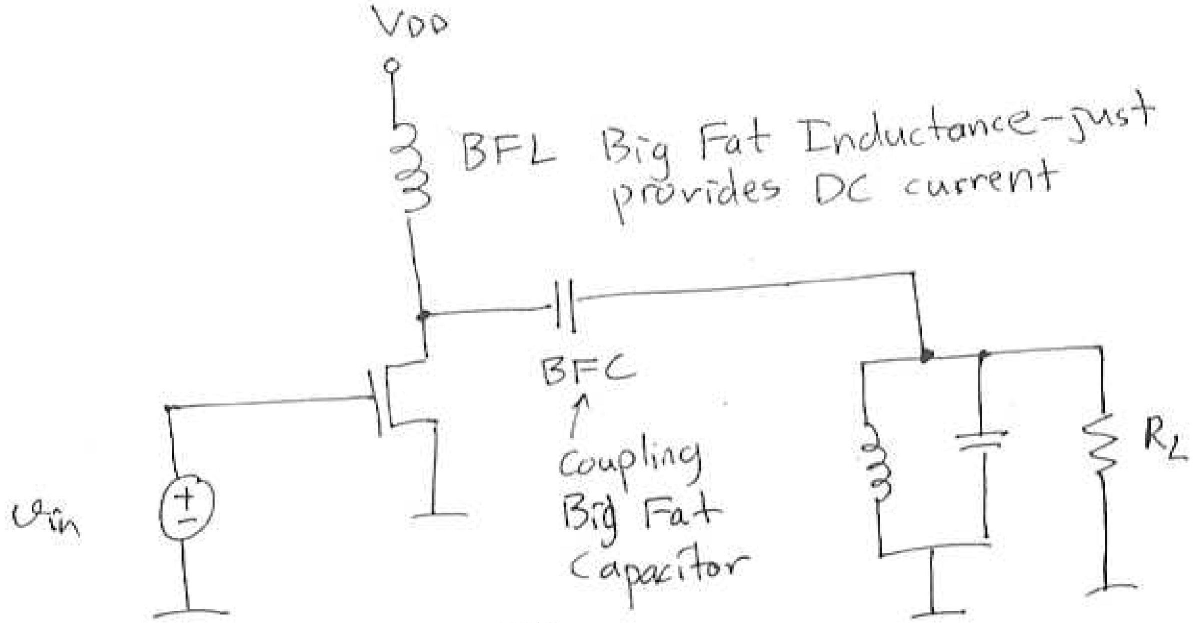
RF POWER AMPLIFIERS

- Short talk on linearity and efficiency



General Power Amplifier Model

Tank Q is high enough to create sinusoidal output voltage

Narrowband power amplifier

BJT → cut-off and saturation is avoided (Linear region)
MOS → always in saturation (pentode region) i.e. the drain current does not depend on the drain voltage, but it is almost solely controlled by Vgs.

Class A, B and C

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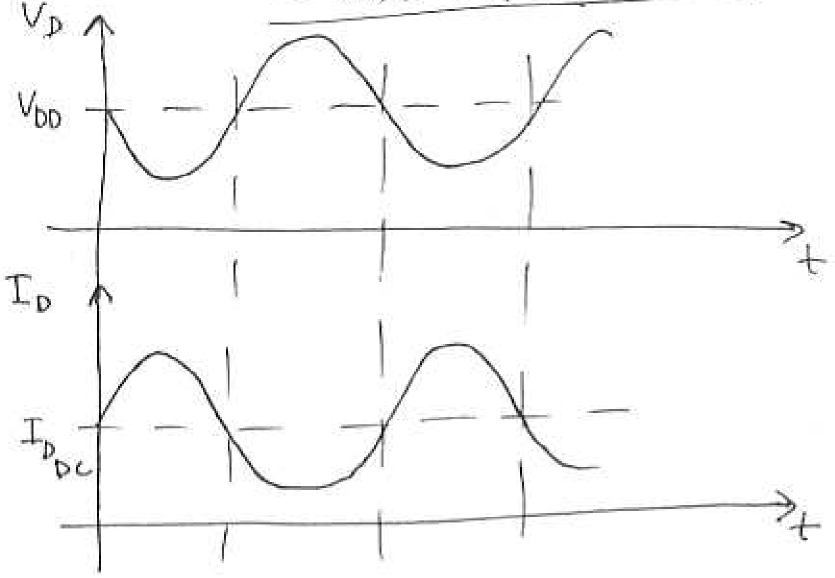
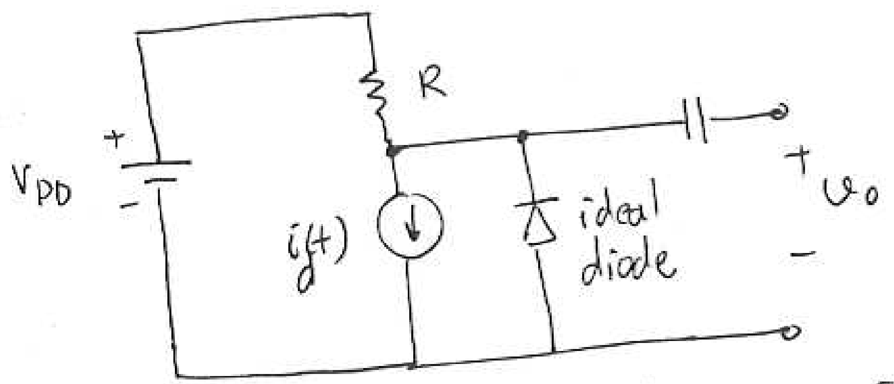
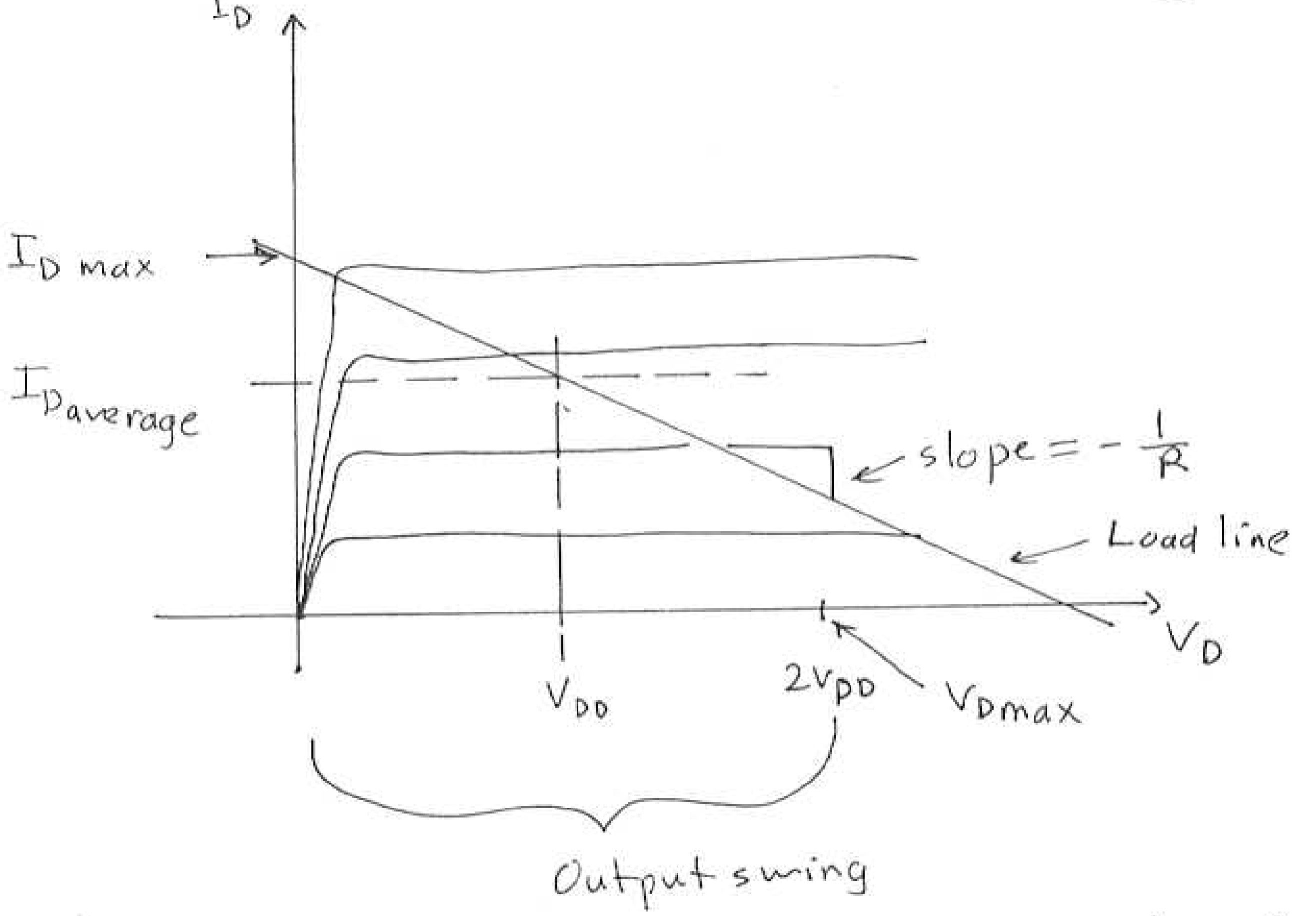


