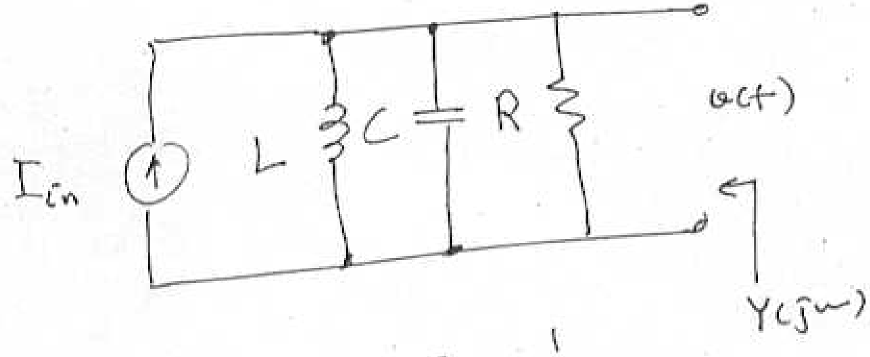
1.10.2003

RLC Circuits and their applications

Bandwidth and Q



$$Y(j\omega) = G + j\omega C + \frac{1}{j\omega L}$$

$$= G + j\left(\omega C - \frac{1}{\omega L}\right)$$

Conductance at the lowest or the impedance is at the highest and it is real at

$$\omega C - \frac{1}{\omega L} = 0 \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

Different definitions of Q around

$$Q = \omega \frac{\text{Energy stored}}{\text{average power dissipated}}$$ dimensionless

Q factor or Quality factor of the tank inductor or capacitor

$$= \frac{2\pi}{T} \frac{\text{Energy stored}}{P_{av}} = 2\pi \frac{\text{energy stored}}{T \times P_{av}}$$

$$= 2\pi \frac{\text{energy stored}}{\text{Energy dissipated in a cycle}}$$

peak voltage $= I_{pk} R$ at resonance

energy stored $= \frac{1}{2} C V_{pk}^2 = \frac{1}{2} C (I_{pk} R)^2$

average diss. pow $= \frac{1}{2} I_{pk}^2 R$

$$Q = w_0 \frac{E_{stored}}{P_{av}} = w_0 \frac{\cancel{\frac{1}{2}} C \cancel{I_{pk}^2} R^{\cancel{2}}}{\cancel{\frac{1}{2}} \cancel{I_p^2} \cancel{R}} = w_0 C R$$

since $w_0 C = \frac{1}{w_0 L}$ at resonance

$$Q = \frac{1}{w_0 L} R = \frac{\sqrt{LC}}{L} \times \frac{R}{L} = R \sqrt{\frac{C}{L}} = \frac{R}{\sqrt{L/C}}$$

also the reactive currents peak at resonance along with the voltage

$$|I_{ind}| = |I_{cap}| = \frac{|V|}{|Z|} = \frac{|I_R \cdot R|}{\underbrace{w L}_{=Q}} = Q |I_R|$$

Therefore branch currents are multiplied by Q at resonance, means that if $Q = 1000$ and you drive 1mA input to the tank, the capacitor and the inductor must be rated at 1 Amps

The approximate 3 dB BW of high Q resonators

let $\omega = \omega_0 + \Delta\omega$ $\quad |\Delta\omega| \ll \omega_0$

$$Y = G + \frac{-j}{\omega L} + j\omega C = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= G + j\frac{C}{\omega}\left[\omega^2 - \frac{1}{LC}\right]$$

letting $\omega = \omega_0 + \Delta\omega$

$$Y = G + j\frac{C}{\omega}\left[\cancel{\omega_0^2} + 2\Delta\omega\,\omega + \cancel{\Delta\omega^2} - \cancel{\frac{1}{LC}}\right]$$

but $\omega_0^2 = \frac{1}{LC}$ and letting $|\Delta\omega| \ll \omega$

$$Y = G + j\frac{C}{\cancel{\omega}}2\Delta\omega\,\cancel{\omega} = G + j2C\Delta\omega$$

