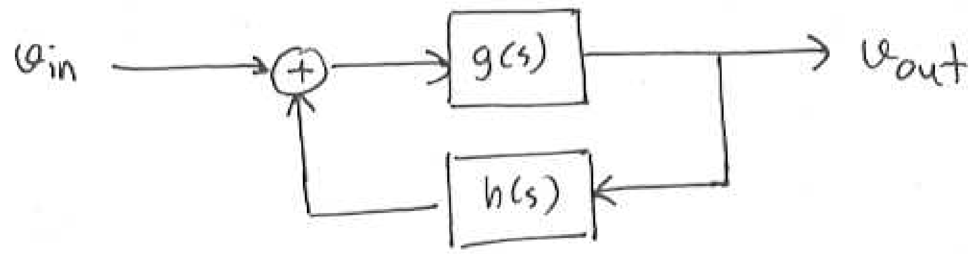
RF Oscillators

stability of an oscillator (or the purposeful instability) can be analysed in various ways. One of the simple methods is to create a feedback system with $0°$ phase margin.



$$v_{out}(s) = \frac{g(s)}{1 - g(s)h(s)} v_{in}(s)$$

The system is unstable when the denominator is equal to zero or $g(s)h(s) = 1$

or $|g(j\omega)h(j\omega)| = 1$ and

$$\angle g(j\omega)h(j\omega) = n2\pi \qquad n = 0, 1, 2, \ldots$$

$\Rightarrow$ an oscillator oscillates at the angular frequency ω at which the above condition is satisfied (i.e. at steady state). Furthermore, if the open loop gain magnitude is greater than one when the angle is equal to $2\pi n$, then the oscillation grows (which implies that the poles of the transfer function are at the right hand plane) and only when the magnitude of the open-loop gain decreased, by some limiting mechanism (which might also shift the angle a little bit, too), to be equal to one where the angle equal to $2\pi n$, the growth of the amplitude ceases and the oscillation amplitude stabilizes (when the poles at the right-hand plane shifts to the imaginary axis).

One point should be emphisized, though. That the open-loop gain is the gain of the loop ($g(s)\,h(s)$) and $h(s)$ is still being loaded at its output. If you just break the loop by physically disconnecting the output of the feedback gain $h(s)$, then the picture might change beacause of the change at the load.

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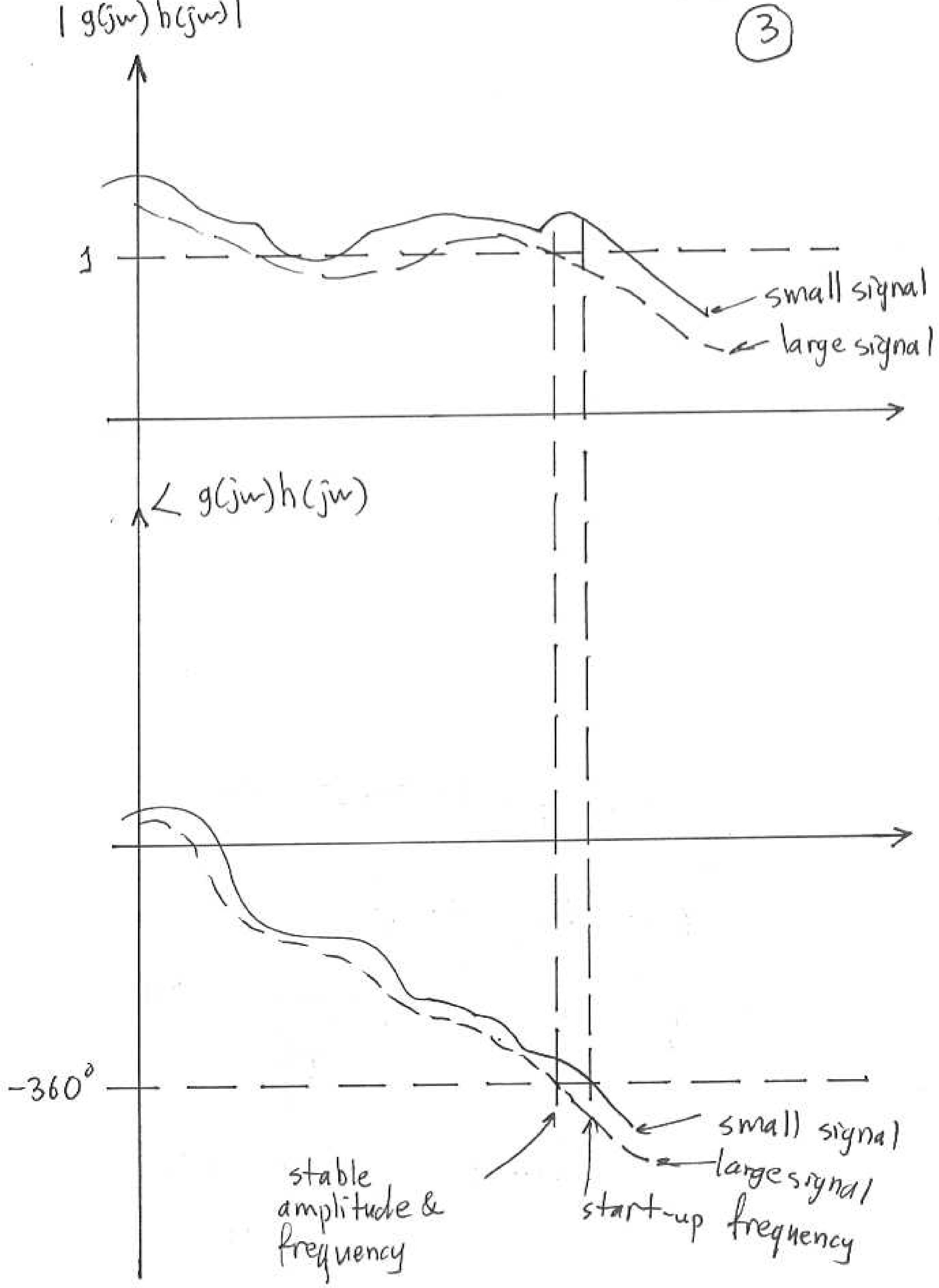
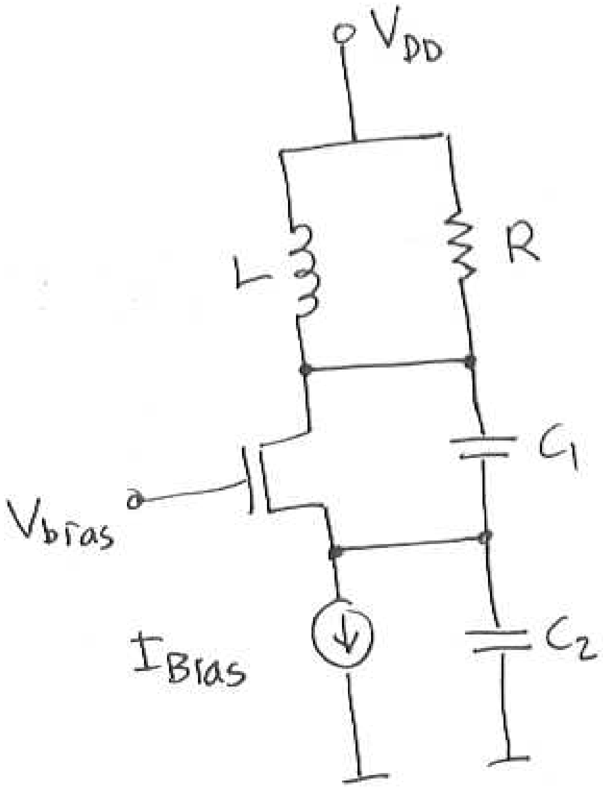