Class-A amplifier resembles the small signal linear tuned amplifier with resistive Load R_L in many respects. If the current drive I_D is assumed to be sinusoidal (it does not necessarily be) The figures are representative.

The circuit can be represented by:



where $i_d(t)$ is always positive



Typical RF Power Amplifier load line for Class A amplifier

At all of the Class A, B and C amplifiers

$V_D = V_{DD} + V_{tank}$, therefore V_D is limited by $2V_{DD}$ since the peak tank voltage is limited by the drain saturation voltage.

$I_{D\,average}$ depends on the Class of the amplifier

$$I_D = I_{DC} + i_{RF} \sin w_0 t \quad \text{if } I_D \geq 0$$
$$= 0 \quad \text{if } I_D < 0$$

$I_{DC} > i_{RF} \Rightarrow$ Class A (no distorsion)

$0 < I_{DC} < i_{RF} \Rightarrow$ Class AB (Transistor starts switching)

$I_{DC} = 0 \Rightarrow$ class B (Transistor conducts at positive halfcycle)

$I_{DC} < 0 \Rightarrow$ class C (Transistor conducts for a portion of the positive half cycle)

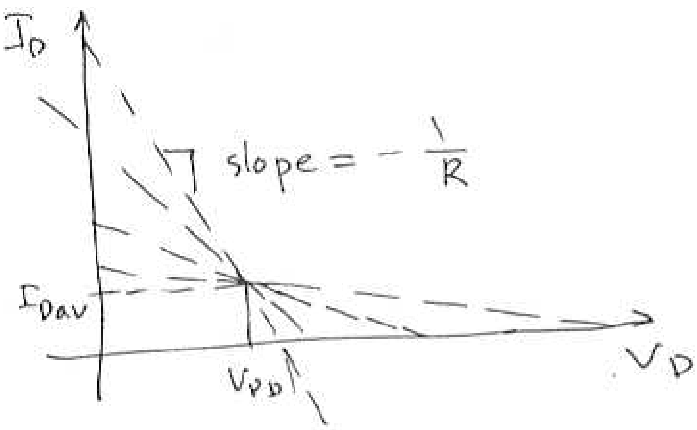
At Class A $\quad I_{D_{average}} = I_{DC}$

At Class B $\quad I_{D_{average}} = \dfrac{i_{RF}}{\pi}$

At Class C $\quad I_{D_{average}} = \dfrac{i_{RF}}{\pi}(\sin\varphi - \varphi\cos\varphi)$

where φ is the conduction angle of the input current.

Class A amplifier



Assuming that the linear region is limited by either $V_D = 0$ or $I_d = 0$

The always correct equation for power output

$$P_{RF} = \frac{I_{peak}^2 R}{2} = \frac{(I_{fundamental})^2 R}{2}$$

and the DC power supplied by the powersupply

$$P_{DC} = V_{DD} \times I_{D_{average}} \quad \text{(constant for class A ampl.)}$$

The voltage swing over the transistor is always symmetric and sinusoidal because of the output tank circuit.