just like a parallel RC circuit with $2C \to C$ & $\Delta\omega \to \omega$

The 3 dB BW of the system is defined by

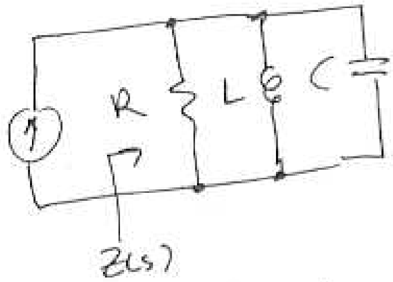
$$G = \pm 2C\Delta\omega \Rightarrow$$

$$\Delta\omega = \pm\frac{G}{2C} \Rightarrow BW = 2\Delta\omega = \frac{G}{C}$$

The normalized BW is given by

$$\frac{BW}{\omega_0} = \frac{2\Delta\omega}{\omega} = \frac{G}{C}\cdot\frac{\sqrt{LC}}{1} = G\sqrt{\frac{L}{C}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{Q}$$

$$\boxed{\frac{\omega_0}{BW} = Q}$$



The natural response is determined by the zeroes of the admittance or the poles of the impedance Z(s)

$$G(s) = \frac{1}{Z(s)} = G + sC + \frac{1}{sL} = \frac{GLs + s^2LC + 1}{sL}$$

$$GLs + LCs^2 + 1 = \underbrace{LC}_{a}s^2 + \underbrace{GL}_{b}s + \underbrace{1}_{c} = 0$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-GL \pm \sqrt{G^2L^2 - 4LC}}{2LC}$$

$$= \frac{-G}{2C} \pm \underbrace{\sqrt{\frac{G^2\cancel{L^2}}{4\cancel{L^2}C^2} - \frac{1}{LC}}}_{\omega}$$

$$= \underbrace{-\frac{1}{2RC}}_{\text{the damping factor}} \pm \underbrace{\sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}}_{\omega}$$

where $\frac{1}{4R^2C^2} - \frac{1}{LC} \ll 0$

or $\frac{1}{LC} \gg \left(\frac{1}{2RC}\right)^2$

$\Rightarrow$ high Q

$$v_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} =$$

$$= B_1 e^{-\frac{t}{2RC}} \sin(\omega t + \theta)$$

The time response can also be written as

$$Q = \frac{w_0}{BW} = RCw_0$$

$$\Rightarrow RC = \frac{Q}{w_0} \Rightarrow \frac{1}{2RC} = \frac{w_0}{2Q} = \frac{\cancel{2}\pi}{T}\frac{1}{\cancel{2}Q}$$