|g(jw) h(jw)|
1
small signal
large signal
< g(jw) h(jw)
-360°
small signal
large signal
stable amplitude & frequency
start-up frequency

Colpitts Oscillator

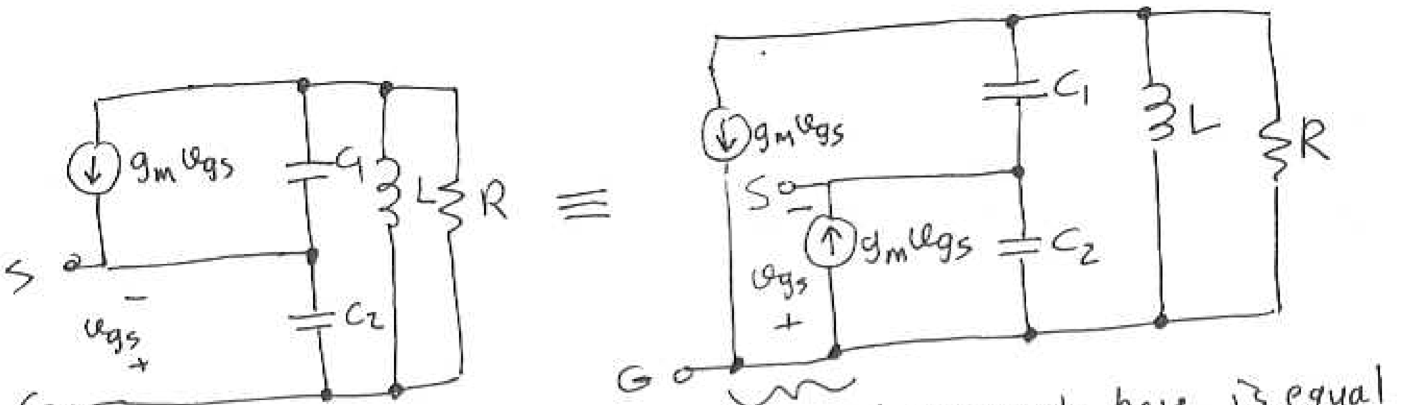
Colpitts is a classical stable oscillator which uses the large-signal gain of the oscillator transistor for stabilizing the amplitude.



The series combination of C_1 & C_2 creates a tank circuit (parallel RLC) with L loaded by R.

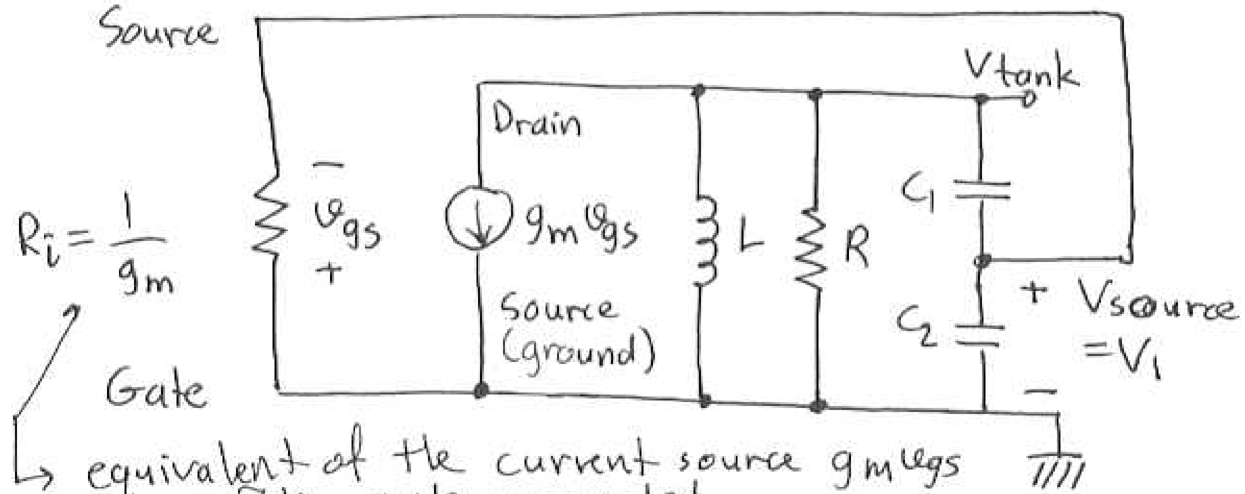
The drain signal fed back to source via the voltage divider created by C_1 & C_2.