The maximum peak swing which is limited on the lower side by $V_D = 0$, dictates a maximum fundamental drain swing $= 2V_{DD}$ or $V_p = V_{DD}$

The current drive at the lowest possible level is limited by $I_D = 0$, therefore $I_{peak} = I_{DC}$ for maximum current drive

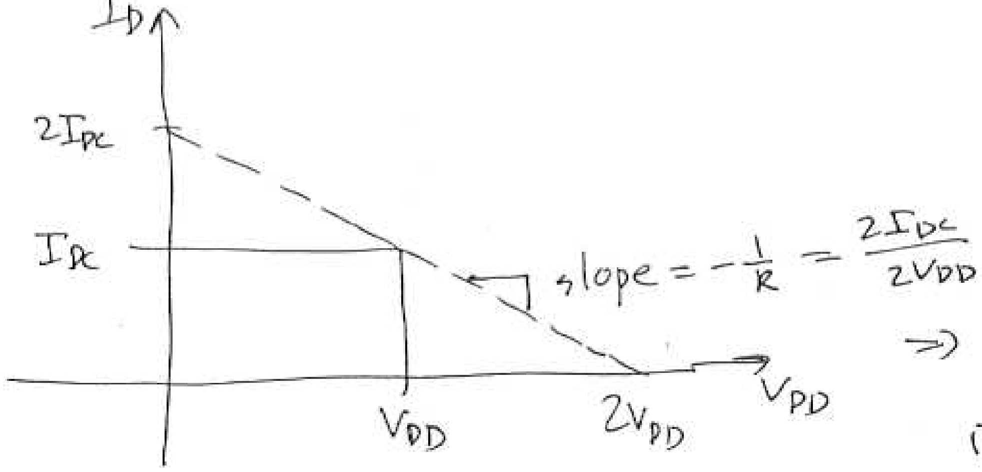
In that case the maximum power output which can drawn from the circuit is

$$P_{RF} = \frac{I_p \times V_p}{2} = \frac{I_{DC} \times V_{DD}}{2}$$

Therefore the maximum efficiency is

$$\eta_{max} = \frac{(\cancel{I_{DC}} \times \cancel{V_{DD}})}{2} \cdot \frac{1}{\cancel{I_{DC}}\cancel{V_{DD}}} = \frac{1}{2}$$

Therefore at the maximum power output



$\Rightarrow R = \frac{V_{DD}}{I_{DC}}$ is the optimum Load resistance.

for $\eta = \frac{1}{2}$

The normalized power output capability for Class A amplifier, which is in general defined by:

$$\frac{P_{o\,max}}{V_{max} \times I_{max}} = \frac{I_{DC} V_{DD}/2}{2I_{DC} \times 2V_{DD}} = \frac{1}{8}$$

Class B amplifier

$i_D = i_{RF} \sin\omega_0 t \quad \text{for } i_D > 0$

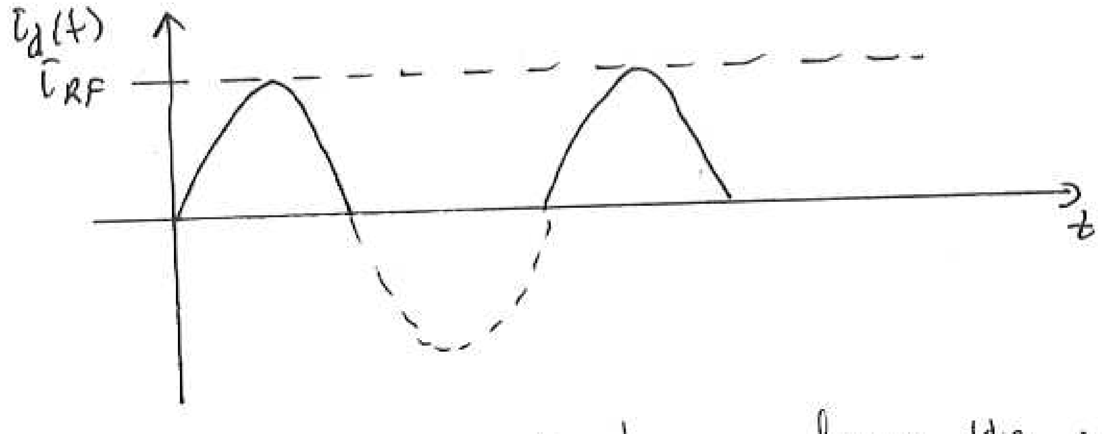
$i_D = 0 \quad \text{for } i_D \leq 0$

$$i_{fund} = \frac{i_{RF}}{2} \Rightarrow v_{out} = \frac{i_{RF}}{2} R \sin\omega_0 t$$

$$i_{fund} = \frac{2}{T} \int_0^{T/2} i_{RF} \sin\omega_0 t \sin\omega_0 t \, dt = i_{RF}/2$$

$$P_{out} = \left(v_{out_p} \times \frac{i_{RF}}{2} \right) \frac{1}{2} = \frac{i_{RF} \times R/2 \times i_{RF}/2}{2} = \frac{i_{RF}^2 R}{8}$$

