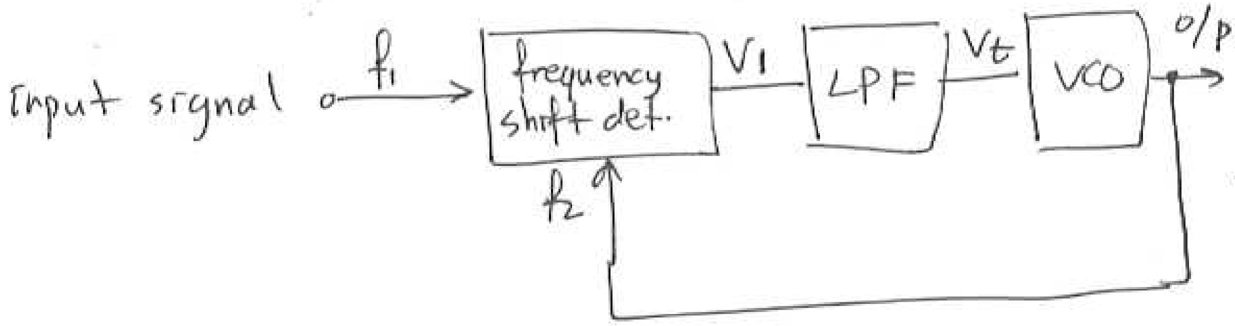
Frequency-locked loop

Historically, there were frequency-locked loop circuits which were used to fine tune L.O. circuits used at reception. They were feedback circuits which compansated for the shifts of the LC tuned local oscillators. The block diagram of a typical frequency-locked loop circuit is given below:

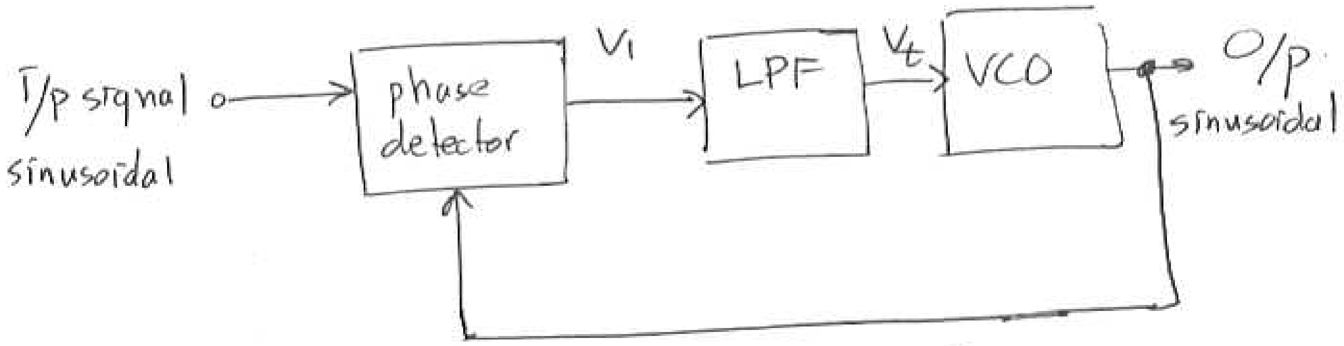


$$V_t = k(f_1 - f_2)$$

A typical application is the "Automatic Frequency Control" feature used at old analog radios. This technique naturally followed the input frequency with some residual frequency error.

Phase-locked loop

If you can lock the phase of the output sinusoidal signal to the phase of the input sinusoidal signal, then the residual frequency error goes to zero, leaving a residual phase error at the most. If we replace the frequency detector with a phase detector, then we obtain a phase-locked loop.



Phase-locked loop $(V_t = (\varphi_{in} - \varphi_0)k)$

In the following analysis, we are going to examine the the behaviour of the phases of the input and output signals and the voltages at the o/p othe phase detector and at the i/p of the VCO (which are proportional to the phase difference), all in the phase domain.