The small signal equivalent circuit is given by:



the net current here is equal to zero, therefore equivalent to no connection.

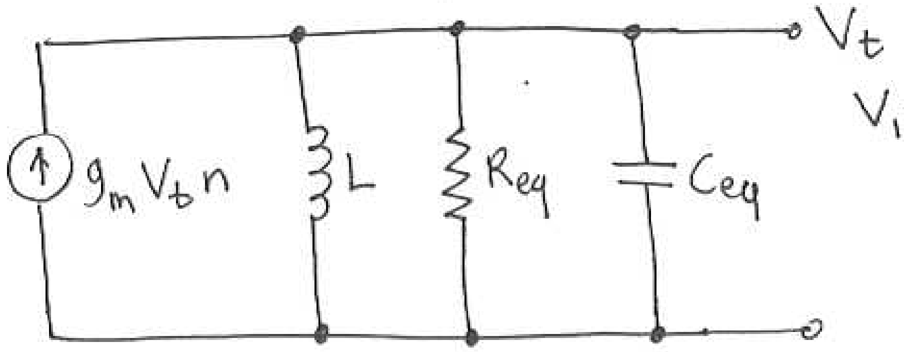
We can draw the small signal equivalent circuit to find out wheather the oscillation conditron is satisfied, i.e. to see if the oscillation starts.



↳ equivalent of the current source $g_m v_{gs}$ with gate grounded.

Source is connected to ground because the other side of a current can be connected to any other place without disturbing the tank circuit.

The circuit can bereduced to



$V_1 = nV_{tank}$
$= nV_t$

$R_{eq} = R // \frac{1}{n^2 g_m}$ $\quad n = \frac{C_1}{C_1 + C_2}$ $\quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$ $\quad w = \frac{1}{\sqrt{L C_{eq}}}$

At resonance the tank is represented by R_{eq}

The open-loop gain is given by

$$g_m n R_{eq}$$

The condition for oscillation becomes,

$$g_m n R_{eq} > 1 \quad (\text{with some margin})$$

Notice that it is loaded (by $\frac{1}{n^2 g_m}$)

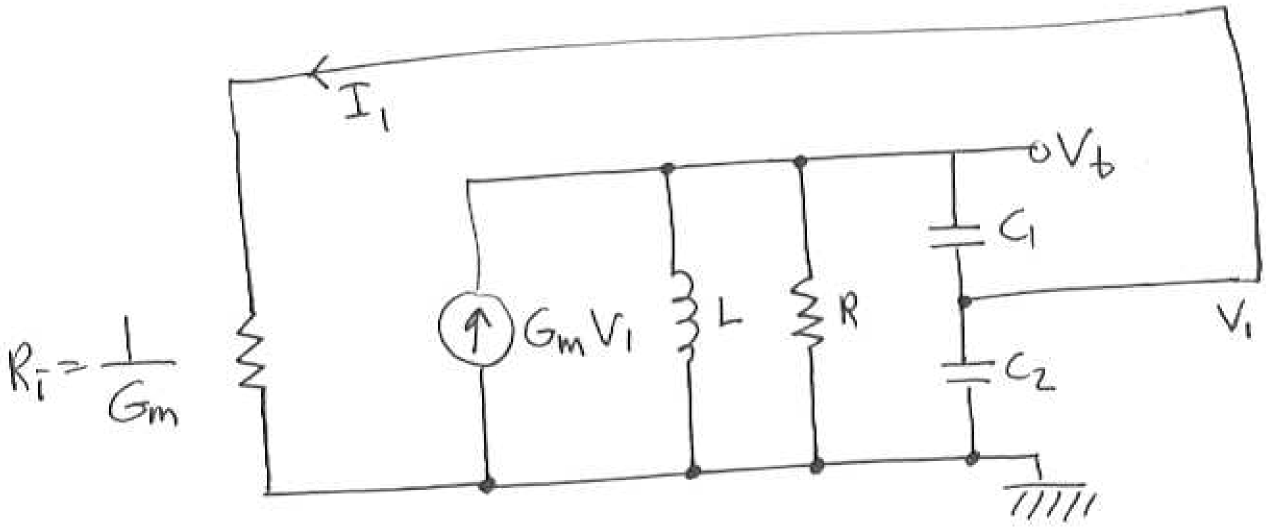
$$g_m n \left[R // \frac{1}{n^2 g_m} \right] > 1$$

$$n g_m \frac{R \frac{1}{n^2 g_m}}{R + \frac{1}{n^2 g_m}} = n g_m \frac{R}{1 + R n^2 g_m} > 1$$

$$n R g_m > 1 + R n^2 g_m \Rightarrow n R g_m (1-n) > 1 \Rightarrow$$

$$g_m > \frac{1}{R n (1-n)} = \frac{1}{R(n-n^2)}$$

In order to find the oscillation amplitude the large-signal equivalent of the circuit must be used.