for any given amplitude of i_{RF}



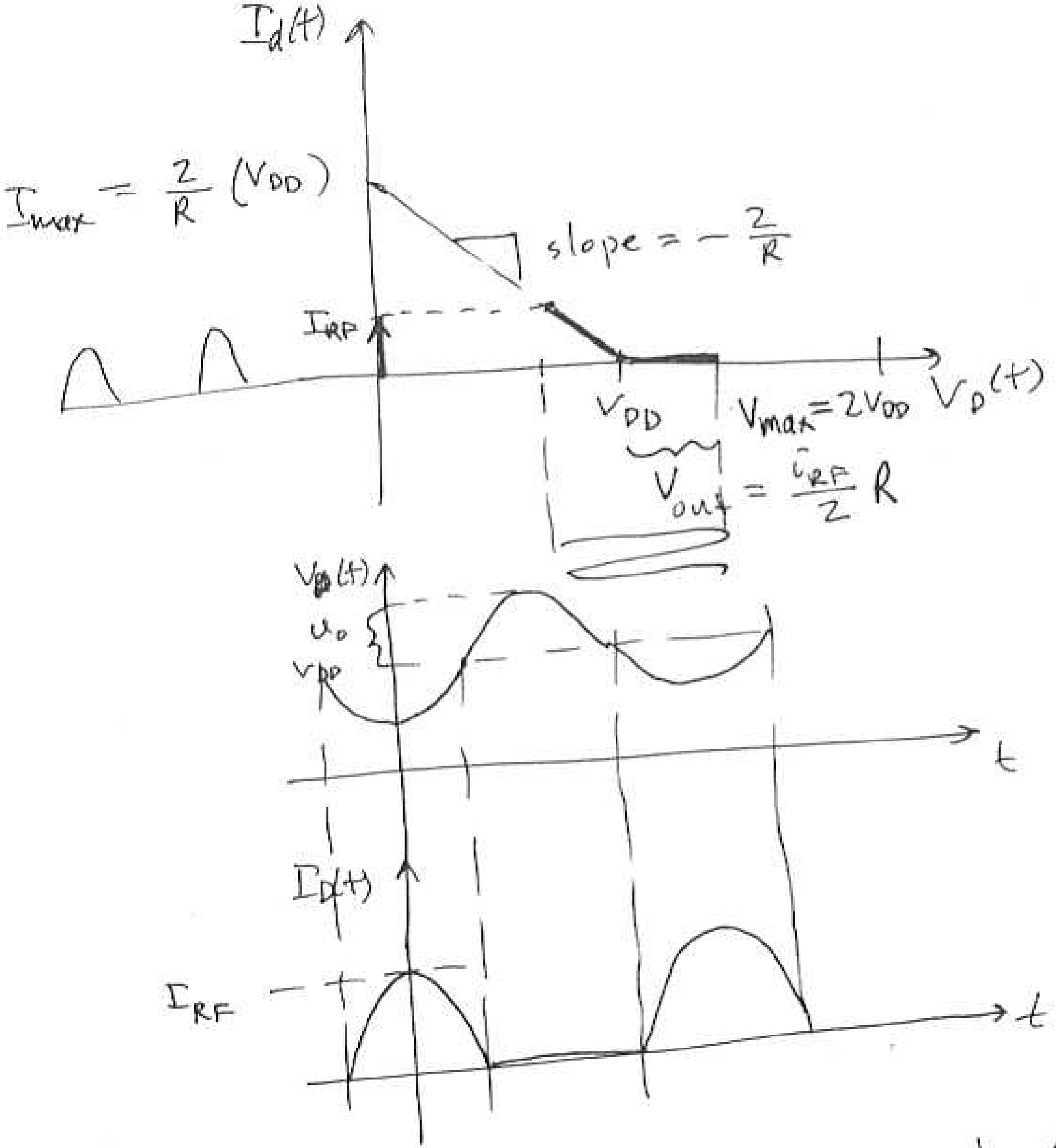
The average current drawn from the power supply is

$$\frac{1}{\pi} i_{RF} \Rightarrow P_{DC} = \frac{V_{DD} \cdot i_{RF}}{\pi} \Rightarrow$$

The efficiency of the class B amplifier at any given drive level i_{RF} is given by

$$\eta = \frac{P_{RF}}{P_{DC}} = \frac{\frac{i_{RF}^2 R}{8}}{\frac{V_{DC}\, i_{RF}}{\pi}} = \frac{i_{RF} R \pi}{8 V_{DC}} = 0.393 \frac{i_{RF} R}{V_{DC}}$$

In order to see the load curve (in contrast to load line of $V_D(t)$ versus $I_D(t)$) of $V_D(t)$ and $I_D(t)$, let us carry out a short analysis:



At any nonlinear current drive with tuned load (parallel) the output swing is determined by the fundamental current and the fundamental voltage

Small signal drive case at Class B ampl.

$$v_{out}(t) = -R\, i_{fund} \cos w_0 t$$

$$= -R \frac{i_{RF}}{2} \cos w_0 t$$

$$I_D(t) = I_{RF}(t) = i_{RF} \cos w_0 t \quad \text{for } I_D(t) \geq 0$$

$$= 0 \quad \text{" } I_D(t) < 0$$

for $\pm \pi/2$ period $I_D(t) = I_{RF} \sin w_0 t$ (1)

$$V_D(t) = V_{DD} - R\, i_{fund}(t) = V_{DD} - R \frac{I_{RF} \sin w_0 t}{2}$$ (2)

Solving for $V_D(t)$ in terms of $I_D(t)$ from equations 1 & 2

$$V_D(t) = V_{DD} - R\,\frac{\cancel{I_{RF}}}{2}\cdot\frac{I_D(t)}{\cancel{I_{RF}}}$$

$$V_D(t) = V_{DD} - \frac{R}{2} I_D(t) \quad \text{or}$$

$$I_D(t) = \frac{2}{R}\left[V_{DD} - V_D(t)\right]$$

The load curve, i.e. the loci of the all possible $V_D(t)$ & $i_D(t)$ pairs is two straight lines extending from the point $V_D(t) = V_{DD}$ and $i_D(t) = 0$ to $V_D(t) = 2V_{DD}$ & $i_D(t) = 0$ and $V_D(t) = 0$ & $i_D(t) = I_{max}$

The span covered is shown as the thick line depending on the drive level. At maximum drive level, which corresponds to maximum efficiency, the current spans 0 to $\frac{2}{R} V_{DD}$ and the voltage spans 0 to $2V_{DD}$ where $i_{RF} = \frac{2V_{DD}}{R}$ & $V_{DC} = V_{DD} \Rightarrow$

$$\eta_{max} = 0.393\,\frac{2\cancel{V_{DD}}}{\cancel{R}}\cdot\frac{\cancel{R}}{\cancel{V_{DD}}} = \frac{\pi}{8}\cdot 2 = \frac{\pi}{4} = 0.785$$

(η_{max} corresponds to the maximum possible i_{RF} since it is proportional with it.)

