EEE 321 Computer Assignment III

FIR filters & Linear Phase Filter Design

Due Date:

July 16, Friday

Q1)

Let

$x(t)=\cos(\pi\alpha t^2)$

and

$x[n]=x(nT_s)$ for n=0,1,2,...,255

Take

$T_s=1/256$

$\alpha=1/(2T_s)$

a) Plot x[n]. What can you say about the frequency of this signal? Comment.

b) Let h[n]=[0.5 0.5] (that is, h[0]=0.5, h[1]=0.5 and h[n]=0 for n≠0,1) denote the impulse response of a system. Pass x[n] through this system (that is, convolve x[n] and h[n]). Let y[n]=x[n]*h[n] denote the resulting output. Compute and plot y[n] for n=0,1,2,...,255. What can you say about this system?Comment.

c) Repeat part b with h[n]=[0.5 -0.5].

d) Repeat part b with h[n]=[0.25 0.25 0.25 0.25].

e) Repeat part b with h[n]=[0.25 -0.25 0.25 -0.25].

f) Compare the impulse response of part b with that of part c, and impulse response of part d with that of part e. Can you recognize a pattern? Comment.

(Note: In the report, for this part, you should provide 5 plots and 6 comments)

Q2)

Again let

$x(t)=\cos(\pi\alpha t^2)$

but this time let

$x[n]=x(nT_s)$ for n=0,1,2,...,4*8192-1

and take

$T_s=1/8192$

$\alpha=1/(8T_s)$

a) Plot x[n]. Store x[n] in an array in Matlab. Let us call this array **x**. Use the command **sound(x)** and listen to the sound that Matlab generates. What can you say about the frequency of the sound that you here? Comment.

b) As in Q1, let h[n]=[0.5 0.5], pass x[n] through this system and compute the resulting output y[n]=x[n]*h[n]. Plot y[n]. Also listen to this output. What can you say about this new sound signal?Comment.

c) Repeat part b with h[n]=[0.5 -0.5].

d) Repeat part b with h[n]=[0.25 0.25 0.25 0.25].

e) Repeat part b with h[n]=[0.25 -0.25 0.25 -0.25].

(Note: In the report, for this part, you should provide 5 plots, and 5 comments)

Q3)

In this part, you are going to design two **REAL VALUED LINEAR PHASE FIR** FILTERs, and you will test your filters with the sound signal that we prepared in Q2.

Below is a quick summary of filter design to mentally aid you during this desing process. If you know what it implies to place a zero or a pole while designing an FIR or IIR filter, you can skip this part quickly. Otherwise study this part and make sure you understand it.

---------------------FILTER DESIGN – BEGINNING OF SUMMARY----------------

FIR or IIR filter design is about choosing the location of zeros and poles on the complex plane. How to choose the location of these poles and zeros? How can we approach the problem? Here is a simple model to help you:

Point 1) As you know, one way to specify filters is through their transfer function H(z). Think about |H(z)| (magnitude of H(z)). This is a real-valued function defined over the complex plane (Real-Imaginary plane). Try to visualize it in your mind. It is like a mountaneous terrain over a flat world. We will design this terrain.

Point 2)Remember that the important part of H(z) is the profile falling on the unit circle ($z=e^{jw}$). This profile determines the frequency characteristics of the filter. Actually, we will design the whole terrain so that the profile on the unit circle behaves as we wish.

Point 3) While designing the terrain as we wish, we have two tools at hand: poles and zeros (for FIR filters, we only have zeros, but for IIR filters, we can use both)

Point 4) If z_0 is a pole of H(z), $|H(z_0)|=\infty$. That is, poles cause |H(z)| to climb to infinity . In other words, when we insert a pole at z_0, we actually place a mountain around z_0. Depending on z_0, this mountain elevates some parts of the unit circle relative to others. For instance, if $z_0=2+j2$, the part of the unit circle in the first quadrant are raised relative to the parts in the other quadrants.

Point 5) If z_0 is a zero of H(z), $H(z_0)=0$. In a sense, adding a zero at z_0 is like creating a crater around that point. Therefore, we supress the vicinity of z_0 by adding a zero there.

Point 6) In this way, we prepare the terrain so that the profile on the unit circle matches our expectations as much as possible.

-------------------------FILTER DESIGN-END OF SUMMARY----------------------------

An M-point FIR filter can be specified with its M+1 coefficients ($h_0,h_1,h_2,\ldots,h_M$). Such a filter has the following transfer function:

$$H(z)=h_0+h_1z^{-1}+h_2z^{-2}+\ \ldots\ +h_Mz^{-M}$$

Notice that H(z) is only undefined at z=0. Therefore, the only pole of the transfer function of an FIR filter occurs at z=0. Also, in general, we can find M different z values for which H(z)=0. (Sometimes a single zero can occur with multiplicity greater than 1, but now we are speaking about a general case)

Therefore, in general, an M-point FIR filter has M zeros and only a single pole at z=0. **Since the pole location is already determined, we can only play with the location of the zeros while designing an FIR filter**, and in this assignment we will do so.

If we were asked to design an **arbitrary** FIR filter, we could place M zeros as we wish. However, we are asked to design a **real-valued linear-phase** FIR filter. We should understand these properties and the constraints such properties **impose on the location of the zeros**.

Study section 7-9 of your textbook. You will notice that,

> For a real-valued linear phase filter, if z_0 is a zero of the transfer function H(z) of the filter, then **$1/z_0$, z_0^* and $1/z_0^*$ are also zeros** of the transfer function.

Taking all of these facts into account,

a) Design a 16-point real valued linear phase FIR lowpass filter (you can choose the cut off as you wish) Determine and provide the filters coefficients. Plot the pole-zero diagram (**zplane** command) and frequency response (**freqz** command).
b) Repeat part a for a 21-point real valued linear phase FIR bandpass filter (you can choose the cut offs as you wish).

Then, process the input that we generated in Q2 with your filters and plot the outputs. Especially compare the output plots to the frequency response plots. Can you recognize a similiarity? Comment.

(Note: In the report, for this part, you should filter coefficients for the two filters, 6 plots, and your comment to the final question)