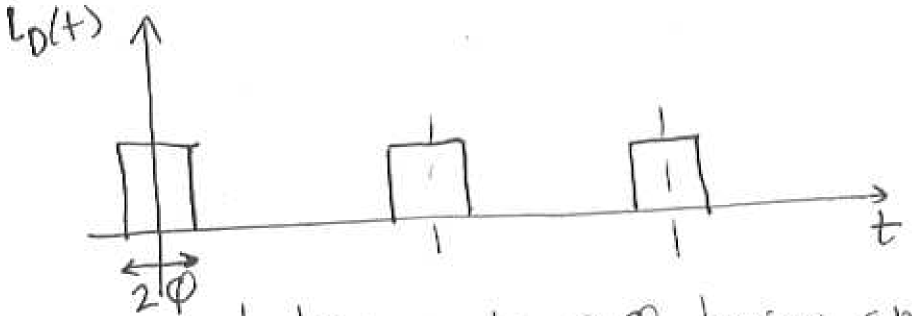


As the amplitude of the oscillation increases, the input impedance (the source impedance) increases, decreasing the large-signal admittance which is equal to the large-signal transconductance. Assuming that, there was enough gain margin at the small-signal model, i.e. the final oscillation amplitude is large enough to switch the transistor on and off at each cycle and the conduction angle is small enough, the bias current and the fundamental current driving the tank circuit can be related as:

$$I_1 = \frac{2}{T}\int_{-T/2}^{+T/2} i_D(t)\cos w_o t\, dt = 2\int_{-T/2}^{+T/2} \frac{i_d(t)}{T}\, dt$$

I_1 being the fundamental current
at small conduction angles $\cos w_o t \approx 1$,
therefore the integral becomes the average current

$$I_1 = 2 I_{Bias}$$

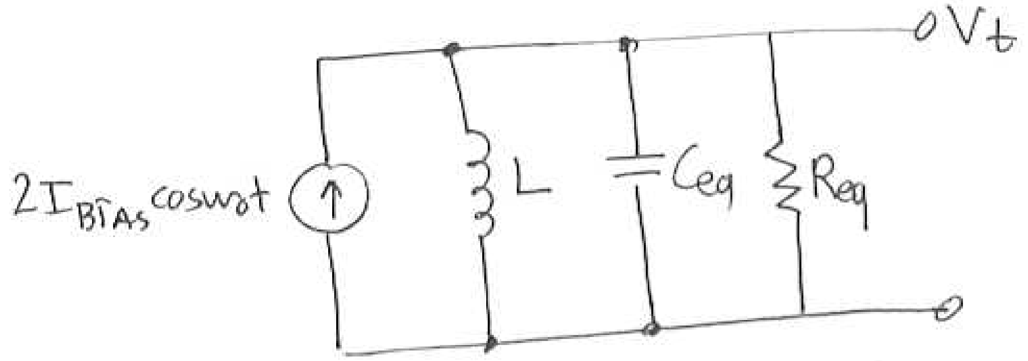


conduction angle 2φ being small

$$G_m = \frac{I_1}{V_1} = \frac{2 I_{Bias}}{V_1}$$

Large-signal average transconductance for constant source bias. Notice it is defined at the fundamental frequency and it is inversely proportional with the i/p. Also notice that harmonic currents are rejected by the tuned circuit.

The circuit can again be reduced



$$G_m = \frac{I_1}{V_1} = \frac{2I_{BIAS}}{V_1} \qquad V_1 = n V_{tank}$$

$$n = \frac{C_1}{C_1 + C_2} \qquad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$R_{eq} = R \,//\, \frac{1}{n^2 G_m}$$

The voltage developed over the tank by the driving current $2I_{BIAS}\cos\omega_0 t$

$$V_{tank} = 2I_{BIAS}\left(R \,//\, R_{eq}\right) = 2I_{BIAS}\left(R \,//\, \frac{1}{n^2 G_m}\right)$$

$$= 2I_{BIAS}\left(R \,//\, \frac{V_1}{n^2\, 2I_{BIAS}}\right)$$

$$= 2I_{BIAS}\left(R \,//\, \frac{\cancel{n} V_{tank}}{n^{\cancel{2}}\, 2I_{BIAS}}\right)$$

$$V_{tank} = 2I_{BIAS}\left(R \mathbin{//} \frac{V_{tank}}{2nI_{BIAS}}\right)$$

$$V_{tank} = 2I_{BIAS}\,\frac{R \times \dfrac{V_{tank}}{2nI_{BIAS}}}{R + \dfrac{V_{tank}}{2nI_{BIAS}}}$$

$$\cancel{V_{tank}} = 2I_{BIAS}\,\frac{R\cancel{V_{tank}}}{2nRI_{BIAS} + V_{tank}}$$

$$2nRI_{BIAS} + V_{tank} = 2I_{BIAS}R$$

$$\boxed{V_{tank} = 2I_{BIAS}R(1-n)}$$

Note that this equation is valid only for high drive levels of the transistor which results in small conduction angles.