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The normalized power output capability which is defined as

$$\frac{P_{o\,max}}{V_{max} \times I_{max}} = \frac{I^2_{fund_{max}} R/2}{V_{max}\, I_{max}} = \frac{\left(\frac{I_{RF\,max}}{2}\right)^2 \cdot \frac{R}{2}}{V_{max} \times I_{max}}$$

$$= \frac{\left(\frac{I_{max}}{2}\right)^2 \cdot \frac{R}{2}}{V_{max} \times I_{max}} = \frac{\frac{I^{\cancel{2}}_{\cancel{max}}}{4} \cdot \frac{1}{2} \cdot \frac{V_{\cancel{max}}}{I_{\cancel{max}}}}{V_{\cancel{max}} \times I_{\cancel{max}}} = \frac{1}{8}$$

Where R_{opt} can be found from

$$I_{max} = \frac{2}{R_{opt}} V_{DD} = \frac{\cancel{2}}{R_{opt}} \cdot \frac{V_{max}}{\cancel{2}} \Rightarrow$$

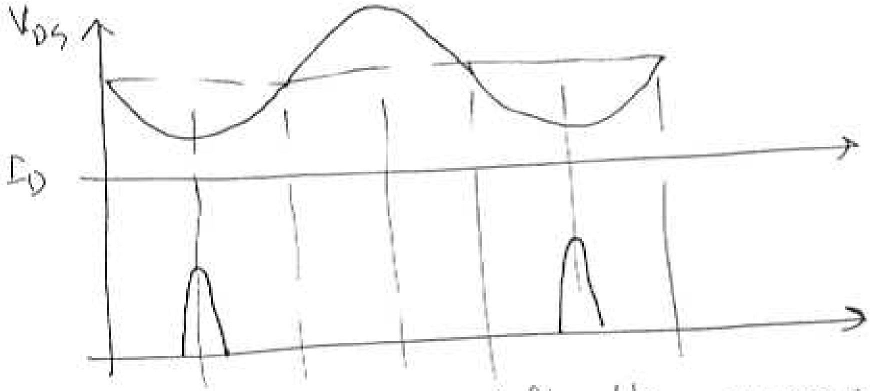
$$\boxed{R_{opt} = \frac{V_{max}}{I_{max}}}$$ for class B amplifiers

Class C amplifier

The transistor conducts for less than the half period

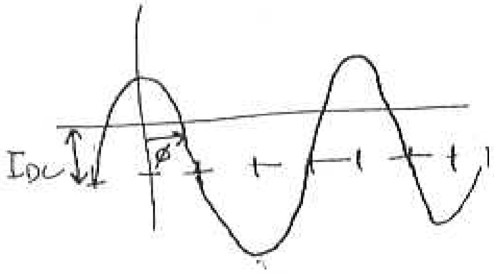
$i_D = I_{DC} + i_{RF} \sin \omega_0 t$, $i_D > 0$

$i_D = 0$ for the rest of the period.



let us shift the current waveform a little bit.

Detailed drain current



let $i_D = i_{DC} + i_{RF} \cos \omega t$, $i_D > 0$

$$2\phi = 2 \cdot \cos^{-1}\left(-\frac{I_{DC}}{i_{RF}}\right) \qquad -I_{DC} = I_{RF} \cos \phi$$

$$\bar{i}_D = \frac{1}{2\pi} \int_{-\phi}^{+\phi} (I_{DC} + i_{RF} \cos\theta)\, d\theta = \frac{1}{2\pi} 2\phi I_{DC} + \frac{i_{RF}}{2\pi} \sin\theta \Big|_{-\phi}^{+\phi}$$

$$\bar{i}_D = \frac{i_{RF}}{\pi} (\sin\phi - \phi \cos\phi)$$

$$i_{fund} = \frac{1}{\pi}\int_{-\varphi}^{+\varphi} (I_{DC} + i_{RF}\cos\theta)\, d\theta \cos\theta$$

$$= \frac{2}{\pi}\int_{0}^{\varphi} (I_{DC} + i_{RF}\cos\theta)\, d\theta \cos\theta$$

$$i_{fund} = \frac{2}{\pi}\left\{ I_{DC}\sin\theta \Big|_0^{\varphi} + \frac{i_{RF}}{2}\int_0^{\varphi}\left[1+\cos 2\theta\right] d\theta \right\}$$

$$= \frac{2}{\pi}\left\{ I_{DC}\sin\varphi + \frac{i_{RF}\varphi}{2} + \frac{i_{RF}}{2}\frac{\sin 2\varphi}{2}\right.$$

$$= \frac{2}{\pi}\left(-i_{RF}\cos\varphi\sin\varphi + \frac{i_{RF}\varphi}{2} + \frac{i_{RF}\sin 2\varphi}{4}\right.$$

$$\cos\varphi\sin\varphi = \frac{\sin 2\varphi}{2}$$

$$= \frac{2 * i_{RF}}{\pi}\left[-\frac{2}{4}\sin 2\varphi + \frac{\varphi}{2} + \frac{\sin 2\varphi}{4}\right]$$

$$= \frac{\cancel{2} i_{RF}}{\pi}\left[\frac{\varphi}{\cancel{2}} - \frac{\sin 2\varphi}{\cancel{4}\,2}\right] = \frac{i_{RF}}{\pi}\left[\varphi - \frac{\sin 2\varphi}{2}\right]$$

$$V_{out_{peak}} = \frac{i_{RF}}{2\pi}\left[2\varphi - \sin 2\varphi\right] R$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{i_{fund}^2 R/2}{V_{DD} \cdot \overline{i_D}}$$

$$= \frac{\left(\frac{i_{RF}}{2\pi}\left[2\varphi - \sin 2\varphi\right]\right) \overbrace{R\,\frac{i_{RF}}{2\pi}\left[2\varphi - \sin 2\varphi\right]}^{V_{peak}}}{2 \times V_{DD}\,\underbrace{\frac{i_{RF}}{\pi}\left[\sin\varphi - \varphi\cos\varphi\right]}_{\text{average current}}}$$

at the maximum efficiency point $V_{peak} = V_{DD}$

$$\eta_{max} = \frac{\frac{\cancel{i_{RF}}}{2\cancel{\pi}}\left[2\varphi - 2\sin\varphi\right] \times \cancel{V_{DD}}}{2 \times \cancel{V_{DD}}\,\frac{\cancel{i_{RF}}}{\cancel{\pi}}\left[\sin\varphi - \varphi\cos\varphi\right]}$$

$$\eta_{max} = \frac{\left[2\varphi - 2\sin\varphi\right]}{4\left[\sin\varphi - \varphi\cos\varphi\right]}$$

