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NOISE

Difference between noise & interference

Broadest definition of noise
- inkluding 50Hz hum
- interference uncorrelated with the signal
  despreaded carrier interference

Audio noise → hiss

Video noise → snowing on the screen

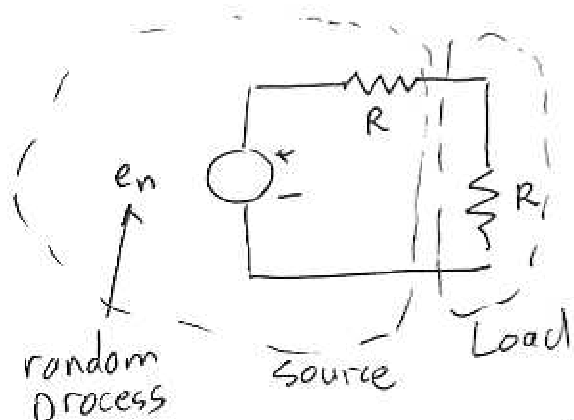
Thermal noise

$P_n = k T_o B$ (available thermal noise power at the output of the resistor R)

B : Bandwidth in Hz

$k = 1.38 \times 10^{-23}$ J/K Boltzmann's constant

white gaussian noise in the practical range of interest (under 80 THz)



with $\overline{e_n}$ being the rms and $\overline{e_n^2}$

$$\frac{\overline{e_n^2}}{4R} = P_n = k T_o B \Rightarrow$$

$$k T_o B \, 4R = \overline{e_n^2}$$

$\left(\overline{e_n^2}\right)^{1/2} = \overline{e_n}$ rms of the noise voltage

The process e_n being ergodic

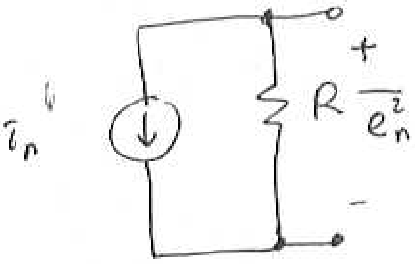
P_n in one hertz BW is about

4×10^{-21} W or -174 dBm at room temp.

or $4\,nV/\sqrt{Hz}$ rms or $16\times10^{-18}\ \frac{V^2}{Hz}$ of 1K resistor

$\equiv$

such that



$$\overline{i_n^2} = \frac{\overline{e_n^2}}{R^2} = \frac{4kT_oBR}{R^2} = 4kT_oGB$$

For 10 kHz BW $\left(\frac{10000\,Hz}{1\,Hz}\right)^{1/2} = 100$

$\Rightarrow \left(\overline{e_n^2}\right)^{1/2}$ i.e. rms noise voltage becomes

$400\,nV_{EMF} \Rightarrow 200\,nV$ on 1K resistor

Example: a) Calculate the rms noise voltage developed on a 50Ω load (receiver) by a 50Ω signal source. The receiver noise BW is 17MHz (IEEE 802.11b receiver) $T = 293\,^oK$

$$\overline{e_n^2} = 4kT_oBR = 4\times1.38\times10^{-23}\times293\times17\times10^{6}\times50$$

$$= 1.375\times10^{-11}\ V^2$$

$$\left(\overline{e_n^2}\right)^{1/2} = 3.7\times10^{-6}\ V_{rms}\ EMF \Rightarrow \text{on } 50\Omega \text{ load } \frac{3.7\times10^{-6}}{2} = 1.85\times10^{-6}\ V_{rms}$$

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b-) calculate the signal-to-noise ratio if the signal has 18.5×10^{-6} Vrms amplitude

$$\frac{18.5\times10^{-6}}{1.85\times10^{-6}} = 10 \Rightarrow 20\,dB \text{ SNR}$$

c-) Express the noise voltage amplitude as a random variable

$$f(x) = \tilde{X}(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\eta)^2}{2\sigma^2}}$$

$$\sigma^2 = \overline{e_n^2} \qquad \sigma : \text{standard deviation} = e_{rms} = \left(\overline{e_n^2}\right)^{1/2}$$

$$\sigma^2 = E\{x^2\} = \overline{X^2} = \overline{e_n^2}$$

one sigma $\rightarrow (0.159 \overset{=F_x(x)}{—} \overset{=F_x(x)}{0.841})$

encompasses $1 - 2\times0.159 = 0.682 \approx 0.7$

two sigma $0.05274 \rightarrow 0.94726$

encompasses $0.89452 \approx 0.9$ (Probability)

three sigma $0.00135 — 0.99865$

encompasses $0.9973 = 1 - 0.0027$

There fore 90% of the time the noise signal is within the $\pm\, 2\times1.85\times10^{-6} = \pm3.7\mu V$ window and for 99.73% of the time it is in the $3\times1.85\mu V$

B (Bandwidth) is which BW?