The representation of the basic PLL circuit shown above in the phase domain (assuming that the phase lock is maintained indefinitely and that the circuit starts from 0 initial conditions, i.e. phase error and frequency error (deviation from f_0) for $t<0$ is equal to 0) with the given VCO and phase detector characteristics, which are given below:

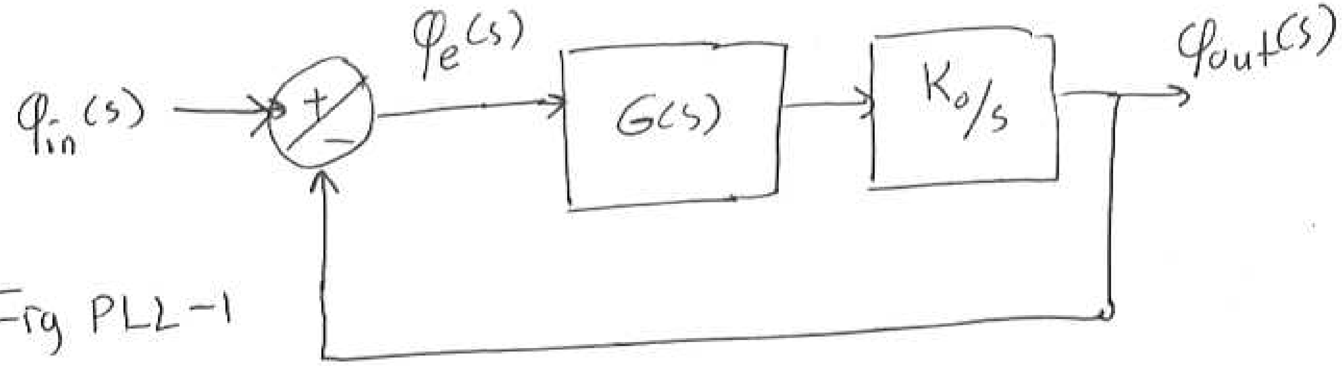


Fig PLL-1

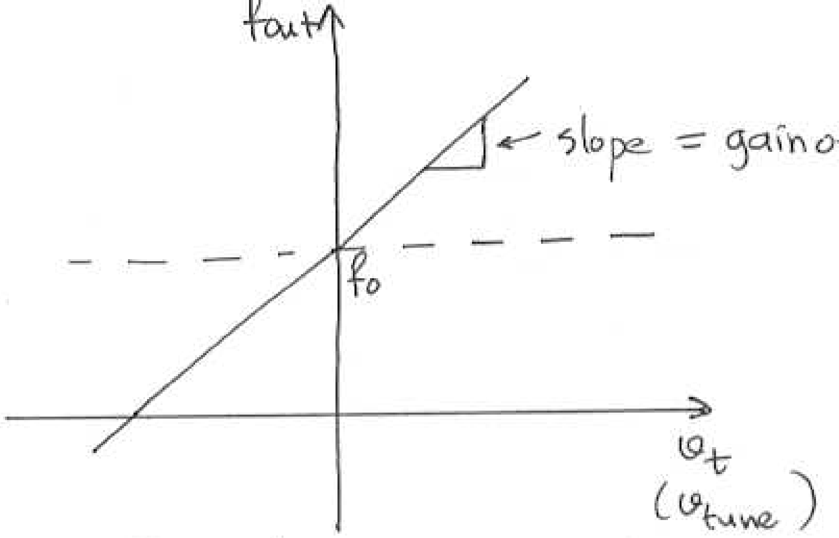
$G(s)$ is the gain function of the loop filter (Note that the book uses $H(s)$)

K_o is the VCO gain (or modulation sensitivity) in rad/sec/volt

K_D is the phase detector sensitivity in Volts/rad and

$$v_e(t) = \varphi_e(t) = K_D\,(\varphi_{in}(t) - \varphi_{out}(t))$$

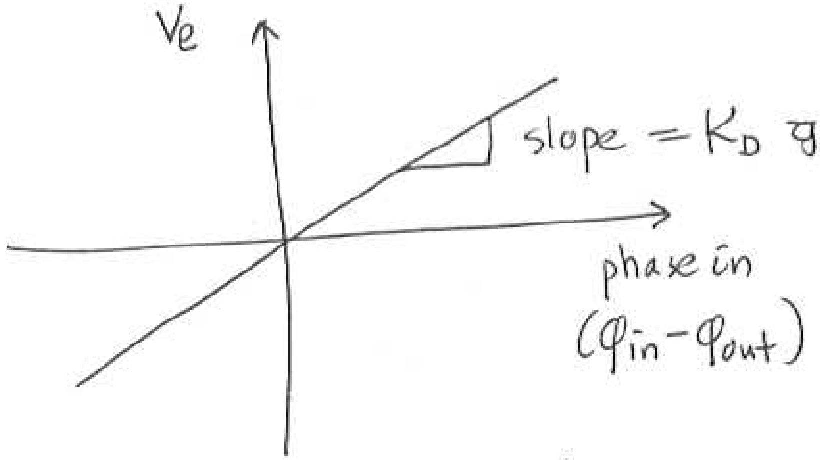
The VCO is represented by K_o/s in the phase domain because phase function is the time integral of the frequency.