φ (rad)	η_{max}
0.01	0.99999034
0.1	0.999
0.2	0.996
0.3	0.991
0.4	0.98
0.5	0.975
1.0	0.9
$\pi/2$	0.7854 → class B amplifier

assuming that R is chosen such that $V_{peak} = V_{DD}$ each time

Talk about switching loss

in practice these values can not even be approached

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$$V_D(t) = V_{DD} - R \; i_f(t)$$

$$V_D(t) = V_{DD} - R \; k \, i_{RF} \cos wt$$

$$= V_{DD} - kR \left[I_D(t) - I_{DC} \right]$$

$$= V_{DD} - kR \; I_D(t) + kR \; I_{DC}$$

$$I_D(t) = I_{DC} + i_{RF} \cos wt$$

$$i_{RF} \cos wt = i_D(t) - I_{DC}$$

where

$$k = \frac{1}{2\pi}\left[2\varphi - \sin 2\varphi\right] = \frac{1}{\pi}\left[\varphi - \frac{\sin 2\varphi}{2}\right]$$

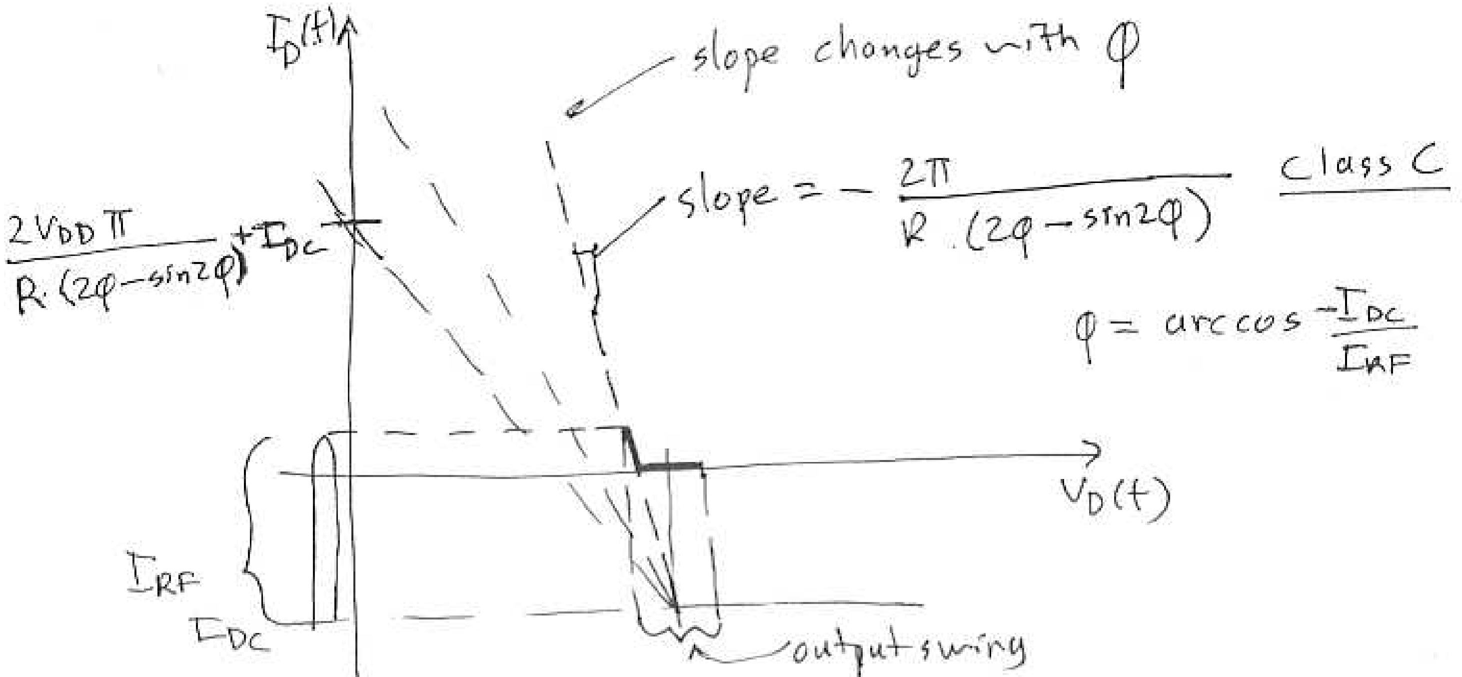
$$kR \; I_D(t) = V_{DD} - V_D(t) + kR \; I_{DC}$$

$$= \left[V_{DD} + kR \; I_{DC}\right] - V_D(t)$$

$$I_D(t) = \frac{V_{DD}}{kR} + I_{DC} - \frac{V_D(t)}{kR}$$

$$I_D(t) = \frac{\left[V_{DD} - V_D(t)\right]}{R} \, \frac{\pi}{\varphi - \underbrace{\frac{\sin 2\varphi}{2}}_{\text{always} > 2}} + I_{DC}$$

where φ is the single sided conduction angle



class C load curve

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As can be observed from the figure for arbitrary selection of R and φ and I_{DC} the instantenous $v_d(t)$ & $i_d(t)$ locus can assume lines with different slopes. But for maximum efficiency & maximum output, the output voltage swing must cover $V_D(t)$ range from 0 to $2V_{DD}$ and current must reach to I_{max} where the load line intersects with the current axis and this point is defined by

$$I_{max} = I_{DC} + \frac{2V_{DD}\pi}{R(2\varphi - \sin 2\varphi)} \quad equ(1)$$