B is the noise equivalent bandwidth of the device in question.

$$\Delta f \equiv \frac{1}{|H_{pk}|^2} \int_0^\infty |H(f)|^2 \, df$$

H_{pk} is the peak voltage transfer function of the device. For an RC lowpass filter

$$\Delta f \equiv \frac{1}{|H_{pk}|^2} \int_0^\infty \left[\frac{1}{(2\pi f RC)^2 + 1} \right] df$$

$$= \frac{1}{2\pi RC} \operatorname{arctg} 2\pi f RC \Big|_0^\infty = \frac{1}{2\cancel{\pi} RC} \cdot \frac{\cancel{\pi}}{2}$$

$$= \frac{1}{4RC} = \frac{\pi}{2} f_{3dB} = 1.57 \, f_{3dB}$$

!!!

$\Rightarrow$ The brickwall BW of a simple RC lowpass filter is 1.57 times its 3dB BW

Types of Noise

Shot Noise

also known as
Schottky Noise

Comes from the granular nature of the electronic charge.

A potential barrier and a flow of charges against it.

$$\overline{i_{rms}^2} = \overline{i_n^2} = 2q I_{DC} \Delta f$$

$q = 1.6 \times 10^{-19}$ Coulomb charge of electron

Δf in Hz

I_{DC} in Amperes

Gaussian distributed

$18\,pA/\sqrt{Hz}$ for a 1-mA DC current.

Flicker Noise

$1/f$ noise, no obvious mechanism

cell membrane potential,
earth's rotation rate
galactic radiation noise
transistor noise

There is a lack of unifying theory.

Just an empirical formula

$$\overline{N^2} = \frac{K}{f^n} \Delta f \quad \text{for small } \Delta f$$

n is usually close to 1

K is an empirical formula.

$$\overline{N^2} = \int_{f_\ell}^{f_h} \frac{K}{f} df = K \ln\left(\frac{f_h}{f_\ell}\right)$$

carbon composites has the highest flicker noise
metal film and wirewound resistors exhibit the smallest amount.

Popcorn noise

$$\overline{N^2} = \frac{K}{1 + \left(f/f_c\right)^2} \Delta f \quad \text{for small } \Delta f$$

More empirical, caused by contamination
burst noise

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Noise factor

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{N_o}{\underbrace{kT_oB}_{N_i}\,G} = \frac{N_o}{G}\cdot\frac{1}{N_i}$$

$$\frac{S_o}{S_i} = G \Rightarrow$$

$$N_i F G = N_o$$

S_i means all of the signals other than the thermal noise generated by the source impedance.

Noise Figure

$$10 \log_{10} F = \text{Noise figure}$$

Noise Figure and Noise Temperature

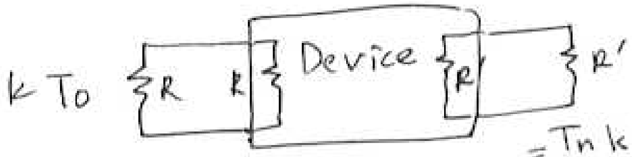
Noise Temperature of an amplifier (or another device) is defined as the increase in the temperature of the source resistance required to account for all of the noise contributors of the amplifier.

$$N_o = F N_i = F k T_o B = kB(T_n + T_o)$$

$$\Rightarrow F T_o = T_n + T_o \Rightarrow T_n = (F-1) T_o$$

N_o = noise o/p referred to the input.

How to measure the Noise Factor (Figure)



$$N_o = N_i F G B = F k T_o B G = \underbrace{\overbrace{(F-1) k T_o B G}^{= T_n k B G}}_{\text{added noise}} + \underbrace{k T_o B G}_{\text{input noise}}$$

If one adds $F k T_o B$ to the input noise, the output noise becomes

$$N_o' = (F-1) k T_o B G + k T_o B G + F k T_o B G$$

$$= F k T_o B G - \cancel{k T_o B G} + \cancel{k T_o B G} + F k T_o B G$$

$$= 2 F k T_o B G \quad \text{(twice the output noise)}$$

If we have a calibrated noise source which adds F times the thermal noise to the input, the o/p noise is doubled.

So we connect a true-rms-voltmeter to the output. Also connect the calibrated noise source to the i/p. We read the scale of the noise source when the o/p increaded by 3dB. It is the Noise figure of the amplifier.

