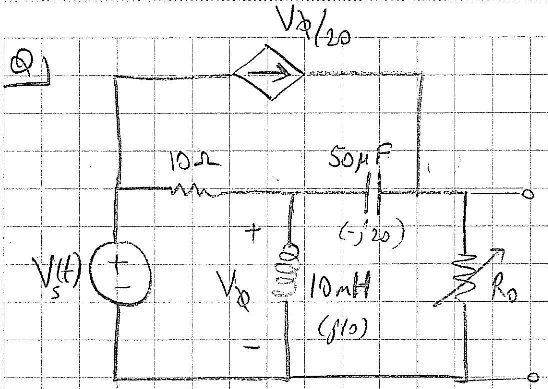
Q)



The variable resistor R_o in the circuit shown in Figure is adjusted until maximum average power is delivered to R_o.

(a) What is the value of R_o?

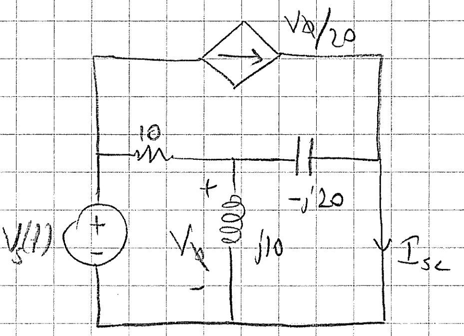
(b) Calculate the average power delivered to R_o

(c) If R_o is replaced with a variable impedance Z_o what is the maximum average power that can be delivered to Z_o

$V_s(t) = 6\cos(1000t)$

(a) $$\frac{V_\phi - 6}{10} + \frac{V_\phi}{j10} - \frac{V_\phi}{20} = 0 \Rightarrow V_\phi\left(\frac{1}{10} - \frac{j}{10} - \frac{1}{20}\right) = \frac{6}{10} \Rightarrow V_\phi = \frac{12}{5}(1+j2)$$

$$V_{oc} = V_\phi + (-j20)\frac{V_\phi}{20} = \frac{12}{5}(3+j)$$



$$I_{sc} = \frac{V_\phi}{-j20} + \frac{V_\phi}{20} = \frac{V_\phi}{20}(1+j)$$

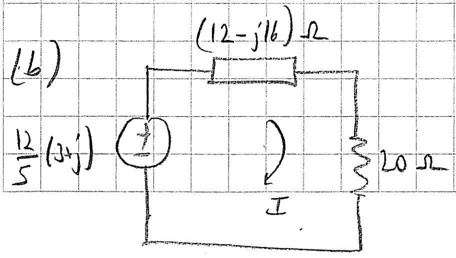
$$\frac{V_\phi - 6}{10} + \frac{V_\phi}{j10} + \frac{V_\phi}{-j20} = 0 \Rightarrow V_\phi = \frac{12}{5}(2+j)$$

$$I_{sc} = \frac{12}{5}(2+j)\frac{1}{20}(1+j) = \frac{3}{25}(1+j3)$$

$$Z_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{\frac{12}{5}(3+j)}{\frac{3}{25}(1+j3)} = 4(3-j4) = 20\angle -53{,}13^\circ$$

$R_o = 20\ \Omega$

(b)



$$I = \frac{3}{20}(1+j3) \Rightarrow i(t) = 0{,}4743\cos(1000t + 71{,}6^\circ)$$

$$p(t) = \frac{i^2(t)}{20} = \frac{1}{20}\left[(0{,}4743)^2\frac{1}{2}\right]\left[1 + \cos(2000t + 143{,}2^\circ)\right]$$

$$P_{ave} = \frac{1}{40}(0{,}4743)^2 = 0{,}0112\ W$$

(c) when $Z_L = (12 + j16)\,\Omega$.

$$I = \frac{12}{5}(3+j)\Big/24 = \frac{1}{10}(3+j) \Rightarrow i(t) = 0{,}3162\cos(1000t + 18{,}43^\circ)$$

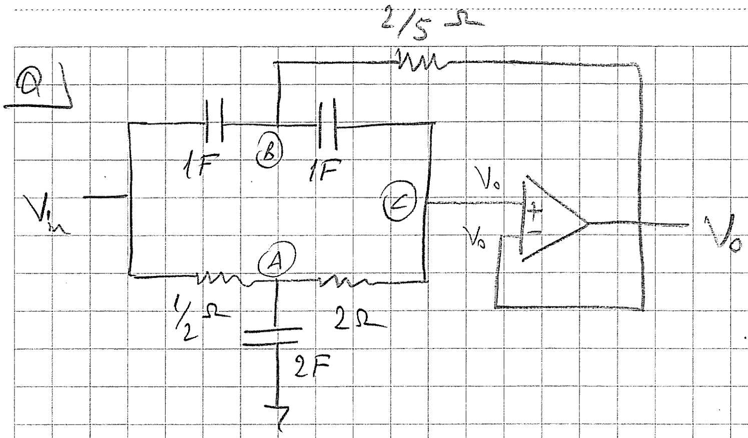
$$V_L = \frac{1}{10}(3+j)(12+j16) = (2+j6) \Longrightarrow V(t) = 6{,}3246\cos(1000t + 71{,}57^\circ)$$

$$p(t) = i(t)\,V(t) = 2\cos(1000t + 18{,}43^\circ)\cos(1000t + 71{,}57^\circ)$$

$$= \left[\cos(-53{,}14^\circ) + \cos(2000t + 90^\circ)\right]$$

$$P_{ave} = 0{,}6 \text{ w.}$$

Q)



