HW#5

due 12 July 2010

IMPORTANT In all the plots you produce, do not forget to label the axes properly. Mention all the units (sec, Hz, etc.). Otherwise, you will lose grades.

IMPORTANT If you feel uncomfortable with Matlab, you can study the Matlab tutorial provided in the Appendix for your textbook.

Question 1 - Reconstruction of a continuous-time signal from its samples

Let

$$x(t) = cos(2\pi t) \tag{1}$$

for $0 \leq t \leq 5$ and $x(t) = 0$ outside of $0 \leq t \leq 5$.

Display $x(t)$ in Matlab using the comment **plot** for $0 \leq t \leq 5$. Use a sufficiently small time increment ($\Delta t = 0.01sec$) so that your plots look like continuous.

Next, consider the discrete signal $x[n]$ obtained by sampling $x(t)$ with sampling period T_s such that

$$x[n] = x(nT_s) \tag{2}$$

for $-\infty < n < \infty$, $n \in \mathcal{Z}$. Note that since $x(t)$ is nonzero only within $0 \leq t \leq 5$, only a finite number of samples will be nonzero. Let us call the number of nonzero samples N.

In this question, you will reconstruct $x(t)$ from its samples $x[n]$ ($n = 0, 1, \cdots, N-1$) using three different interpolators.

Write a Matlab program which takes T_s as the input and

- finds N (the number of nonzero samples)
- displays $x[n]$ using the command **stem**
- (**zero order interpolator**) computes and displays (using **plot** and $\Delta t = 0.01$) the following continuous signal for $0 \leq t \leq 5$:

$$x_1(t) = \sum_{n=0}^{N-1} x[n]\phi_1(t - nT_s) \tag{3}$$

where $\phi_1(t) = 1$ for $-\frac{T_s}{2} \leq t \leq \frac{T_s}{2}$ and $\phi_1(t) = 0$ for the given duration. (Also display $\phi_1(t - 2.5)$ within $0 \leq t \leq 5$ separately using **plot** and $\Delta t = 0.01$.)

- (**first order interpolator**) computes and displays (using **plot** and $\Delta t = 0.01$) the following continuous signal for $0 \leq t \leq 5$:

$$x_2(t) = \sum_{n=0}^{N-1} x[n]\phi_2(t - nT_s) \tag{4}$$

where $\phi_2(t) = \frac{t+T_s}{T_s}$ for $-T_s \leq t \leq 0$, $\phi_2(t) = \frac{-t+T_s}{T_s}$ for $0 \leq t \leq T_s$, and $\phi_2(t) = 0$ for the outside of the given duration. (Also display $\phi_2(t - 2.5)$ within $0 \leq t \leq 5$ separately using **plot** and $\Delta t = 0.01$.)

- (**sinc interpolator**) computes and displays (using **plot** and $\Delta t = 0.01$) the following continuous signal for $0 \leq t \leq 5$:

$$x_3(t) = \sum_{n=0}^{N-1} x[n]\phi_3(t - nT_s) \tag{5}$$

where

$$\phi_3(t) = \frac{\sin(\frac{\pi t}{T_s})}{\frac{\pi t}{T_s}} \tag{6}$$

(Also display $\phi_3(t - 2.5)$ within $0 \leq t \leq 5$ separately using **plot** and $\Delta t = 0.01$.)

Run your code and **display** the results for

- $T_s = 0.1$
- $T_s = 0.5$
- $T_s = 1$
- $T_s = 2$

Comment on your observations.
Note: In the report, for this part, you should provide your code, $1 + 4 \times 7 = 29$ plots and your comments.

Question 2 - Nyquist Rate, Aliasing

Let

$$f(t) = \cos(2\pi t) \tag{7}$$

and

$$g(t) = \cos(8\pi t). \tag{8}$$

Display both signals using **plot** for $0 \leq t \leq 4$ and take $\Delta_t = 0.01$. Determine the signal whose frequency is higher?

Notation: Let $x[n]$ denote the discrete signal obtained by sampling a continuous time signal $x(t)$ with sampling period T_s such that

$$x_{T_s}[n] = x(nT_s) \tag{9}$$

for $-\infty < n < \infty$, $n \in \mathcal{Z}$.

Find the Nyquist sampling rates for $f(t)$ and $g(t)$. Let the corresponding Nyquist sampling periods be T_f and T_g, respectively.

- Display $f_{T_f}[n]$ and g_{T_g} for $0 \leq n \leq 100$ using **stem**. What do you observe? Comment.
- Display $f_{1/3}[n]$ and $g_{1/3}$ for $0 \leq n \leq 100$ using **stem**. What do you observe? Comment. Which signal(s) is/are undersampled?
- Display $f_{1/4}[n]$ and $g_{1/4}$. What do you observe? Which discrete signal appears to have the higher frequency? Can you easily find a continuous signal which produces $g_{1/4}[n]$ when sampled with $T_s = 1/4$ but has lower frequency than $f(t)$? Comment.

Note: In the report, for this part, you should provide your code, eight plots and your comments.