At such a configuration I_{max}, I_{DC}, I_{RF} & φ become related to each other as

$$\cos\varphi = \frac{-I_{DC}}{I_{RF}} = \frac{-I_{DC}}{I_{Max} - I_{DC}} \Rightarrow$$

$$I_{max}\cos\varphi - I_{DC}\cos\varphi = -I_{DC} \Rightarrow$$

$$I_{max}\cos\varphi = I_{DC}(\cos\varphi - 1) \Rightarrow$$

$$I_{DC} = I_{max}\frac{\cos\varphi}{\cos\varphi - 1}$$

Now rearranging equation 1 to express Roptimum

$$R_{opt} = \frac{V_{max}}{I_{max} - I_{DC}} \cdot \frac{\pi}{2\varphi - \sin 2\varphi} \quad \text{where } V_{max} = 2V_{DD}$$

and substituting I_{DC}, R_{opt} becomes

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$$R_{opt} = \frac{V_{max}}{I_{max}\left[1 - \frac{\cos\varphi}{\cos\varphi - 1}\right]} \cdot \frac{\pi}{2\varphi - \sin\varphi}$$

$$= \frac{V_{max}}{I_{max}} \cdot \frac{\pi}{\left[\frac{\cancel{\cos\varphi} - 1 - \cancel{\cos\varphi}}{\cos\varphi - 1}\right]} \cdot \frac{1}{2\varphi - \sin\varphi}$$

$$= \frac{V_{max}}{I_{max}} \cdot \frac{\pi(1 - \cos\varphi)}{2\varphi - \sin 2\varphi}$$ where $\varphi = \arccos \frac{I_{DC}}{I_{DC} - I_{max}}$

$$R_{opt} = \frac{V_{max}}{I_{max}} \cdot \frac{\pi(1 - \cos\varphi)}{2\varphi - \sin 2\varphi} \quad \text{(equ 2)}$$

which is the expression for Ropt in terms of Vmax, Imax and φ which is half the conduction angle. The conduction angle can also be expressed as

$$\cos\varphi = \frac{I_{DC}}{I_{DC} - I_{max}}$$ or as in the equation stated

a while ago as

$$I_{DC} = I_{max} \frac{\cos\varphi}{\cos\varphi - 1} \quad \text{(equ 3)}$$

Therefore if a transistor with a set of Vmax & Imax is given or if a set of values Imax & Vmax are given as the required drive levels then for an amplifier with optimum efficiency and maximum output with those levels, 10.12_2003

13 - C

then, R_{opt} & I_{DC} must be selected using (equ 2) & (equ 3) after selecting the conduction angle 2φ.

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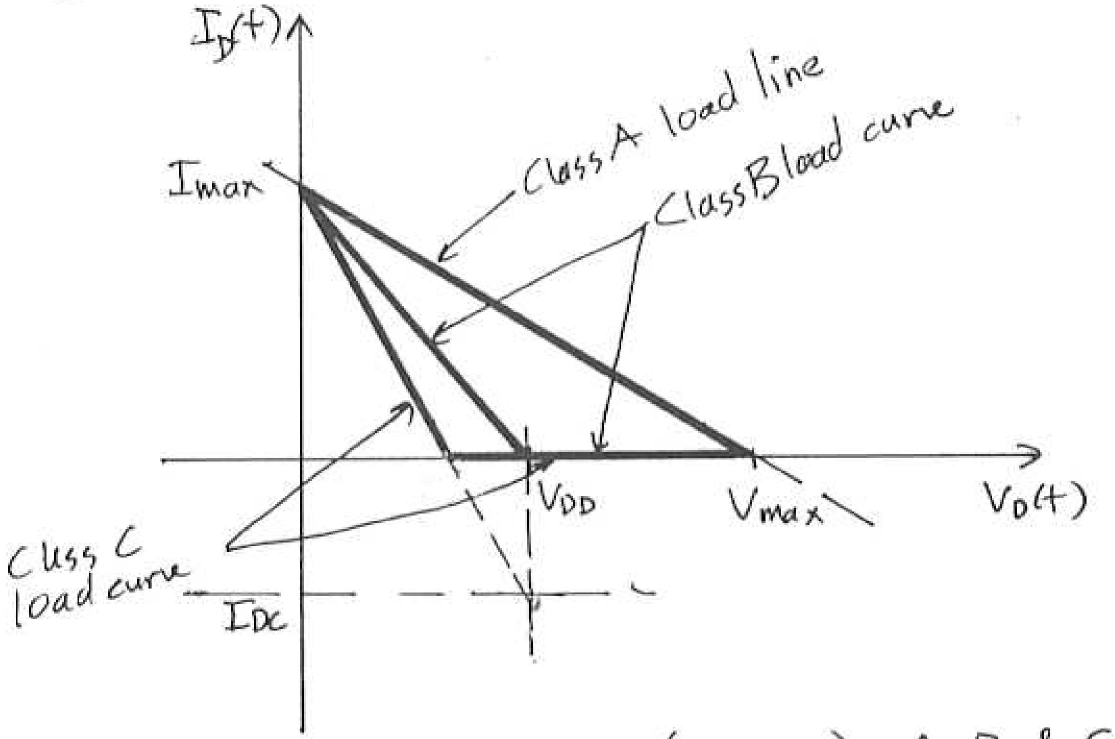
Summary of Class A, B & C type amplifiers for optimum solutions



Optimum load lines (curves) A, B & C class of amplifiers

$$\boxed{R_{opt} = \frac{V_{max}}{I_{max}} = \frac{V_{DD}}{I_{fund}} = \frac{V_{DD}}{I_{RF}} \quad \text{for Class A}}$$

$$\boxed{R_{opt} = \frac{V_{max}}{I_{max}} = \frac{V_{max}}{I_{RF}} = \frac{2V_{DD}}{I_{RF}} = \frac{\cancel{2}V_{DD}}{\cancel{2}I_{fund}} \quad \text{for Class B}}$$

$$R_{opt} = \frac{V_{max}}{I_{max}} \cdot \frac{\pi(1-\cos\varphi)}{2\varphi - \sin 2\varphi} = \frac{2V_{DD}}{I_{RF}+I_{DC}} \cdot \frac{\pi(1-\cos\varphi)}{(2\varphi - \sin 2\varphi)}$$

where $\varphi = \arccos \frac{-I_{DC}}{I_{RF}}$ for Class C

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Or alternatively R_{opt} for Class C can be expressed as by substituting $\cos\varphi = \frac{-I_{DC}}{I_{RF}}$

$$R_{optimum} = \frac{2V_{DD}}{I_{RF}+I_{DC}} \cdot \frac{\pi\left(1 - \frac{-I_{DC}}{I_{RF}}\right)}{2\varphi - 2\sin\varphi}$$

$$= \frac{2V_{DD}}{I_{RF}+I_{DC}} \cdot \frac{\pi \frac{I_{RF}+I_{DC}}{I_{RF}}}{2\varphi - 2\sin\varphi}$$