$$\Rightarrow \frac{1}{2RC} = \frac{\pi}{TQ}$$

$$v_n(t) = B_1 e^{-\frac{\pi}{TQ}t} \sin(\omega t + \theta)$$

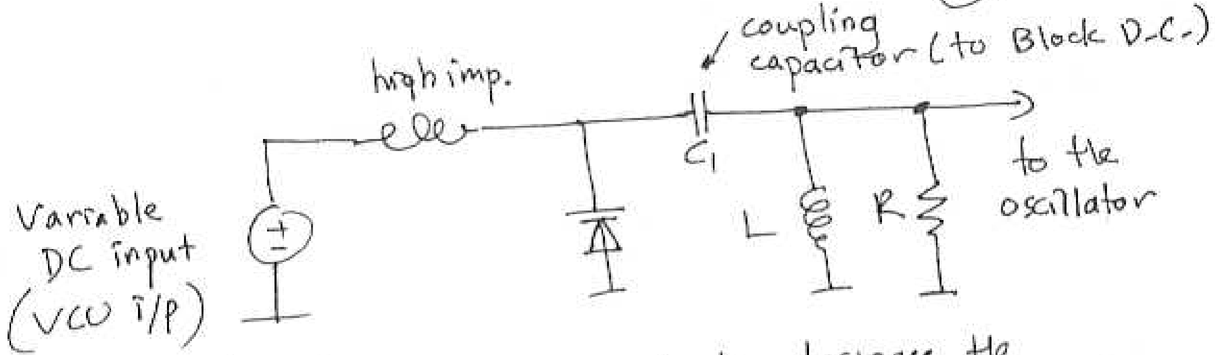
letting

$$e^{-\frac{\pi}{TQ}t} = e^{-1}$$

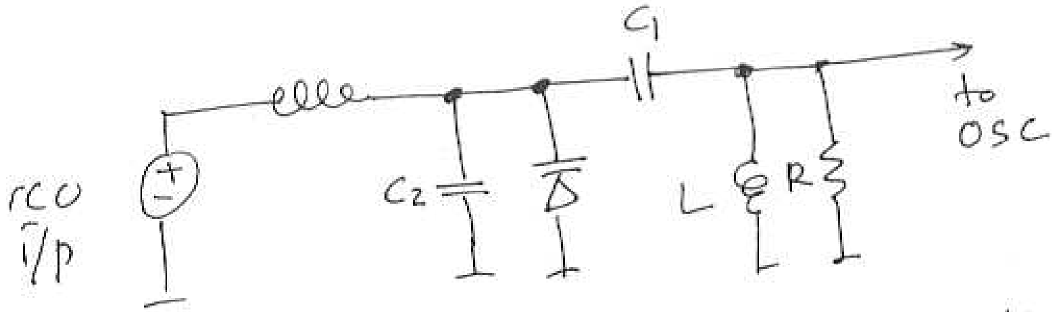
$\frac{\pi}{TQ}t = 1 \Rightarrow t = \frac{TQ}{\pi}$ or $\frac{Q}{\pi}$ cycles is the time constant of decay

if we say that 4% is the vanishing poing

$0.04 \approx e^{-\pi} \Rightarrow Q$ gives us the no of cycles of ringing - A useful tool in estimating Q from the impulse response



Sometimes C_1 is used to decrease the capacitor range to a suitable value and also to absorbe some of the voltage on the varactor to limit the A-C voltage on the voractor. A more practical circuit is



Suitable values of C_1 & C_2 maps the required capacitor voractor range to the required capacitor range whilst manupulating the A-C swing on the varactor to minimize the effects of the nonlinearity of the varactor.

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Example:

Design a VCO (voltage controlled oscillator) tank circuit centered at 100 MHz using a 3-20pF varactor diode (Centered means the geometric mean of the lowest & highest frequencies). The Q of the circuit must at least be 100. Find L & the minimum parallel resistance of the tank and draw the circuit.

$$w_{low} = \frac{1}{\sqrt{C_{high} L}} \quad , \quad w_{high} = \frac{1}{\sqrt{C_{low} L}}$$

$$w_{low} \times w_{high} = w^2_{midrange} = \left(2\pi\ 100\times10^6\right)^2 = \left(6.28\times10^8\right)^2$$

$$\frac{1}{\sqrt{20\times10^{-12} \times 3\times10^{-12}}} \cdot \frac{1}{L} = 3.94\times10^{17}$$

$$\frac{1}{\sqrt{60}\times10^{-12} \times 3.94\times10^{+17}} = L$$

$$L = \frac{1}{7.75\times3.94\times10^5} = 3.27\times10^{-7} = 0.327\mu H = 327 nH$$

$$Q = \frac{R}{w_0 L} = R w_0 C$$

$$80 = R\times2\pi\times10^8 C = \frac{R}{2\pi\times10^8 L}$$

$$\Rightarrow R = Q w_0 L = 80\times6.28\times10^8\times0.327\times10^{-6} = 16{,}453\ \Omega$$

$$\boxed{R = 16.5 k\Omega \text{ min}}$$

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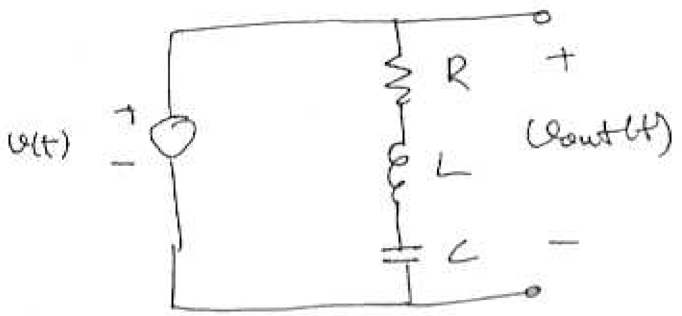
Series RLC networks