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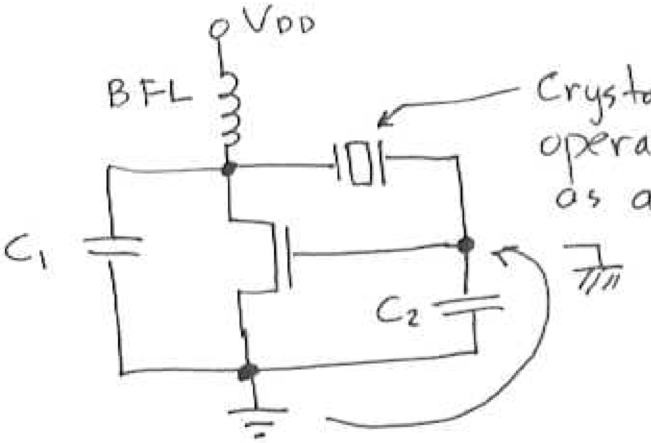
Equivalence of differently named oscillators

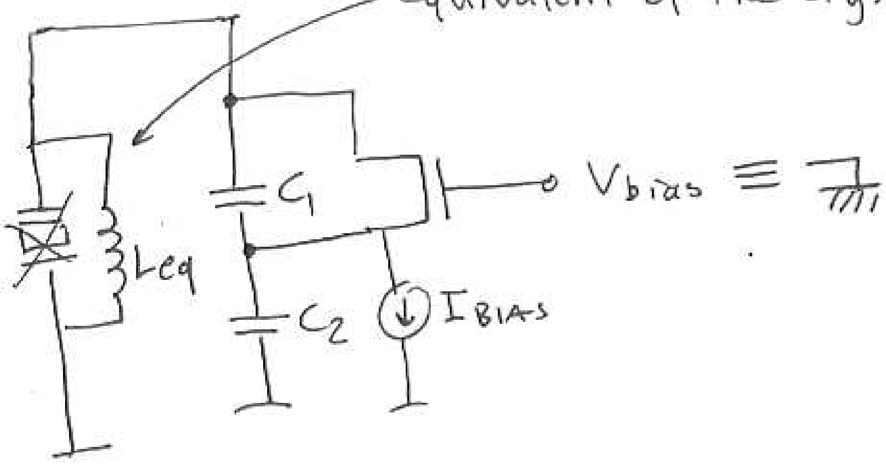
Pierce oscillator



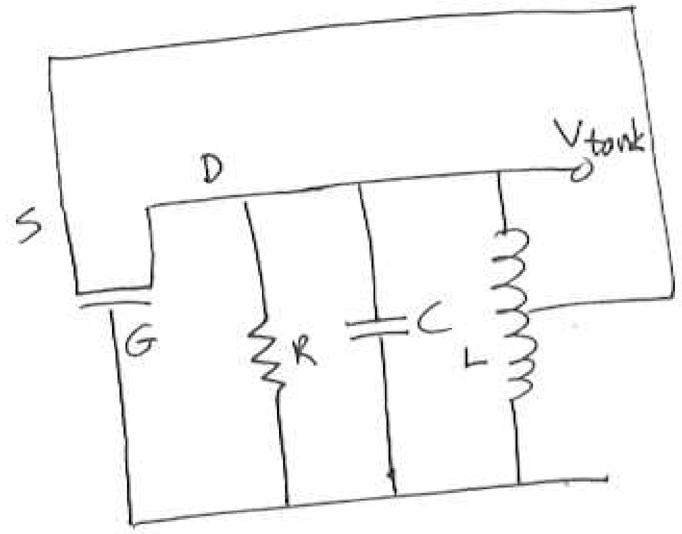
Crystal frequency reference operated such that it acts as an inductor. Biasing not shown.

The circuit is grounded at the source. If the ground is moved to the gate, which does not effect the operation of the circuit, then circuit becomes a colpitts oscillator as shown below.

equivalent of the crystal

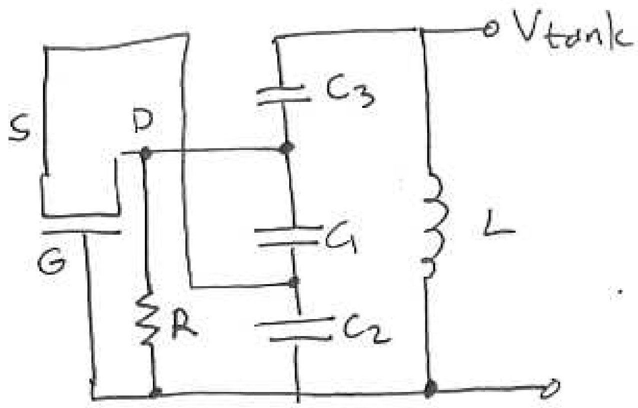


Hartley



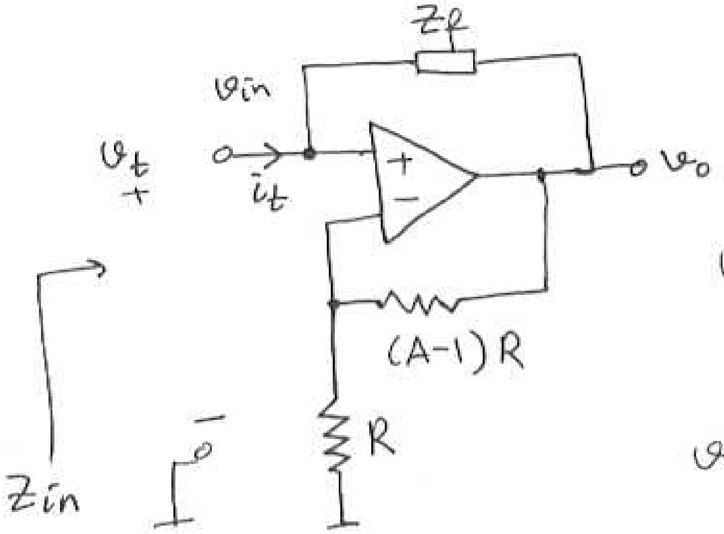
Hartley is a colpitts like oscillator where an inductive divider instead of a capacitive voltage divider is used.

Clapp oscillator



Clapp is a modified colpitts oscillator where the tank has an additional capacitive divider. The Q of the tank must be calculated taking the additional divider into account.