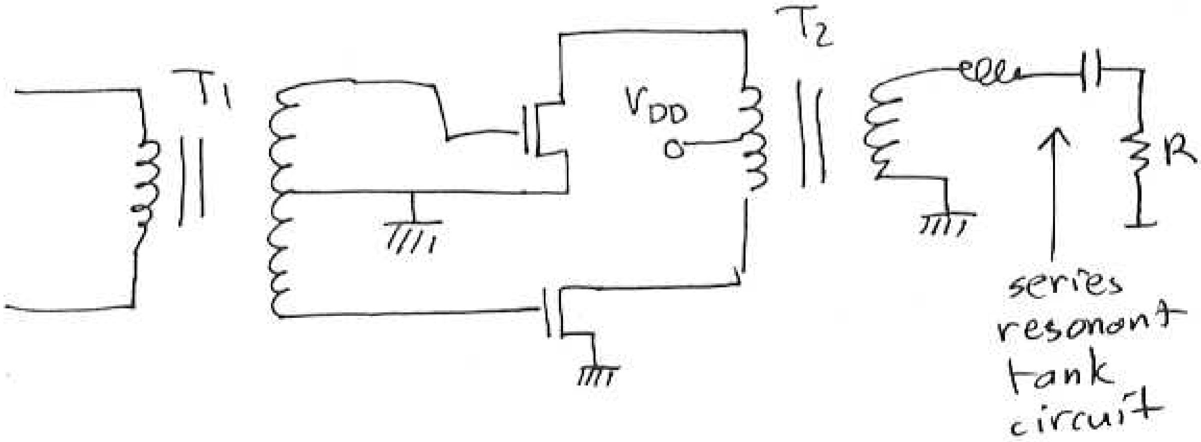
$$R_{opt} = \frac{V_{DD}}{I_{RF}} \cdot \frac{2\pi}{(2\varphi - 2\sin\varphi)}$$

One can therefore also choose V_{DD}, I_{RF} and conduction angle 2φ and can then determine R_{opt} accordingly.

Class AB amplifier

Class AB amplifier is a driving method between class A & class B. Class AB is a some what linear amplifier with a smaller bias current so that at large drive levels the current drive is limited at the lower end.

Class D amplifier



The transistors act like switches, as in a push-pull amplifier, but they remain on for one half cycle and remain off for the other half cycle (alternately)

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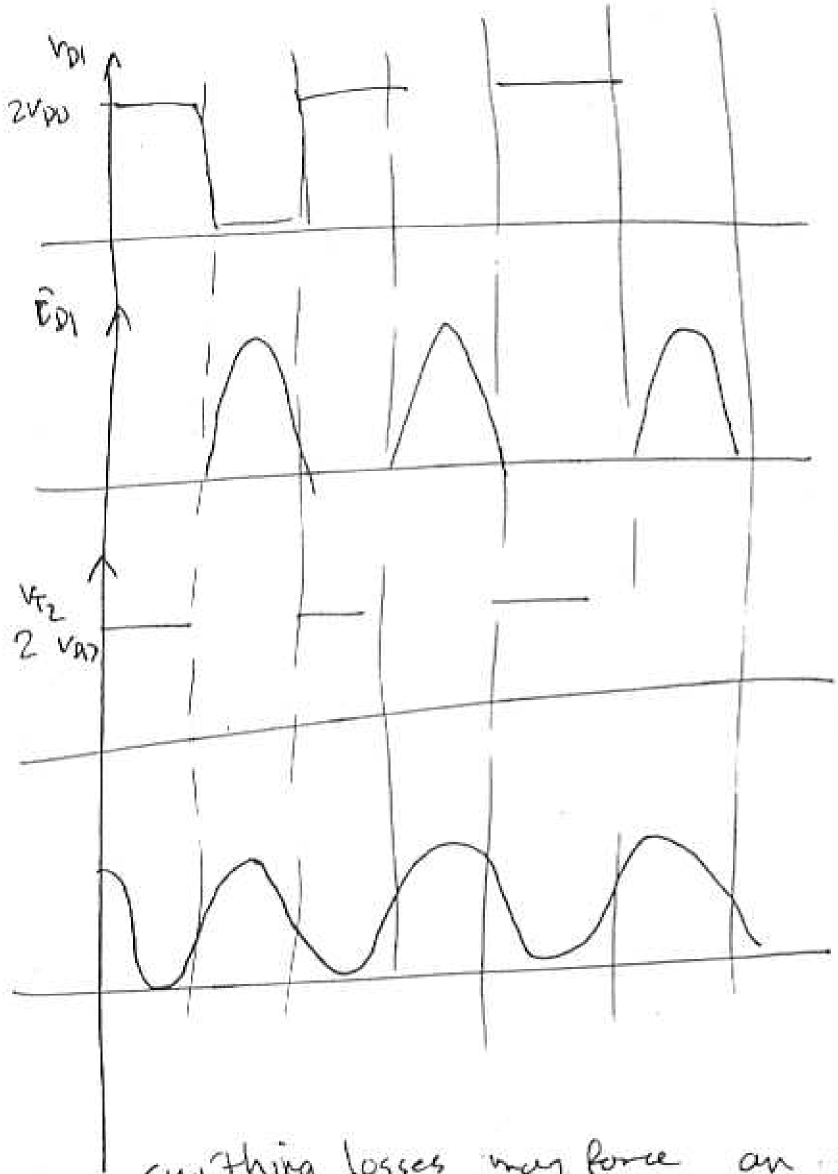

square wave voltage on primary and secondary ⇒ therefore only switching losses

If you drive the transistors hard enough the transformer acting as inductor will saturate the transistor when at the active state. Once one transistor stays in sat for one half period, the other one will have twice the voltage of VDD on its drain.

switching losses may force an efficiency down to 60–70%