← slope = gain of the VCO in Hz/volt

= cycles/sec/volt

The characteristics of an ideal VCO assumed at the analysis.



slope = K_D gain of the phase det. in Volts/rad

The characteristics of an ideal Phase Detector assumed at the analysis. The relationship between the phases and frequencies are defined as follows:

$$F_{out}(t) = \left(f_{out}(t) - f_o\right) \times 2\pi.$$

$$F_{in}(t) = \left(f_{in}(t) - f_o\right) \times 2\pi$$

$$\varphi_{out}(t) = \int_0^t F_{out}(t)\,dt \quad \& \quad \varphi_{in}(t) = \int_0^t F_{in}(t)\,dt$$

assuming that initially $\varphi_{out}(t)\Big|_{t=0} = \varphi_{in}(t)\Big|_{t=0} = 0$

and consequenty

$$w_{out}(t) = \frac{d}{dt}\varphi_{out}(t) = 2\pi\left(f_{out}(t) - f_0\right)$$

$$w_{in}(t) = \frac{d}{dt}\varphi_{in}(t) = 2\pi\left(f_{in}(t) - f_0\right)$$

As the summary in the analysis of the feedback system shown at FIG PLL-1,

we mean the angular frequency deviation from $2\pi f_0$, when we talk about frequency, and we mean the phase deviation from the sinusoidal signal $\cos 2\pi f_0 t$, when we talk about phase.

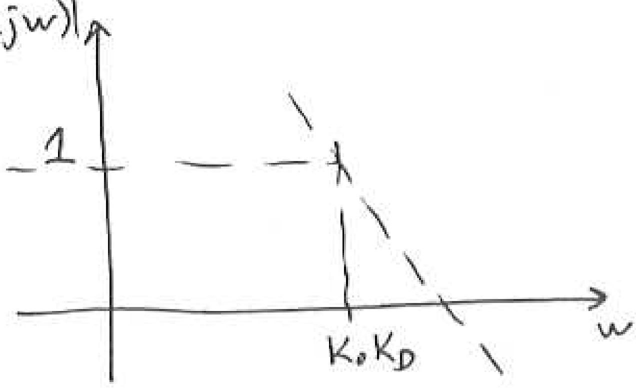
First order loop

$G(s) = K_D \Rightarrow$

$$T(s) = \frac{G'(s)}{1 + H'(s)\,G'(s)}$$

where $G'(s)$ is the forword gaind and $H'(s)$ is the feedback gain

$$T(s) = \frac{K_0 K_D / s}{1 + K_0 K_D / s} = \frac{K_0 K_D}{K_0 K_D + s} = \frac{1}{1 + \frac{s}{K_0 K_D}}$$



Note that $G(s)$ is used to represent the gain of the phase detector

$$\varphi_{out}(s) = \frac{1}{1 + \frac{s}{K_o K_D}} \varphi_{in}(s)$$

$$\varphi_{out}(s) = \frac{F_{out}(s)}{s} \qquad \varphi_{in}(s) = \frac{F_{in}(s)}{s} \Rightarrow$$

$$F_{out}(s) = \frac{1}{1 + \frac{s}{K_o K_D}} F_{in}(s) \quad \text{by substitution}$$

~~Which means that if there are frequency and phase deviations from the input signal at a first order phase-locked loop, the loop acts as a low-pass filter with $w_{3db} = K_o K_D$ to these changes~~

Which means that if there are frequency and phase deviations at the input signal with reference to the reference signal $\cos 2\pi f_0 t$, then these changes are transferred to the output signal with the PLL acting as a simple low pass filter with the breakpoint $w_{3db} = K_o K_D$

Error voltage (phase error) depending on the type of phase input function

The error voltage as a function of the input can be expressed as

$$\varphi_e(t) = \varphi_{in}(t) - \varphi_o(t) \text{ or}$$

$$\varphi_e(s) = \varphi_{in}(s) - \varphi_o(s) = \varphi_{in} - T(s)\varphi_{in}(s)$$

$$= \varphi_{in}(s)\left[1 - T(s)\right) = \varphi_{in}(s)\left[1 - \frac{K_oK_D}{K_oK_D + s}\right]$$

$$\varphi_e(s) = \varphi_{in}(s)\left[\frac{\cancel{K_oK_D} + s - \cancel{K_oK_D}}{K_oK_D + s}\right] = \varphi_{in}(s)\left[\frac{s}{K_oK_D + s}\right]$$