Negative Resistance Devices



Without Z_f

$$v_{in} = \frac{R}{(A-1)R + R} v_{out}$$

$$v_{in} = \frac{R}{AR - \cancel{R} + \cancel{R}} v_{out}$$

$$\frac{v_{in}}{v_{out}} = \frac{\cancel{R}}{A\cancel{R}} \Rightarrow G = A$$

To find the input impedance we connect a test source in order to relate the input voltage & current.

$$i_t = \frac{v_t - v_o}{Z_f} = \frac{v_t - A v_t}{Z_f} = v_t \frac{(1-A)}{Z_f} \Rightarrow$$

$$\frac{v_t}{i_t} = \frac{Z_f}{1-A} = Z_{in}$$

If an ordinary resistance R_f is connected in the place of Z_f and A is set to be 2, then the input impedance becomes $-R_f$

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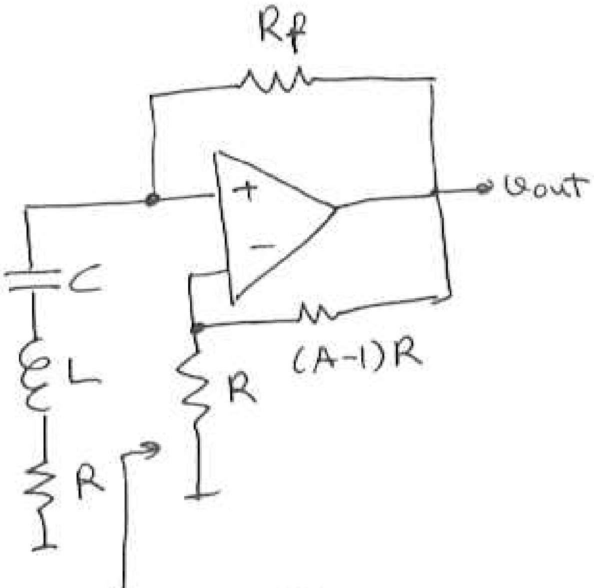
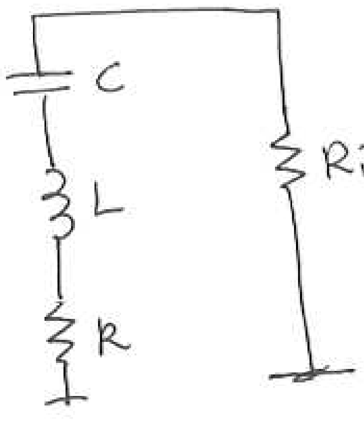
Oscillator using a series RLC circuit



$R_{in} = -R_f$

This circuit is equivalent to

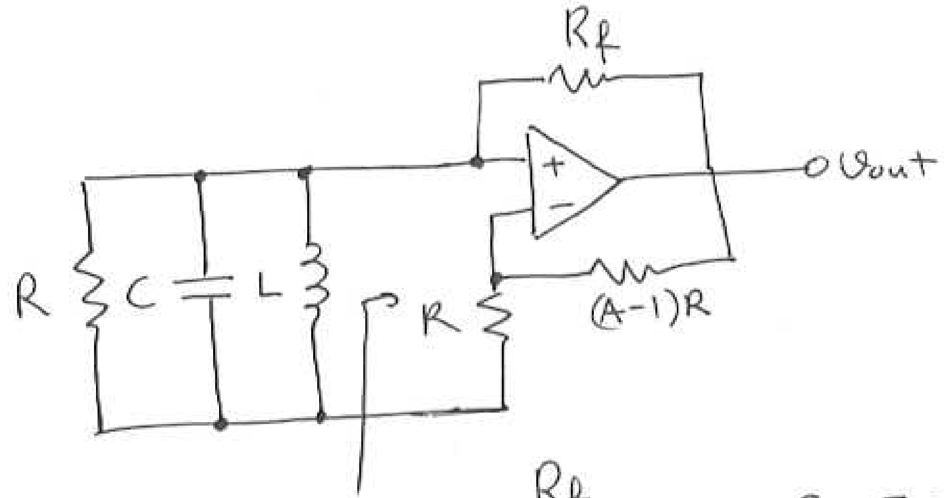


$R_{eq} = R + R_{in}$

The circuit is going to have a growing oscillation if $R_{in} + R < 0 \Rightarrow$

$R < -R_{in}$

Oscillator using a parallel RLC circuit



$$R_{in} = \frac{R_f}{1-A} \quad \text{or} \quad G_{in} = \frac{1-A}{R_f}$$

If the conductance loading the tank circuit is negative then an oscillation starts. Therefore:

$$G_{total} = G + G_{in} < 0 \quad \text{where } G = \frac{1}{R} \ \& \ G_{in} = \frac{1}{R_{in}}$$

$$\frac{1}{R} + \frac{1}{R_{in}} < 0 \Rightarrow \frac{1}{R} < -\frac{1}{R_{in}} \Rightarrow R_{in} < -R$$

or

$$\frac{1}{R_{in}} < -\frac{1}{R} \Rightarrow \boxed{R < -R_{in}}$$ where $-R_{in}$ is positive

The deduction is:

1. At a series RLC circuit resistances must add up to a negative number in order to have an oscillation
2. At a parallel RLC circuit conductances must add up to a negative conductance in order to have an oscillation.