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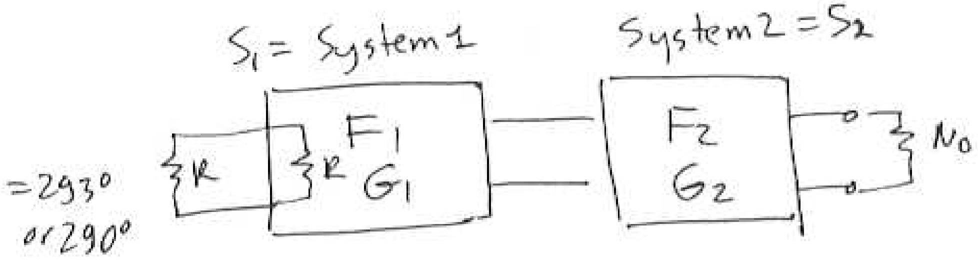
Noise Factor of Cascaded Systems



$$F = \frac{N_o}{k T_o B G} = \frac{k T_o B G}{k T_o B G} + \frac{\Delta N}{k T_o B G}$$

$$= 1 + \frac{\Delta N}{k T_o B G} \Rightarrow (F-1) = \frac{\Delta N}{k T_o B G}$$

F_{12} is the overall noise factor

$$N_o = F_{12} G_1 G_2 k T_o B$$

$$= \underbrace{F_1 k T_o B G_1 G_2}_{S_1 \text{ o/p noise}} + \Delta N_2 = F_1 k T_o B G_1 G_2 + (F_2 - 1) k T_o B G_2$$