Step input

If we apply a step function as $\varphi_{in}(s)$ which has the laplace transform A/s

$$F(t) = A \quad \text{if} \quad t \geq 0$$
$$= 0 \quad \text{if} \quad t < 0$$

then $\varphi_e(s)$ becomes

$$\varphi_e(s) = \frac{A}{\cancel{s}} \cdot \frac{\cancel{s}}{s + K_oK_D} = \frac{A}{s + K_oK_D}$$

But the inverse laplace transform of $\varphi_e(s)$ is given by

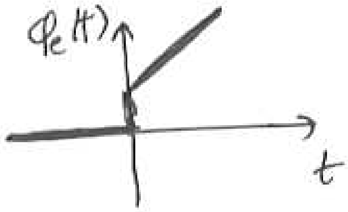
$$\varphi_e(t) = Ae^{-K_oK_Dt} \quad \forall\, t \geq 0$$

As can be seen from the equation, the phase error of a first order phase-locked loop goes to zero as time goes to infinity. Therefore a first order PLL responds to a step input, i.e. a sudden change at the input phase by a exponentially decaying the phase error.

Ramp input

Let the input phase function to be defined as

$$\varphi_{in}(t) = w_i t + \theta \quad \forall\, t \geq 0$$
$$= 0 \quad \forall\, t < 0$$



$$\varphi_{in}(s) = \frac{w_i}{s^2} + \frac{\theta}{s}$$

$$\varphi_e(s) = \frac{s}{s+K_o K_D}\left[\frac{w_i}{s^2} + \frac{\theta}{s}\right] = \frac{w_i}{s(s+K_o K_D)} + \frac{\theta}{s+K_o K_D}$$

$$\varphi_e(s) = \frac{w_i}{K_o K_D}\cdot\frac{K_o K_D}{s(s+K_o K_D)} + \frac{\theta}{s+K_o K_D}$$

Now

$$L^{-1}\left\{\frac{a}{s(s+a)}\right\} = 1 - e^{-at}, \text{ therefore}$$

$$\varphi_e(t) = \frac{w_i}{K_o K_D}\left[1 - e^{-K_o K_D t}\right] + \theta e^{-K_o K_D t}$$

$$= \frac{w_i}{K_o K_D} + \left[\theta - \frac{w_i}{K_o K_D}\right] e^{-K_o K_D t}$$

It can be seen that the steady state phase error for a ramp input, i.e. an input with a different frequency than f_0, is given by:

$$\frac{w_i}{K_0 K_D}$$

You can also find the same result intuitively by reasoning that the VCO must shift to the frequency of the input, i.e. to follow the input frequency $w_i = 2\pi f_i$. The VCO input voltage required to shift the VCO to this frequency is given by $\frac{w_i}{K_0}$, and the phase shift required to produce the voltage $\frac{w_i}{K_0}$ is $\frac{w_i}{K_0 K_D}$.

Higher order input

Any input with a component higher order than t^1 results in unbounded phase error, which means that such input functions can not be tracked by first order inputs.

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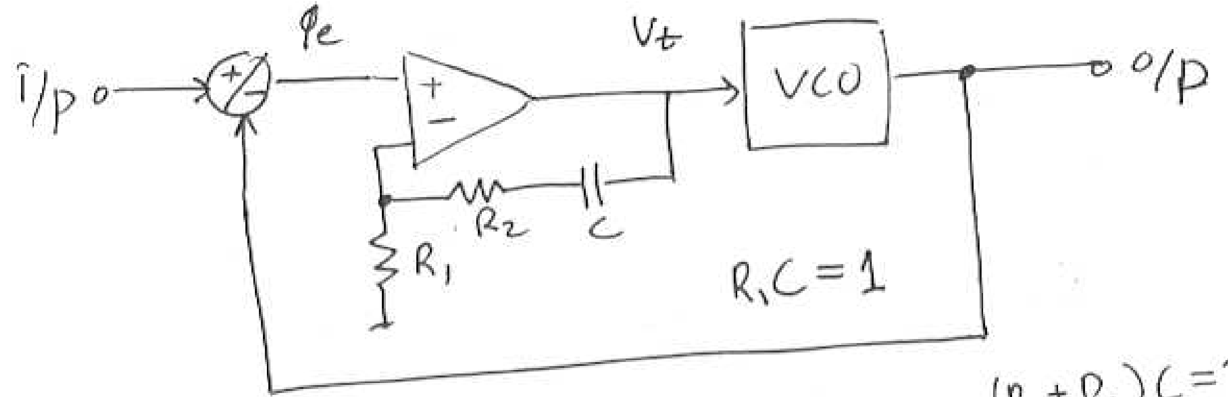
PLL-10