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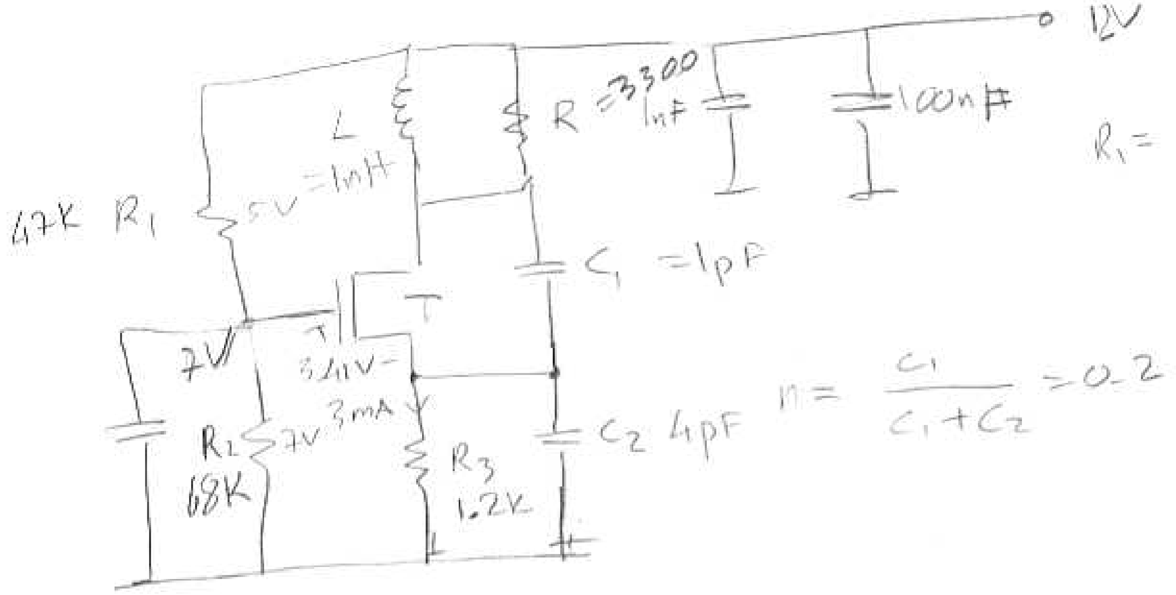
Examples: I



$R_1 =$

$n = \frac{C_1}{C_1 + C_2} = 0.2 \Rightarrow C_1 = 0.2C_1 + 0.2C$

$0.8C_1 = 0.2C_2$

$C_2 = 4C_1$

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{gs} - V_t)^2 = K_D(V_{gs} - V_t)^2$$

$$g_m = 2K_D(V_{gs} - V_t) = 2 \times 1.5 \times 10^{-3}(3.41 - 2)$$
$$= 4.23 \times 10^{-3} \text{ siemens}$$

$V_t = 2\ V \qquad K_D = 1.5\ mA/V^2$

$3\ mA \overset{\times 10^{-3}}{=} 1.5 \times 10^{-3}(V_{gs} - 2)^2 \Rightarrow 2 = (V_{gs} - 2)^2 \Rightarrow \sqrt{2} + 2 = V_{gs}$

$V_{gs} = 3.41\ V$

$$R_3 = \frac{7 - 3.41}{3\ mA} = \frac{3.41\ V}{3\ mA} = 1.13\bar{6}\ K\Omega \approx 1.2K$$

$$g_m = 6 \times 10^{-3} > \frac{1}{Rn(n-1)} \Rightarrow$$

$$Rn(1-n) > \frac{1}{g_m} = \frac{1}{6 \times 10^{-3}} = 166.\bar{6}$$

let $n = 0.2 \Rightarrow n(1-n) = 0.2 \times 0.8 = 0.16$

$R > 1041$

$\frac{1.2}{n^2} = 30000 \qquad R // 30000 > 1041 \Rightarrow R = 2200$

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Example 2 continued

First, find the D.C - bias

$K_D = 1.5 \times 10^{-3}$

$V_G = 12 \frac{68}{47+68} = 7.095 = 7V$

2

$(V_G - V_S - 2)^2 \times 1.5 \times 10^{-3} = \frac{V_S}{1.2 \times 10^3}$

$V_G - V_S{}^2 = (5 - V_S)^2 = \frac{V_S}{1.8} = 0.555\,V_S$

$25 - 10\,V_S + V_S{}^2 = 0.555\,V_S$

$25 - 10.555\,V_S + V_S{}^2 = 0$

$\Rightarrow V_S = 7 - 3.41 = 3.59$

$R' = 3300 // \frac{1200}{n^2} = 3300 // \frac{1200}{0.2^2} = 3300 // 30000$

$R' = 2972$

$2972 \cdot 0.2 \times 0.8 = 475.6\Omega$

$\frac{1}{4756} \cong 0.0021 < \frac{1}{g_m} = 2 \times K_D (V_{gs} - V_t)$

$\frac{1}{g_m} = 2 \times 1.5 \times 10^{-3} (3.41 - 2) = 0.00423$

$V_{tank} = 2 I_{BIAS} R(1-n) = 2 \times 3 \times 10^{-3} \times 2972 \times 0.8$
$= 14.2656\,V_{peak} \Rightarrow$ too large