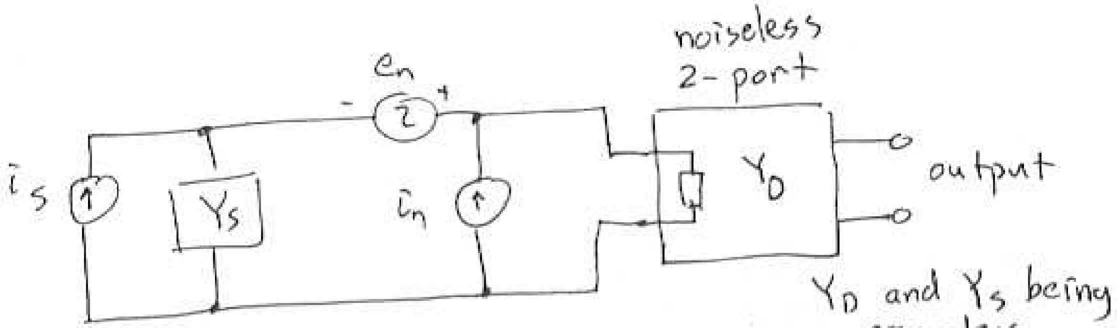
Divide both sides by $k T_o B G_1 G_2$

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1}$$

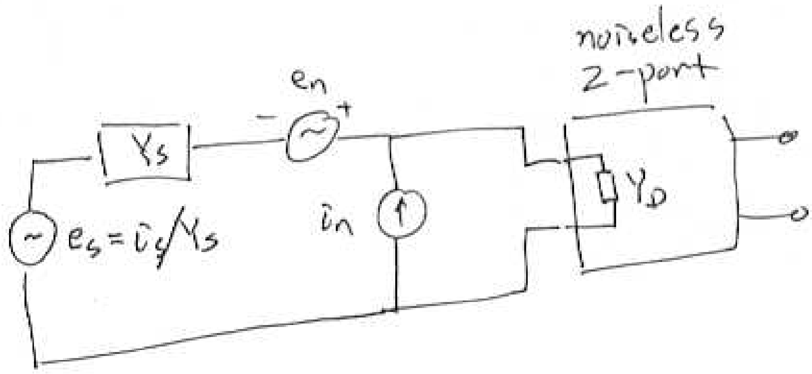
For the general case

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

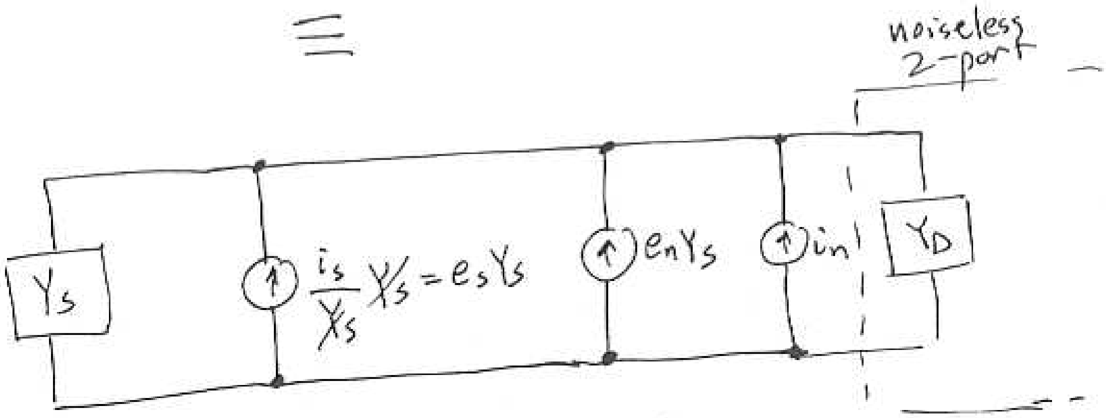
Classical 2-port noise theory



is equivalent to



$\equiv$



where Y_s is the source impedance
and Y_D is the input impedance of the device
i_s represents the noise current of the source

e_n and i_n represent correlated device noise sources referred to the input of the device.

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The noise factor of the device can be written as

$$F = \frac{\text{The total noise refferred to the input}}{\text{The input noise}}$$

and since i_s is uncorrelated with i_n & e_n

$$F = \frac{\overline{i_s^2} + \overline{|i_n + Y_s e_n|^2}}{\overline{i_s^2}}$$

We can always decompose i_n into 2 components as:

$i_n = i_c + i_u$ where i_c is the completely correlated component of i_n with e_n (i.e. proportional with it), therefore we can write

$i_n = Y_c e_n + i_u$ where Y_c is known as the "correlation admittance"

$$\Rightarrow F = \frac{\overline{i_s^2} + \overline{|i_u + (Y_c + Y_s) e_n|^2}}{\overline{i_s^2}} = 1 + \frac{\overline{i_u^2} + (Y_c + Y_s)^2 \overline{e_n^2}}{\overline{i_s^2}}$$

Now letting

$$R_n \equiv \frac{\overline{e_n^2}}{4kT\Delta f}$$

$$G_u \equiv \frac{\overline{i_u^2}}{4kT\Delta f}$$

$$G_s \equiv \frac{\overline{i_s^2}}{4kT\Delta f}$$

note that R_n, G_u, and G_s are real, but the source admittance Y_s and the correlation admittance Y_c are complex

F becomes

$$F = 1 + \frac{G_u + |Y_c + Y_s|^2 R_N}{G_s}$$

letting $Y_c = G_c + jB_c$ and $Y_s = G_s + jB_s$

$$F = 1 + \frac{G_u + (G_c+G_s)^2 R_n + (B_c + B_s)^2 \cdot R_n}{G_s}$$

As it can be seen both from the equation and and the noise model, $Y_s = G_s + jB_s$ affects both $\overline{i_s}$ and the voltage developed by $\overline{i_n}$ and the noise current developed by $\overline{e_n}$. Therefore their ratios (F) is affected by Y_s.

Optimum source impedance (independent of the signal gain)

In order to see the effect of G_s and B_s, let us try to find the optimum source impedance by finding the optimum value of G_s and B_s which minimizes F.

$(B_s + B_c)^2 = 0$ obviously defines the optimum value for $\boxed{B_s = -B_c} = B_{opt}$.

$$\frac{\partial F}{\partial G_s} = 2(G_c + G_s)\frac{R_n}{G_s} - \frac{1}{G_s^2}\left[G_u + (G_c + G_s)^2 R_n + \underbrace{\cancel{(B_c + B_s)^2}}_{=0} R_n\right]$$

because $B_{opt} = -B_c$

equating $\frac{\partial F}{\partial G_s}$ to 0.

$$2(G_c + G_s)\frac{R_n}{\cancel{G_s}} = \frac{1}{G_s^{\cancel{2}}}\left[G_u + (G_c + G_s)^2 R_n\right]$$

$$R_n(G_c + G_s)\left[\cancel{2}G_s - G_c - \cancel{G_s}\right] = G_u$$

$$G_s^2 - G_c^2 = \frac{G_u}{R_N} \qquad G_s = \sqrt{\frac{G_u}{R_n} + G_c^2} = G_{opt}$$

$$R_N G_s^2 - R_N G_c^2 = G_u \Rightarrow R_n G_{opt}^2 = G_u + R_n G_c^2$$

$$R_N G_s^2 = R_N G_c^2 + G_u \Rightarrow R_n G_c^2 = R_n G_{opt}^2 - G_u$$

$$F_{min} = 1 + \frac{G_u + [G_{opt}\ G_c]^2 R_n}{G_{opt}}$$

but $G_u = R_n [G_{opt} - G_c][G_{op} + G_c]$

$$\Rightarrow F_{min} = 1 + \frac{R_n [G_{op} + G_c][G_{opt} - \cancel{G_c} + G_{op} + \cancel{G_c}]}{G_{opt}}$$