Almost the same kind of equations are used except Q is given in a different form as the dual of the parallel RLC circuit

$$Q = \frac{\sqrt{L/C}}{R} = \frac{\sqrt{L}}{R\sqrt{C}} = \frac{\sqrt{L}}{R\sqrt{C}} \times \frac{\sqrt{L}}{\sqrt{L}} = \frac{L w_0}{R}$$

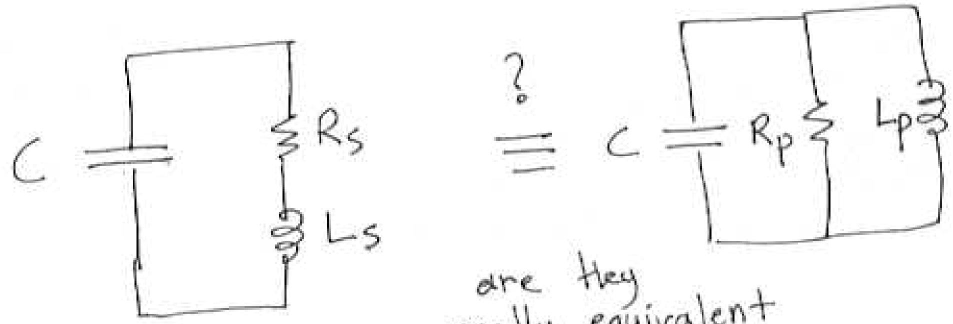
$$= \frac{\sqrt{L}}{R\sqrt{C}} \cdot \frac{\sqrt{C}}{\sqrt{C}} = \frac{\sqrt{LC}}{RC} = \frac{1}{w_0 RC}$$

reciprocal of the parallel tank formulas

You can work them out if you wish to.

Other RLC circuits

How do we account for series losses of the energy storing elements.



are they really equivalent or equivalent under what condition

$$R_s + j\omega L_s = R_p // j\omega L_p = \frac{R_p\, j\omega L_p}{R_p + j\omega L_p}$$

$$= \frac{R_p\, j\omega L_p\left[R_p - j\omega L_p\right]}{R_p^2 + \omega^2 L_p^2}$$

$$R_s + j\omega L_s = \frac{\omega^2 R_p L_p^2 + j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

equating real and imaginary parts

$$R_s = \frac{\omega^2 R_p L_p^2}{R_p^2 + \omega^2 L_p^2} \quad \text{and} \quad \omega L_s = \frac{\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

But $\frac{R_p}{w_0 L_p} = Q$ or $R_p^2 = w_0^2 L_p^2 Q^2$

$L_p = \frac{R_p}{w_0 Q}$

notice w becomes w_0
so at resonance when $w = w_0$

$w_0^2 L_p^2 = \frac{R_p^2}{Q^2}$

$$R_s = R_p \cdot \frac{R_p^2/Q^2}{R_p^2 + R_p^2/Q^2}$$

$$\boxed{R_s = R_p \cdot \frac{1}{Q^2} \cdot \frac{1}{1 + \frac{1}{Q^2}} = R_p \frac{1}{Q^2+1}}$$

$$\boxed{R_s (Q^2+1) = R_p}$$

and

$$w L_s = \frac{w L_p R_p^2}{R_p^2 + w^2 L_p^2} = w L_p \frac{R_p^2}{R_p^2 + \frac{R_p^2}{Q^2}}$$

$$L_s = L_p \frac{1}{1 + \frac{1}{Q^2}} = L_p \frac{Q^2}{Q^2+1}$$

$$L_s \frac{Q^2+1}{Q^2} = L_p$$

inductance changes slightly and series resistance is multiplied by Q^2+1 to be transferred to parallel
... the same Q

