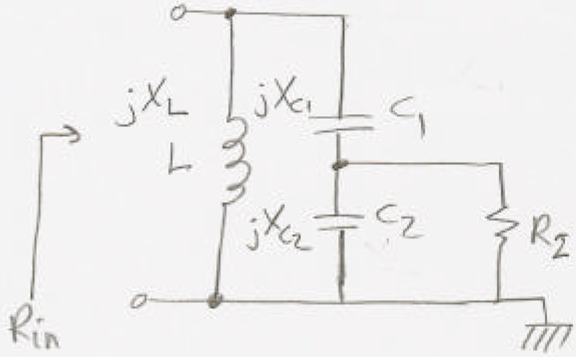
A note on resonant impedence transformer



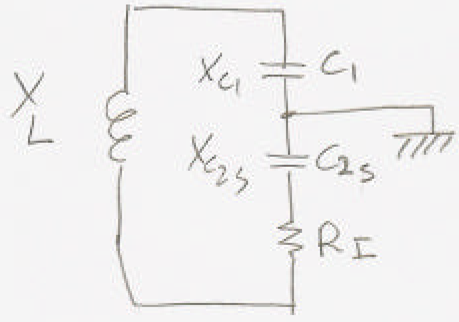
$X_{C_{2S}}$

In this circuit we would like to find the impedance seen in parallel with the inductor, nam R_{in}. The problem is similar to Π-match, with th differences:

a-) The ground is at a different position

b-) We are trying to find the equivalent resistance seen in parallel with L instead of in parallel with C_1

The above circuit is equivalent to grounding changed



$$R_I = R_L / (Q_R^2 + 1)$$

$$X_{C_{2S}} = \frac{Q_R^2}{Q_R^2 + 1} X_{C_2}$$

The circuit resonates at the frequency w_0 defined by

Our aim is to transform the resistance R_L to R_{in} at resonance. If we let

$$K = \frac{R_{P_L}}{R_2} = \frac{R_{in}}{R_2}$$ where R_{P_L} is the equivalent resistance in parallel with L

$$R_{in} = R_{P_L} = \frac{R_I}{(Q^2+1)}$$ where $Q = Q_L + Q_R$

$$(Q^2+1) R_I = R_{in} \quad Q$$

$$(Q_R^2+1) R_I = R_2$$

diving the 2 equations

$$\frac{Q^2+1}{Q_R^2+1} = \frac{R_{in}}{R_2} = K \Rightarrow$$

$$Q^2+1 = K Q_R^2 + K \Rightarrow K Q_R^2 = Q^2+1-K \Rightarrow$$

$$Q_R = \sqrt{\frac{Q^2+1-K}{K}}$$

$$Q_L = Q - Q_R = Q - \sqrt{\frac{Q^2+1-K}{K}}$$

After finding Q_L & Q_R for the required Q & K

R_I can be found by

$$R_I = \frac{R_2}{Q_R^2+1}$$ and $Q_L = Q - Q_R = \frac{X_{C_1}}{R_I}$ and $Q = \frac{R_{in}}{X_L}$

$$(Q^2+1)R_I = R_{in} \Rightarrow R_I = \frac{R_{in}}{Q^2+1}$$

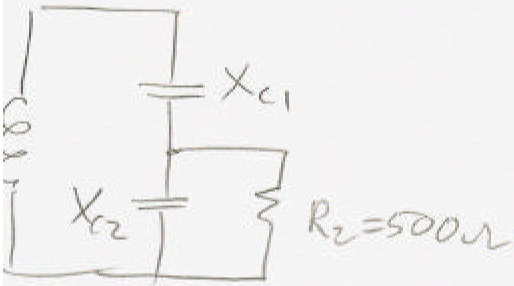
$$(Q_R^2+1)R_I = R_2 \Rightarrow Q_R^2 = \frac{R_2}{R_I} - 1$$

$$Q - Q_R = Q_L = \frac{X_{c1}}{R_I}$$ and finally

$$Q = \frac{R_{in}}{X_L}$$ all unknowns being found.

Example:

Find the reactances of the upward transformer which converts 500Ω into 1000Ω employ[ing] a $Q = 3$.



$$(Q^2+1)R_I = R_{in}$$

$$(3^2+1)R_I = 1000$$

$$\Rightarrow R_I = \frac{1000}{10} = 100\Omega$$

$$(Q_R^2+1)R_I = 500\Omega \Rightarrow$$

$$+1 = \frac{500}{100} = 5 \Rightarrow Q_R = 2$$

$$= 3-2 = Q_L = 1 \Rightarrow X_{c1} = -100j$$

$$2 = \frac{500}{X_{c1}} \Rightarrow X_{c1} = \frac{-500}{2} = -250j$$

$$= \frac{R_{in}}{Q} = \frac{1000}{3} = 333.33j$$