$$= 1 + 2R_n [G_{op} + G_c]$$

$$= 1 + 2R_n \left[\sqrt{\frac{G_u}{R_n} - G_c^2} + G_c\right]$$

Now it is time to calculate $F - F_{min}$ in terms of the optimum and the nonoptimum source parameters; $G_s, G_{op}, B_s, B_{opt}$

$$F - F_{min} = \cancel{1} + \frac{G_u + (G_c + G_s)^2 R_n + (B_c + B_s)^2 R_n}{G_s} - \cancel{1} - 2R_n [G_{opt} + G_c]$$

bearing in mind that $B_{opt} = -B_c$ and

$R_n G_c^2 = R_n G_{opt}^2 - G_u$, $F - F_{min}$ becomes

$$\text{min} = \left[\frac{G_u + R_n G_c^2 + R_n \cancel{2G_c} G_s + R_n G_s^2 + (B_s - B_{opt})^2 R_n - 2R_n G_s G_{opt} - 2\cancel{R_n G_s} G_c}{G_s}\right]$$

$$\text{min} = \frac{1}{G_s}\left[\cancel{G_u} + R_n G_{opt}^2 - \cancel{G_u} + R_n G_s^2 - 2R_n G_s G_{opt} + R_n (B_s - B_{opt})^2\right]$$

$$F = F_{min} + \frac{R_n}{G_s}\left[(G_{opt} - G_s)^2 + (B_s - B_{opt})^2\right]$$

$$F = F_{min} + \frac{R_n}{G_s}\left[(G_s - G_{opt})^2 + (B_s - B_{opt})^2\right]$$

For a given F_{min}, B_{opt}, G_{opt} R_n and the locus constant F is a circle distorted by $\frac{1}{G_s}$ for all values of non zero G_s and large G_{opt} we can say that in the vicinity of G_{opt} and B_{opt} constant F locus is a circle.

Our aim is usually to optimize the signal to noise at the output of the front-end amplifier. One way to reach the target is to maximize the signal by showing the maximum-power-transfer impedance. The other way is to minimize the noise by minimizing the noise factor of the amplifier. As B_{opt} for noise is equal to $-B_c$ which is not generally equal to the reactive part of the source impedance. The same is true for G_s. For arbitrary values of G_u, R_n and G_c which are all transistor parameters, the optimum source impedance for F_{min}, namely G_s is not necessarily the optimum source impedance for the maximum power transfer.

In communication receivers, the optimum match between the antenna and the receiver input can be driven by the noise temperature of the antenna (the noise coming from the environment) rather than both the optimum power match and the minimum F requirement. The noise received and delivered by the antenna may be so high that the actual signal-~~to~~-noise ratio is driven by the environment rather than the receiver, which is the case in HF communications. In HF receivers the intermodulation characteristics of the receiver has the utmost importance and the rest of the requirements have a minimal effect on the design. In VHF and UHF, the man-made noise is usually dominant in urban areas and still dominates the design.

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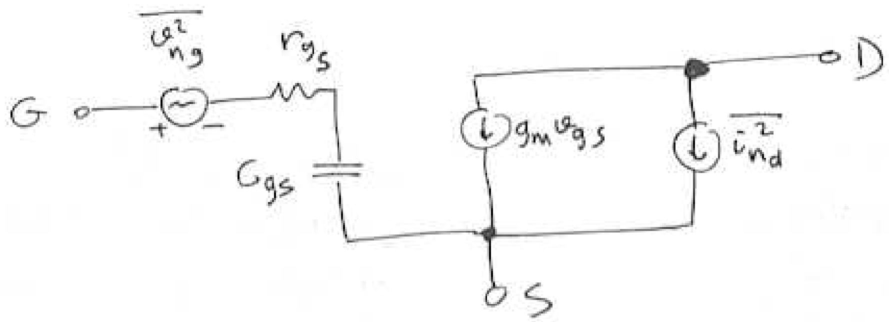
⑦

LNA (Low Noise Amplifier) Design

The MOSFET noise model



$$\overline{i_{nd}^2} = 4kT\gamma g_{do}\Delta f + \frac{K_1}{f}\Delta f$$

$$\overline{v_{ng}^2} = 4kT\delta r_g \Delta f \quad \text{or} \quad \overline{i_{ng}^2} = 4kT\delta g_g \Delta f$$

where $r_{gs} = \dfrac{1}{5g_{do}}$ $\qquad g_g = \dfrac{w^2C^2}{5g_{do}}$

where g_{do} is the drain conductance at $V_{ds} = 0V$.

γ is at the order of 1 to 3.

δ could be around 4-6 at short channel devices.