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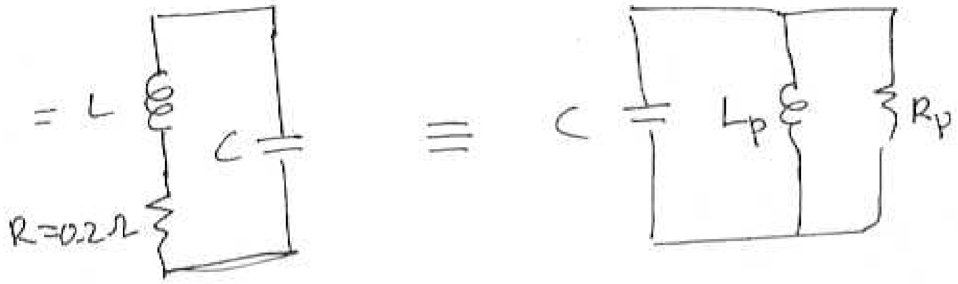
Example 1

let $w = 10^9$ $Q = 40$



$R_p = R_s (Q^2+1) = 0.2 \times (40^2+1) = 0.2 \times 1601$

$\cong 320\Omega$

$L_s = 10nH$ $w^2 = \frac{1}{LC}$ $C = \frac{1}{w^2 L} = 100pF$

$L_p = L_s \frac{Q^2+1}{Q} = 10.00625$

Talk about the effective series resistance of a capacitor

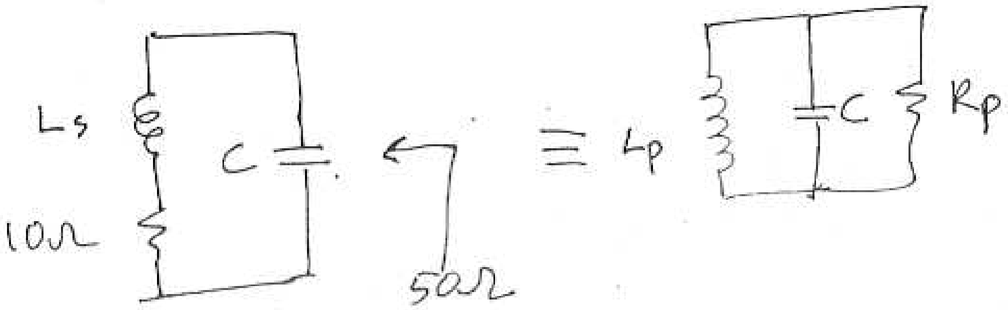
Example 2

Impedance transformation

$R_p = 50\Omega$, $R_s = 10\Omega$ $f_0 = 10MHz$

Find L & C such that R_s is transformed to parallel load R_p

$Q^2+1 = \frac{R_p}{R_s} = 5 \Rightarrow Q^2 = 4$, $Q = 2$



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(11)

$Q = \frac{w_0 L}{R_s}$ series circuit

$2 = \frac{2\pi \times 10^7}{10\Omega} L \Rightarrow L = \frac{\cancel{2} \times 10}{\cancel{2} \times \pi \times 10^7}$

$$\boxed{L = \frac{1}{\pi \times 10^6} = \frac{10^{-6}}{\pi} = 3.18 \times 10^{-7} = 318\, nH}$$