Second order PLL



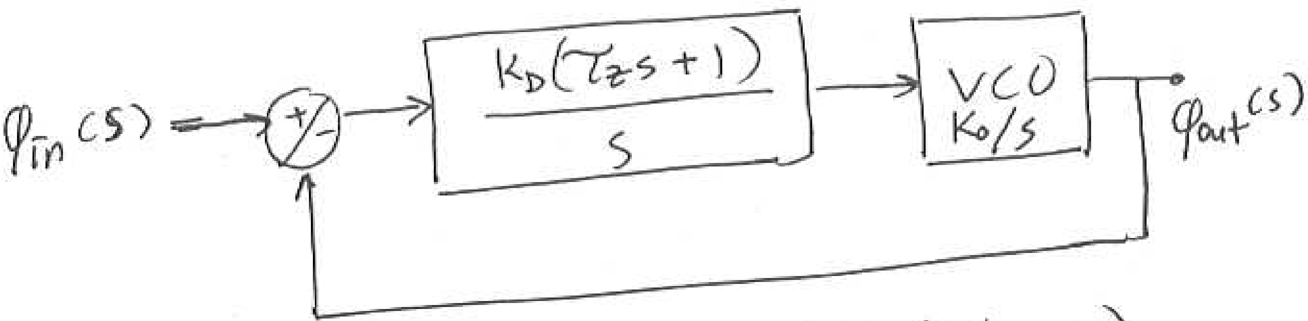
$(R_1+R_2)C = \tau_z$

$$\varphi_e = v_t \cdot \frac{R_1}{\frac{1}{sC} + R_2 + R_1} \Rightarrow$$

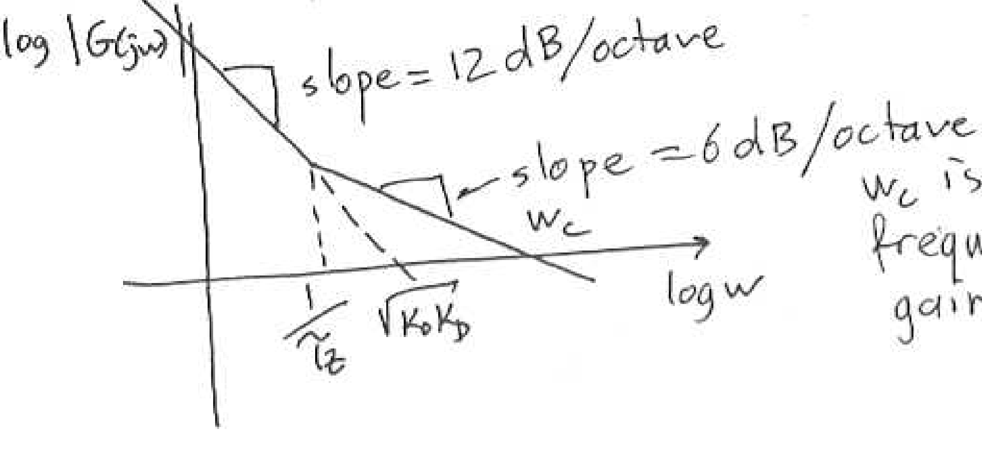
$$\frac{v_t}{\varphi_e} = \frac{1 + sC(R_2+R_1)}{R_1 C s} = \frac{1 + \tau_z s}{s}$$