The gate noise has a correlated component with the drain noise which is instantenously proportional with the drain noise.

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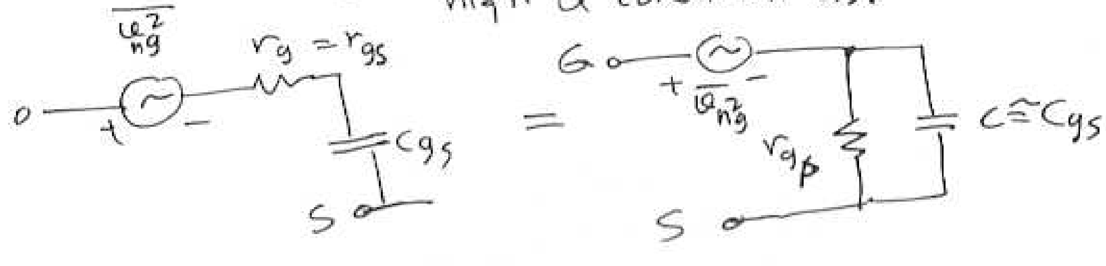
The gate circuit in the MOSFET model can be converted to the parallel form by first converting the series circuit to parallel under high Q condition as:

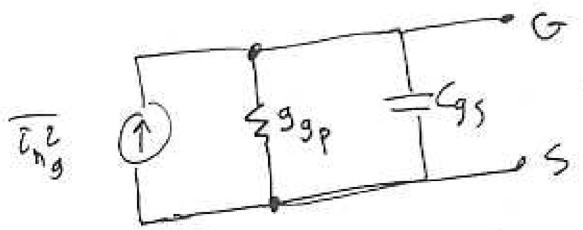


where $r_{gp} = (Q^2+1)\, r_{gs} \approx Q^2 r_{gs}$ and $r_{gs} = \frac{1}{5 g_{do}}$

$$Q = \frac{1}{r_{gs}\,\omega C_{gs}} = \frac{g_{gs}}{\omega C_{gs}}$$

$$\frac{1}{g_{gp}} = r_{gp} = \frac{g_{gs}^{\cancel{2}}}{\omega^2 C_{gs}^2 \cancel{r_{gs}}} \Rightarrow g_{gp} = \frac{\omega^2 C_{gs}^2}{g_{gs}} = \omega^2 C_{gs}^2 r_{gs}$$

The thevenin's equivalent of the circuit on the right hand side is



where

$$\overline{i_{ng}^2} = \overline{v_{gn}^2}\, |Y|^2$$

$$Y = g_{gp} + j\omega C_{gs}$$

$$\overline{i_{ng}^2} = \overline{v_{gn}^2}\,(g_{gp}^2 + \omega^2 C_{gs}^2)$$

$$= 4kT\delta\Delta f\, r_{gs}\,[g_{gp}^2 + \omega^2 C_{gs}^2]$$

but $g_{gp}^{2} = \frac{w^4 C_{gs}^4}{g_{gs}^2} = w^4 C_{gs}^4 r_{gs}^2 \Rightarrow$

$\overline{i_{ng}^2} = 4kT\delta\Delta f\, r_{gs}\left[w^4 C_{gs}^4 r_{gs}^2 + w^2 C_{gs}^2\right]$

$= 4kT\Delta f\,\delta\, r_{gs} w^2 C_{gs}^2\left[w^2 C_{gs}^2 r_{gs}^2 + 1\right]$

under high Q condition where $Q = \frac{1}{wC_{gs}r_{gs}} \gg 1$

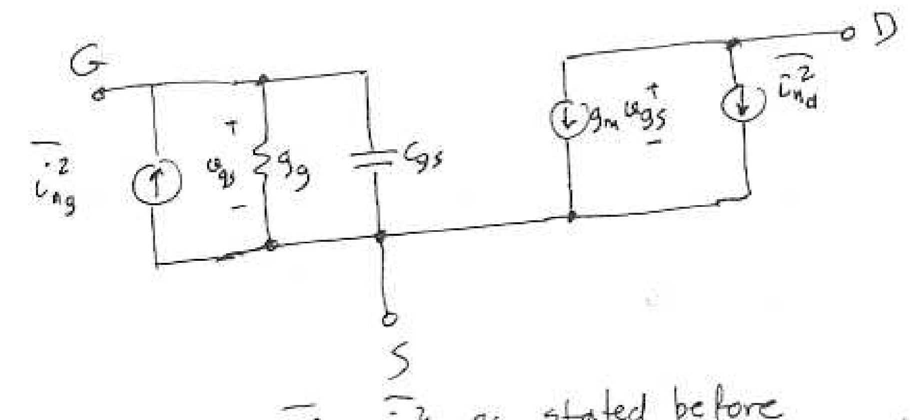
$w C_{gs} r_{gs} \ll 1 \Rightarrow w^2 C_{gs}^2 r_{gs} \ll 1$