$Q = w_0 R_p C = 2\pi \times 10^7 \times 50 \times C \Rightarrow$

$C = \frac{Q}{2\pi \times 10^7 \times 50} = \frac{\cancel{2}}{\cancel{2}\pi \times 10^7 \times 50} = 6.36 \times 10^{-10} F$

$$\boxed{C = 636\, pF}$$

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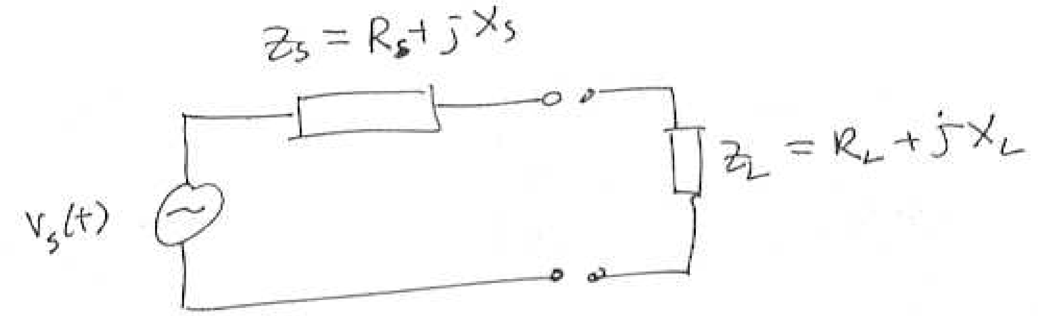
Maximum power transfer

The correct question is



Find Z_L such that maximum power is transferred to the load for a given Z_S

The opposite is not true.

$$P_{R_L} = \frac{|V_{R_L}|^2}{R_L} = \frac{R_L \qquad |V_s|^2}{(R_L+R_S)^2 + (X_L+X_S)^2}$$

obviously $X_L + X_S = 0$ is the optimum

find the maximum by finding the derivative of the dissipated power by R_L

$\Rightarrow$ $R_L = R_S$ $\qquad X_S + X_L = 0$ is the optimum solution.

1-10-2007

12 A

$$P_{R_L} = |V_S|^2 \cdot \frac{R_L}{(R_L+R_S)^2}$$

$$\frac{d\,P_{R_L}}{d\,R_L} = |V_S|^2 \left[1 \times \frac{1}{(R_L+R_S)^2} - \frac{2\,R_L}{(R_L+R_2)^3} \right]$$

$$\frac{(R_L+R_S) - 2\,R_L}{(R_L+R_S)^3} = 0$$

$$R_L + R_S = 2\,R_L$$

$$\boxed{R_S = R_L}$$

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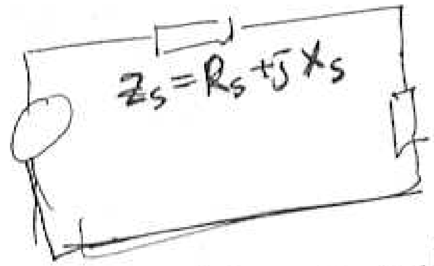
13

The wrong question is

Find the optimum source resistance R_s for a given R_L for maximum power transfer

V_s



$Z_L = R_L + jX_L$

The optimum solution is naturally!!
$X_L + X_s = 0$ and $R_s = 0$!

The reason we use $R_s = R_L$ is that we use $\boxed{R_s = R_L = R_{characteristic}}$ to prevent reflections and mismatch when a transmission line is involved.

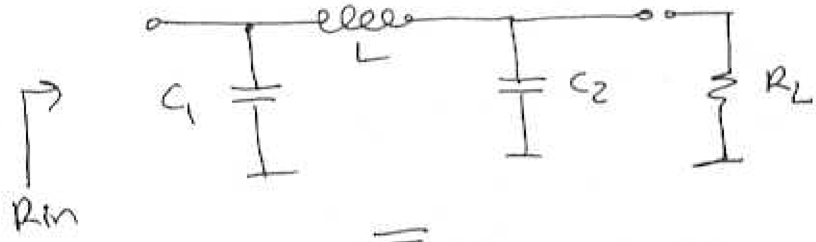
8.10.03

(1)

The Π Match

In order to be able to choose the transformation ratio and the Q of the Π-Match independantly, we can combine 2 L-Match circuits to arrive at an Π-match circuit