since $R_1 C = 1$ & $(R_1+R_2)C = \tau_z$

The s-domain equivalent of the phase is



The open-loop gain $G(s) = \dfrac{K_o K_D(\tau_z s + 1)}{s^2}$

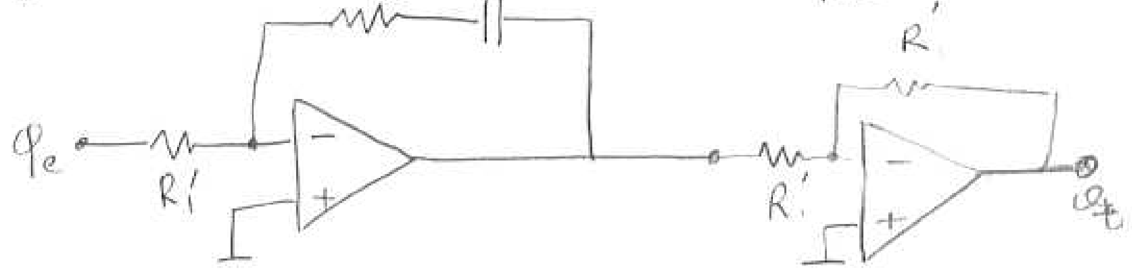


w_c is the cross over frequency of the open-loop gain

Alternate secondorder PLL Configuration

6-1-2005

PLL-10A



$$\frac{v_t}{R_2' + \frac{1}{sC'}} = \frac{\varphi_e}{R_1'} \Rightarrow \frac{v_t}{\varphi_e} = \frac{R_2' + \frac{1}{sC'}}{R_1'}$$

$$= \frac{R_2' s C' + 1}{sC'R_1'} = \frac{1 + R_2'C's}{C'R_1's}$$

$$R_1'C' = \tau_1 \qquad \tau_2 = R_2'C$$

The crossover frequency of the openloop gain

$$\left|\frac{K_0K_D(\tau_z s+1)}{s^2}\right|^2_{s=jw} = 1^2 = 1$$

$$K_0^2K_D^2(\tau_z^2 w^2+1) = w^4$$

let $K_0K_D = w_n^2$ and $w_z = \frac{1}{\tau_z}$

$$\frac{w_n^4}{w_z^2}w^2 + w_n^4 = w^4 \Rightarrow w^4 - \frac{w_n^4}{w_z^2}w^2 - w_n^2 = 0$$

let $w_n^4 = e$ $\quad w^2 = d$

$$w^4 - \frac{e}{w_z^2}w^2 - e = 0$$

$a = 1 \quad b = -\frac{e}{w_z^2} \quad c = -e$

$$d_{1,2} = \frac{-b \pm\sqrt{b^2-4ac}}{2a} = -\frac{b}{2a} \pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$$

$$= +\frac{e}{2w_z^2} \pm\sqrt{\frac{e^2}{4w_z^4}+e} = \frac{w_n^4}{2w_z^2} \pm\sqrt{\frac{1}{4}\left(\frac{w_n^4}{w_z^2}\right)^2 + w_n^4}$$

$$= \frac{w_n^4}{2w_z^2} \pm w_n^2\sqrt{\frac{w_n^4}{4w_z^4}+1}$$

$$w_{1,2} = d_{1,2}^{1/2} = \left[\frac{w_n^4}{2w_z^2} \pm w_n^2\sqrt{\frac{w_n^4}{4w_z^4}+1}\right]^{1/2}$$

$$W_{1,2} = \left[\frac{W_n^4}{2W_z^2} \pm W_n^2 \sqrt{\frac{W_n^4}{4W_z^4}\left(1 + \frac{4W_z^4}{W_n^4}\right)} \right]^{1/2}$$

$$= \left[\frac{W_n^4}{2W_z^2} \pm \frac{W_n^2 W_n^2}{2W_z^2} \sqrt{1 + \frac{4W_z^4}{W_n^4}} \right]^{1/2}$$

$$W_{co} = \frac{W_n^2}{\sqrt{2}W_z} \left[1 + \sqrt{1 + \frac{4W_z^4}{W_n^4}} \right]^{1/2}$$ (crossover freqrency)

The negative quadratic con not be used because it gives rise to an imaginary crossover frequency. Without the lead in angle caused by the term $j\tau_z w + 1$, the loop would have oscillated since the 180° phase shift caused by $(-jw)^2$ at the denominator added to the 180° phase shift created by the negative feedback would have completed the 360° phase shift required at w_c for oscillation (remember w_c is the frequency where the magnitude of the open-loop gain is equal to 1).

Now we can write the phase angle of the term

$$\angle\, j\tau_z w_c + 1 \;=\; \text{phase margin}$$

$$\text{arctg}\; \tau_z w_c \;=\; \text{phase margin}$$

$$\text{arctg}\; \frac{w_c}{w_z} \;=\; \text{phase margin}$$

$$\text{phase margin} = \text{arctg}\left[\frac{w_n^2}{\sqrt{2}\,w_z^2}\left(1+\left(1+\frac{4\,w_z^4}{w_n^4}\right)^{1/2}\right)^{1/2}\right]$$

letting $\frac{w_z}{w_n} = k$

$$\text{phase margin} = \text{arctg}\,\frac{w_n^2}{\sqrt{2}\,k^2\,w_n^2}\left(1+\left(1+4k^4\right)^{1/2}\right)^{1/2}$$

$$PM =$$

$$\text{tg}\,PM = \frac{1}{\sqrt{2}\,k^2}\left[1+\left(1+4k^4\right)^{1/2}\right]^{1/2}$$

where PM denotes the phase margin. Here

$$PM = \text{arctg}\left\{\frac{\left[1+\left(1+4k^4\right)^{1/2}\right]^{1/2}}{\sqrt{2}\,k^2}\right\}$$

can be plotted against k and the necessary phase margin can be selected from the graph.