(a) Find the transfer function

$$T_V(s) = \frac{V_o(s)}{V_{in}(s)}$$

(b) Find the natural poles

(c) Find the $V_o(t)$ when $V_{in}(t) = \sin(t)\,u(t)$

(a)

Node A

$$\frac{V_A - V_{in}}{1/2} + \frac{V_A}{1/2s} + \frac{V_A - V_o}{2} = 0 \Rightarrow V_A\overbrace{\left(2 + 2s + \frac{1}{2}\right)}^{\frac{5+4s}{2}} = V_{in}(2) + V_o\left(1/2\right) \quad (1)$$

Node B

$$\frac{V_B - V_{in}}{1/s} + \frac{V_B - V_o}{1/s} + \frac{V_B - V_o}{2/5} = 0 \Rightarrow V_B\overbrace{\left(s + s + \frac{5}{2}\right)}^{\frac{5+4s}{2}} = V_{in}(s) + V_o\left(5/2 + s\right) \quad (2)$$

Node C

$$\frac{V_o - V_B}{1/s} + \frac{V_o - V_A}{2} = 0 \Rightarrow V_A(1/2) + V_B(s) = V_o\left(s + \frac{1}{2}\right) \quad (3)$$

from (1)

$$V_A = V_{in}\frac{4}{5+4s} + V_o\frac{1}{5+4s}$$

from (2)

$$V_B = V_{in}\frac{2s}{5+4s} + V_o\frac{5+2s}{5+4s}$$

from (1), (2) and (3)

$$V_{in}\underset{(2)}{\frac{2}{5+4s}} + V_o\frac{1}{10+8s} + V_{in}\underset{(2)}{\frac{2s^2}{5+4s}} + V_o\underset{(2)}{\frac{5s+2s^2}{5+4s}} = V_o\underset{(5+4s)}{\left(\frac{2s+1}{2}\right)}$$

$$V_o\left(\frac{4s^2 + 10s + 1 - 8s^2 - 14s - 5}{10+8s}\right) = -V_{in}\frac{(4s^2+4)}{10+8s}$$

$$V_o\left(-4s^2 - 4s - 4\right) = -V_{in}\left(4s^2 + 4\right)$$

$$\frac{V_o}{V_{in}} = \frac{s^2+1}{s^2+s+1}$$

(b) poles: $s_1 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = s_2^*$

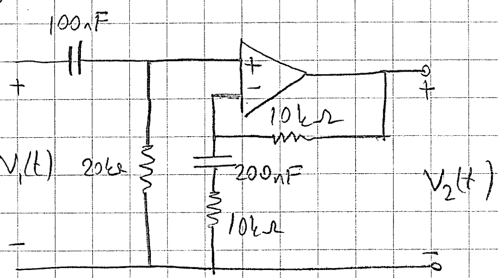
(c) $V_{in}(s) = \dfrac{1}{s^2+1}$

$$V_o(s) = \frac{1}{s^2+s+1} = \frac{j\frac{1}{\sqrt{3}}}{s + \frac{1}{2} + j\frac{\sqrt{3}}{2}} + \frac{-j\frac{1}{\sqrt{3}}}{s + \frac{1}{2} - j\frac{\sqrt{3}}{2}}$$

$$V_o(t) = e^{-\frac{1}{2}t}\,\frac{2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)u(t)$$

Q

(a) Find the voltage transfer function $T_V(s) = V_2(s)/V_1(s)$ of the cascode connection in Figure given below.



$$T_{V_1}(s) = \frac{s}{s+500} \qquad T_{V_2}(s) = 2\frac{s+250}{s+500}$$

$$T_V(s) = \frac{s}{s+500}\, 2\frac{s+250}{s+500}$$

(b) Find $V_2(t)$ when $V_1(t) = \delta(t)$

$$V_2(s) = \frac{2s(s+250)}{(s+500)^2} = 2\left[1 - \frac{750s + 250\,10^3}{(s+500)^2}\right] = 2\left[1 + \frac{\overset{-750}{A}}{s+500} + \frac{\overset{125\,10^3}{B}}{(s+500)^2}\right]$$

$$V_2(t) = 2\delta(t) - 1500\, e^{-500t}\, u(t) + 25\,10^4\, t\, e^{-500t}\, u(t)$$

(c) Find $V_2(t)$ when $V_1(t) = u(t - 2\,10^{-3})$

$$V_2(s) = 2\frac{s(s+250)}{(s+500)^2}\,\frac{1}{s}\, e^{-2\,10^{-3}s} = 2\frac{s+250}{(s+500)^2} e^{-2\,10^{-3}s} = 2e^{-2\,10^{-3}s}\left(\frac{\overset{1}{A}}{s+500} + \frac{\overset{-250}{B}}{(s+500)^2}\right)$$

$$V_2(t) = 2\left[e^{-500(t-2\,10^{-3})} - 250(t - 2\,10^{-3})\, e^{-500(t-2\,10^{-3})}\right] u(t - 2\,10^{-3})$$