$\Rightarrow \overline{i_{ng}^2} = 4kT\delta\Delta f\, r_{gs} w^2 C_{gs}^2$ where $r_{gs} = \frac{1}{5g_{do}}$

or $\overline{i_{ng}^2} = 4kT\Delta f\,\delta\, g_g$

where $g_g = \frac{w^2 C_{gs}^2}{g_{gs}} = \frac{w^2 C_{gs}^2}{5g_{do}}$

Therefore our noise model becomes:



with $\overline{i_{ng}^2}$ $\overline{i_{nd}^2}$ as stated before

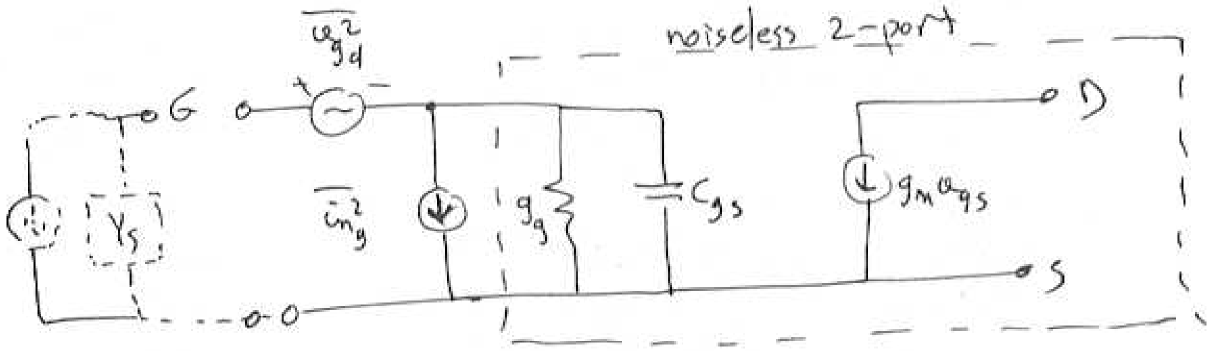
$\overline{i_{ng}^2} = 4kT\Delta f\,\delta\, g_g$ where $g_g = \frac{w^2 C_{gs}^2}{5g_{do}}$

$\overline{i_{nd}^2} = 4kT\Delta f\,\gamma\, g_{do} + \cancel{\frac{K_1}{f}\Delta f}$ neglecting 1/f noise at high freq.

Referring the output noise $\overline{i_{nd}^2}$ to the input by

$$\overline{e_{g\,drain}^2} = \frac{\overline{i_{nd}^2}}{g_m^2} = \frac{4kT\Delta f\,\gamma\, g_{do}}{g_m^2}$$

the equivalent circuit becomes



Therefore arriving at the noise equivalent circuit used in the derivation of the classical noise theory. where (replacing the FET parameters) one obtains:

$$R_n = \frac{\overline{e_n^2}}{4kT\Delta f} = \frac{\gamma g_{do}}{g_m^2} = \frac{\gamma}{\alpha}\cdot\frac{1}{g_m} \qquad \text{where } \alpha = \frac{g_m}{g_{do}}$$

$$G_u = \frac{\delta\,\omega^2 C_{gs}^2 (1-|c|^2)}{5 g_{do}}$$

where c is the correlation coefficient between i_{ng} and i_{nd}

$$G_c \cong 0$$

$$B_c = \omega C_{gs}\left(1 + \alpha |c| \sqrt{\frac{\delta}{5\gamma}}\right)$$

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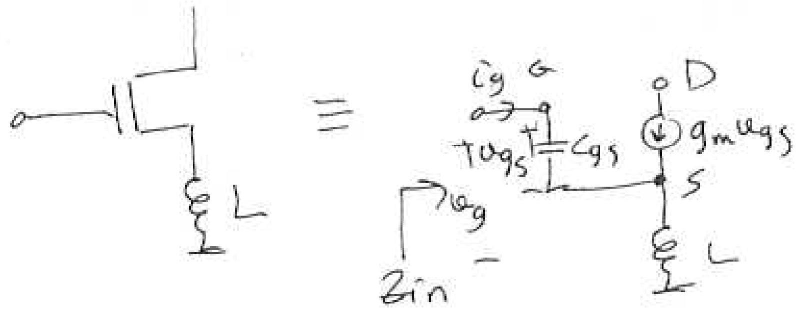
Topology for LNA

Using a common source wideband amplifier biased by resistors or a shunt-series amplifier again biased by resistor would result in higher noise figures because the resistors would significantly contribute to the overall noise. In that case a configuration which contains only a transistor and reactive components would do the trick. A circuit with inductive source degeneration is such a circuit and it also shows real input impedance at resonance.