The cross over frequencies of the closed loop gain of the second order PLL

Now let us find the frequencies for which the closed loop gain is equal to one. These points signify the frequency between which the output phase variations of the 2nd order PLL are greater than the phase variations of the input. This means that phase jitter of phase noise of the input is amplified between these frequencies. First let us calculate the closed-loop transfer function.

$$T(s) = \frac{G(s)}{1 + H(s)\,G(s)} = \frac{\dfrac{K_O K_D(\tau_Z s + 1)}{s^2}}{1 + \dfrac{K_O K_D(\tau_Z s + 1)}{s^2}}$$

$$= K_O K_D \frac{\tau_Z s + 1}{s^2 + K_O K_D(\tau_Z s + 1)}$$

$$= \frac{\tau_Z s + 1}{\dfrac{s^2}{K_O K_D} + \tau_Z s + 1}$$

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$$|T(j\omega)|^2 = 1$$

$$\frac{+\tau_z^2 w^2 + 1}{\left(-\frac{w^2}{K_o K_D} + 1\right)^2 + \tau_z^2 w^2} = 1$$

$$\cancel{\tau_z^2 w^2} + \cancel{1} = \cancel{1} - \frac{2w^2}{K_o K_D} + \frac{w^4}{K_o^2 K_D^2} + \cancel{\tau_z^2 w^2}$$

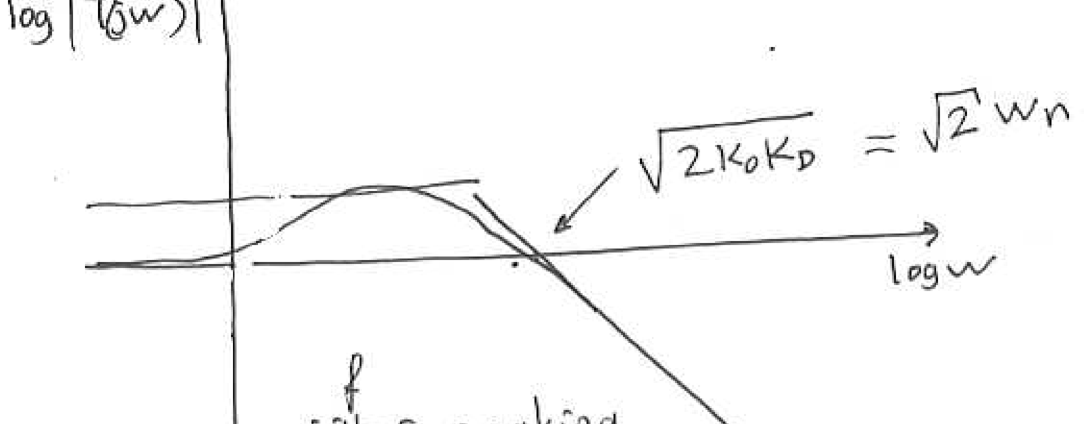
$$\frac{w^4}{K_o^2 K_D^2} - \frac{2w^2}{K_o K_D} = 0 \quad \Rightarrow$$