$12 - 3.6 = 8.4V$ is the actual swing

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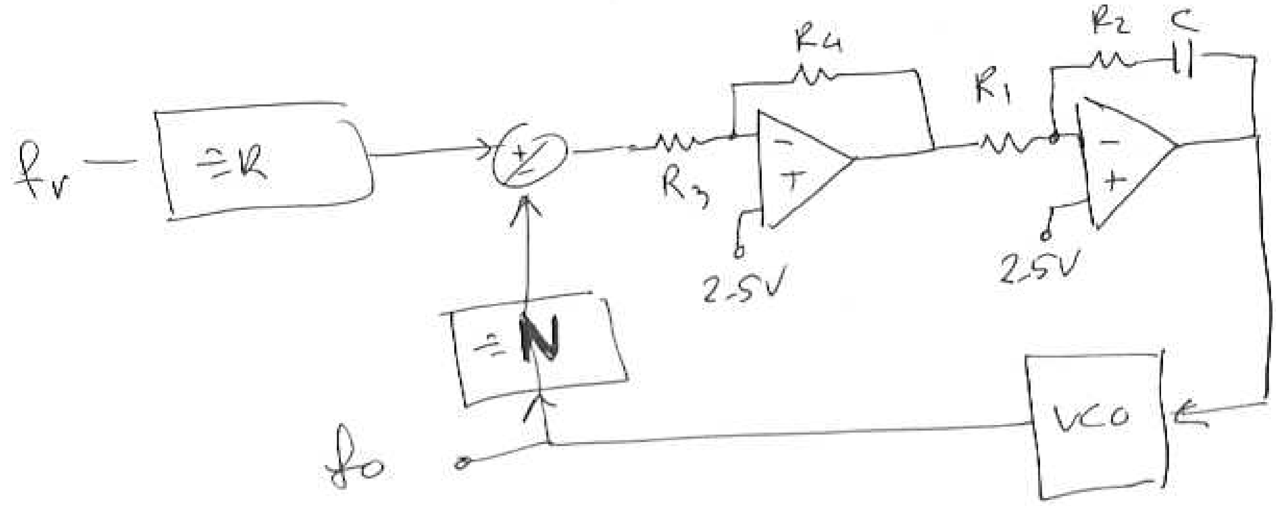
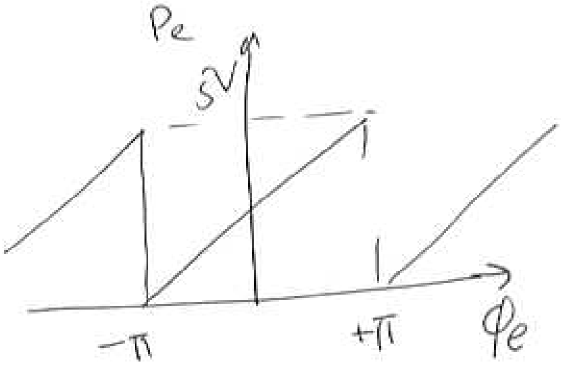
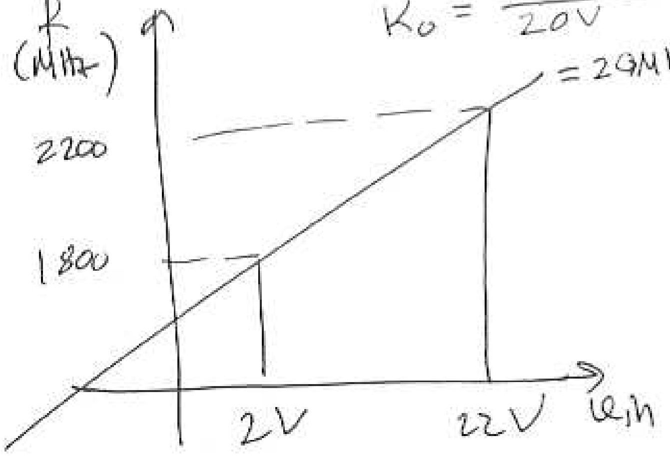
$$G = \frac{R_4}{R_1}$$





$$K_D = \frac{5V}{2\pi} = 0.796\ V/rad$$

$$K_o = \frac{400\ MHz}{20V} = 20\ MHz/V$$



Design the circuit so that the crossover frequency of the closed loop is 1KHz & the phase margin is 30^0

Choose $f_{ref} = 1MHz$

$$\tau_z = R_2 C \qquad R_1 C = \tau_1$$

$$tg 30^0 \qquad 0.577 \Rightarrow K \cong 2 = \frac{w_z}{w_n}$$

$$\Rightarrow w_z = 2 w_n$$

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$$w_c \text{ closed loop} = \sqrt{2}\, w_n = 2\pi \times 10^3 \text{ rad/sec}$$

$$\Rightarrow w_n = \frac{2\pi \times 10^3}{\sqrt{2}} \text{ rad/sec} = 4443 \text{ rad/sec}$$

$$w_z = 2 w_n = 2 \times 4443 = 8886 \text{ rad/sec}$$

$$\tau_z = R_2 C = \frac{1}{w_z} = \frac{1}{8886 \text{ rad/sec}} = 1.125 \times 10^{-4} \text{ sec}$$
$$= 0.1125 \text{ msec}$$

$$w_n^2 = (4443)^2 = \frac{K_o K_p G}{R_1 C N}$$

$$19.74 \times 10^6 = \frac{0.796 \times 20 \times 10^6 \times 2\pi}{2000} \cdot \frac{G}{R_1 C}$$

$$\frac{19.74 \times 10^6 \times 2000}{0.796 \times 20 \times 10^6 \times 2\pi} = \frac{G}{R_1 C} = \frac{1239.9}{\pi} = \frac{1240}{\pi} = 394.7$$

$$\frac{G}{R_1 C} = \frac{1239.9}{\pi} \simeq \frac{1240}{\pi} = 394.7$$

$$\frac{R_1 C}{G} = 0.00253 \text{ sec}$$

$$R_2 C = 1.125 \times 10^{-4} \text{ sec.}$$

$$\frac{R_1}{R_2 G} = 22.52$$

Let $R_2 G = 1K \Rightarrow R_1 = 22.52 K\Omega$ & Let $G = 1$

choose $C = \frac{\tau_z}{R_2} = \frac{0.1125 \times 10^{-3}}{1K} = 0.1125 \mu F$
$= 112.5 nF$