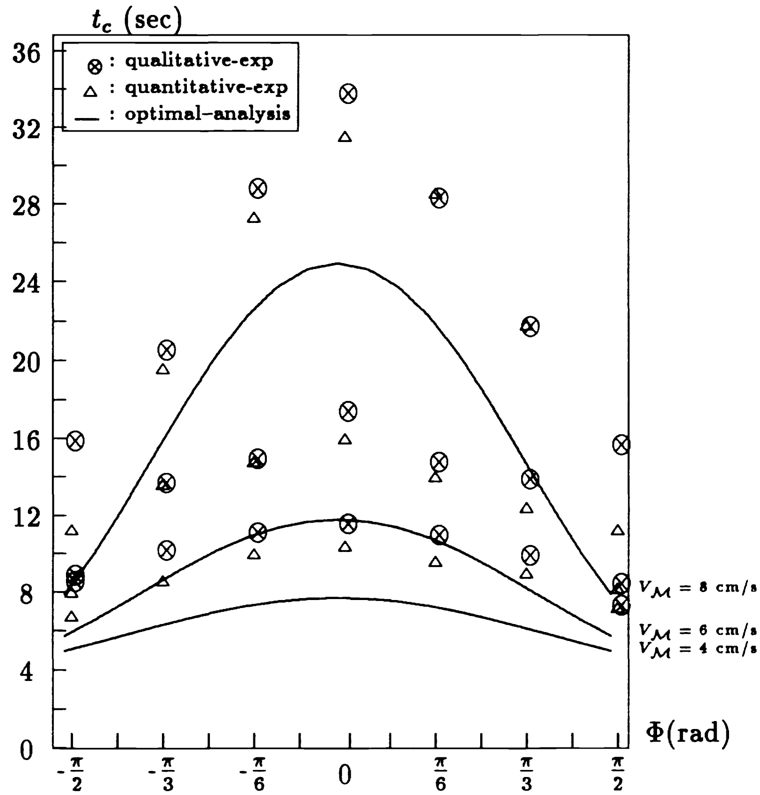


Fig. 11. Experimental results showing capture time $t_c$ as a function of $\Phi$ for $V_{\mathcal{R}}$=9.8 cm/s and $r$=80 cm. The solid line corresponds to the optimal solution.

experimental results were then averaged over $\Phi$. Since five trials were performed for each of the seven $\Phi$ values, the average for a given moth speed is over a total of 35 trials. These results are shown in Fig. 12 as a function of $V_{\mathcal{M}}$. The mean capture time obtained from the qualitative method is only slightly higher than the quantitative method. On the average, prey capture took 2–4 s longer than analytically predicted minimum capture time.

## VIII. SUMMARY

Two different strategies, based on quantitative and qualitative data, were compared for a robotic system to accomplish the prey-capture task, in which a mobile robot equipped with a wide-beam sonar system detects, pursues and captures a second mobile robot. Two measures of performance were considered: the capture probability and the mean capture time when capture occurs. The lower bound for the mean capture time was analytically derived by using the game theory result that assumes complete information about the prey. The most important parameters for the success of prey capture are the speed ratio of the prey to the predator and the scan interval at which the information about the prey location becomes available. It was observed that although binary information about the prey direction was sufficient for capture, quantitative information increased the capture probability and reduced the mean capture time. Both systems exhibited comparable success when $\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}$ <0.5. For faster moths, however, the penalty for qualitative information exhibited itself not in a significant increase in capture time, but rather in a reduced capture probability.

## IX. ACKNOWLEDGMENTS


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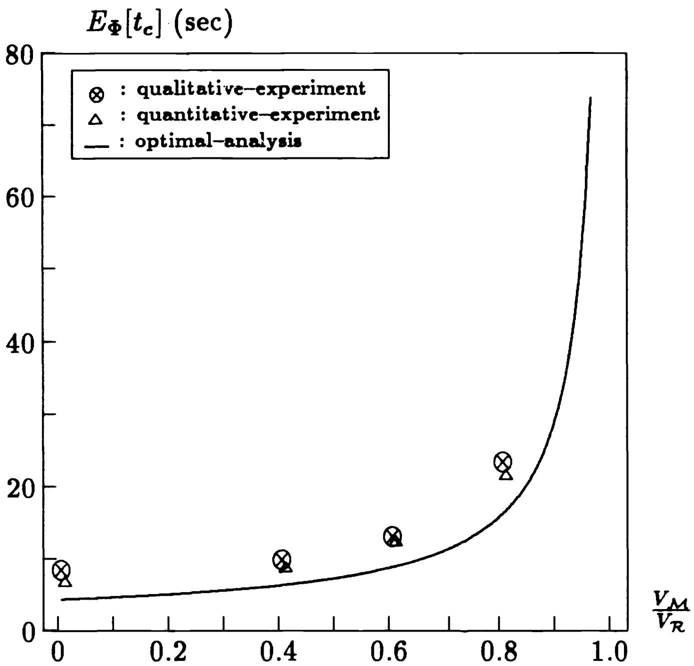


Fig. 12. Experimental results showing mean capture time vs. $\frac{V_{\mathcal{M}}}{V_{\mathcal{R}}}$ for $V_{\mathcal{R}}$=9.8 cm/s, $T_o$=1 s, $r$=80 cm, $\theta=0^o$, and $\beta=20^o$.

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