$$\frac{w^4}{K_o K_D} = 2w^2$$

The above equation has 2 roots

$w^2 = 0$ & $w^2 = 2K_o K_D$

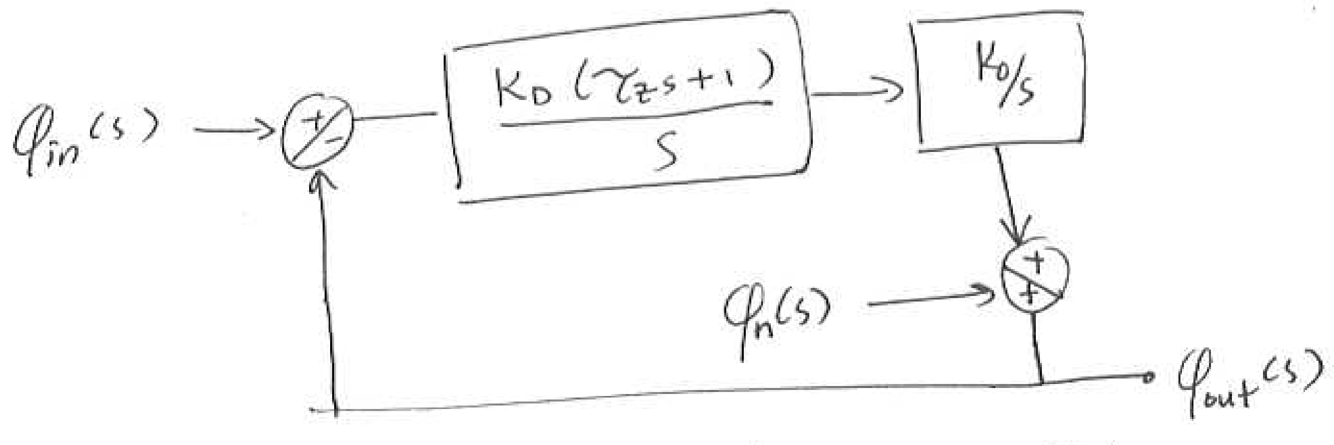
If we plot the closed-loop transfer function w.r.t w,



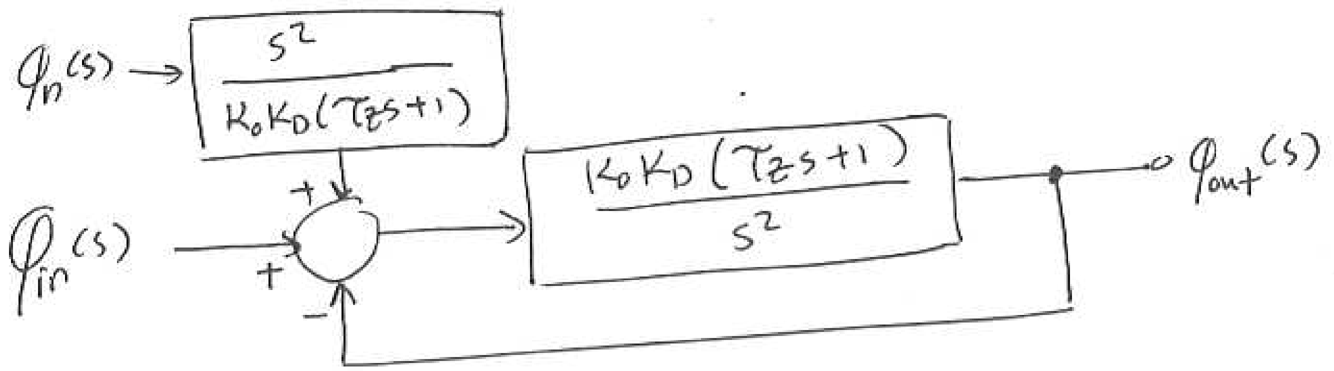
jitter peaking at all frequencies below $\sqrt{2K_oK_D} = \sqrt{2}\,w_n$

At frequencies between 0 and $\sqrt{2K_oK_D}$ the 2nd order PLL emphesizes the input phase noise and disturbances. For frequencies beyond $\sqrt{2K_oK_D}$ it suppresses the input phase noise and actually acts as a low pass filter to input phase noise.

Now let us also see how a 2nd order PLL reacts to VCO phase noise.



Here $\varphi_n(s)$ denotes the phase noise added by VCO. By block diagram algebra,



$$\varphi_{out}(s) = T(s)\,\varphi_{in}(s) + T(s)\,\frac{s^2}{K_oK_D(\tau_z s+1)}\,\varphi_n(s)$$

$$\varphi_{out}(s) = \frac{\tau_z s + 1}{\frac{s^2}{K_oK_D} + \tau_z s + 1}\,\varphi_{in}(s) + \frac{\cancel{(\tau_z s+1)}\,s^2}{\left(\frac{s^2}{K_oK_D} + \tau_z s + 1\right)K_oK_D\cancel{(\tau_z s+1)}}\,\varphi_n(s)$$

$$\varphi_{out}(s) = \frac{\tau_z s+1}{\frac{s^2}{K_oK_D} + \tau_z s + 1}\,\varphi_{in}(s) + \frac{s^2}{s^2 + K_oK_D\tau_z s + K_oK_D}\,\varphi_n(s)$$

The above equation clearly shows that the 2nd order loop acts as a high-pass filter for the noise added by VCO and act as alow pass filter for the input noise.