The input impedance Zin is given by

$$Zin = \frac{v_g}{i_g} = \frac{v_{gs} + Z_L(i_g + g_m v_{gs})}{v_{gs}\, sC_{gs}} = \frac{\cancel{v_{gs}}(1 + Z_L(sC_{gs} + g_m))}{\cancel{v_{gs}}\, sC_{gs}}$$

$$= \frac{1}{sC_{gs}} + sL + \frac{g_m}{C_{gs}} L$$

Obviously at resonance $Zin = \frac{g_m}{C_{gs}} L$

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Example : Design an LNA to operate with 50Ω input impedance with a transistor $g_m = 12\times10^{-3}$ and $C_{gs} = 0.22\,pF$

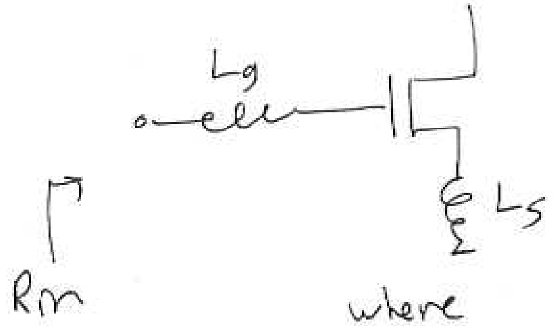
$$Z_{in} = \frac{g_m}{C_{gs}} L \Rightarrow L = \frac{Z_{in}\times C_{gs}}{g_m} = \frac{50\times0.22\times10^{-12}}{12\times10^{-3}}$$

$\Rightarrow L = 0.916\,nH$ and

$$w_0 = \frac{1}{(L C_{gs})^{1/2}} = \frac{1}{(0.916\times10^{-9}\times0.22\times10^{-12})^{1/2}} = 2\pi\times 1.12\times10^{10}\ rad/sec$$

$f_0 = 11.2\ GHz$

In order to decouple the input impedance and the resonant frequency one can add a gate inductance (again a lossless component) so that (to decrease the resonance frequency, not to incerase)



where

$$in = \frac{1}{sC_{gs}} + s(L_g + L_s) + \frac{g_m}{C_{gs}} L_s$$

Now Lg can be chosen to obtain the right frequency.

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The gain at resonance

From the A.C. model
v_{gs} can be found by multiplying the gate current with $\frac{1}{sC_{gs}}$

$$i_g = \frac{v_{in}}{\frac{1}{sC_{gs}} + sL_s + \frac{g_m}{C_{gs}} L_s + sL_g}$$

at resonance i_g reduces to

$$i_g = \frac{v_{in}}{\frac{g_m}{C_{gs}} L_s} \Rightarrow$$

$$i_d = g_m v_{gs} = \cancel{g_m} \frac{v_{in}}{\frac{\cancel{g_m}}{\cancel{C_{gs}}} L_s} \cdot \frac{1}{s\cancel{C_{gs}}} = \frac{v_{in}}{sL_s} \Rightarrow$$

$$G = \frac{i_d}{v_{in}} = \frac{1}{sL_s}$$ but $$R_{in} = \frac{g_m}{C_{gs}} L_s$$ in resonance

$\Rightarrow$

$$L_s = \frac{R_{in} C_{gs}}{g_m}$$

$$G = \frac{1}{s \frac{R_{in} C_{gs}}{g_m}} = \frac{g_m}{s R_{in} C_{gs}} \Rightarrow$$

$$|G| = \frac{g_m}{w_0 R_{in} C_{gs}} = g_m Q$$

where Q is the Q of the LNA unloaded by the source resistance

Alternatively C_{gs} can be increased by adding a shunt capacitor. But, as can be seen from the equation given above for a given frequency and input resistance, the maximum gain can be obtained using the minimum gate-to-source capacitance; namely C_{gs}.

Here, one must careful about the way Q is calculated. Since the gain is defined as

$\frac{i_d}{v_{in}}$, v_{in} measured at the i/p of the amplifier (not the EMF voltage)

in the calculation of Q of the series circuit only the loss component of the load (R_{in}) is taken into account. But when calculating the BW of the amplifier, the whole loss component, i.e.

$$R_s + R_{in} = R_s + \frac{g_m}{C_{gs}} L_s$$

must be taken into account. Which means that the Q of the LNA loaded by the source impedance is half of the Q of the LNA circuit at a properly designed one.