1



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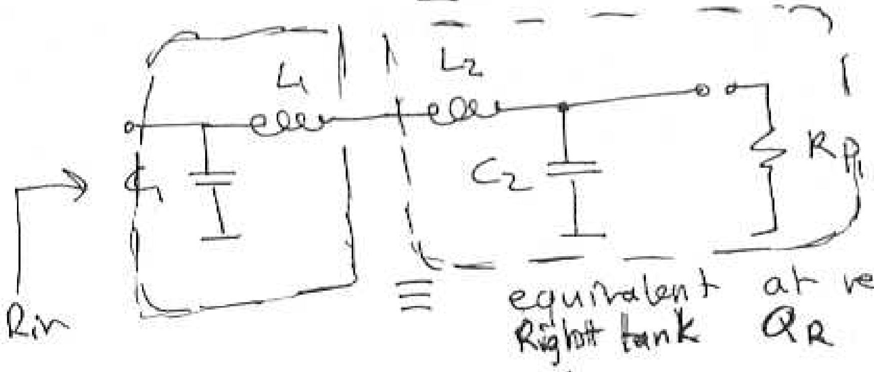
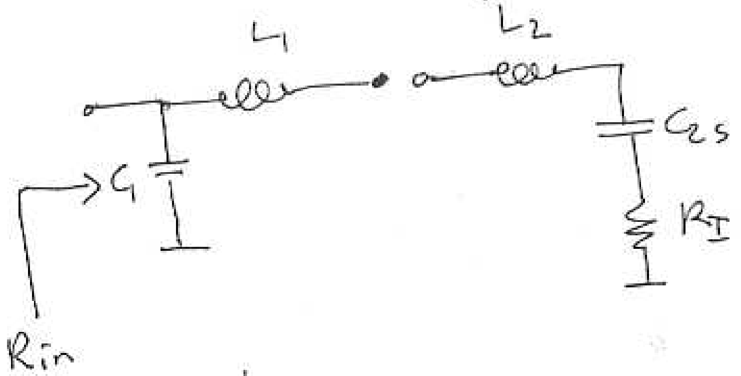


2 L-match circuits in series

≡ equivalent at resonance
Right tank Q_R

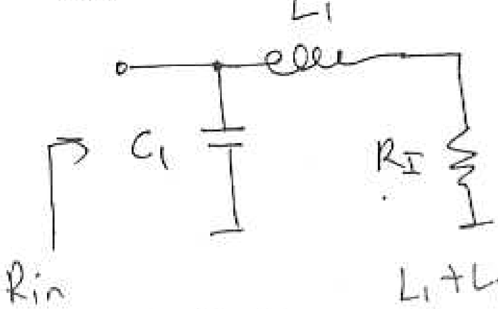
3



where $R_I = \frac{R_L}{Q_R^2+1}$

and since L_2 & C_{2s} are in series resonance

4



$R_{in} = R_I\,(Q_L^2+1)$

back to parallel again

$L_1 + L_2 = L_T$

Please note that here the only lossy element is R_L!!!

8.10.03

(02)

Q_L is given by

$$Q_L = w_0 C_1 R_{in} = \frac{w_0 L_1}{R_I}$$

$$Q_R = w_0 C_2 R_L = \frac{w_0 L_2}{R_I}$$

$$Q_{Total} = \frac{w_0 (L_1 + L_2)}{R_I}$$ from Figure 3

Here it must be emphesized that Rin is the impedance from the terminals of Rin, i.e. Q_{Total} is the Q of the circuit driven by an ideal current source. when talking about the Q of a circuit drien by a 50Ω like 50Ω must also be taken into account. If the loss of the inductor L_I is represented by R_{LOSS} (serres loss) and letting

$$R_I' = R_I + R_{LOSS}$$

Rin becomes

$$R_{in} = (R_I + R_{LOSS})(Q_L^2 + 1)$$

where $Q_L = \dfrac{w_0 L_1}{R_I + R_{LOSS}}$

8.10.03

(3)

$$Q_T = Q_L + Q_R = \frac{\omega(L_1+L_2)}{R_I}$$

$$= \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_L}{R_I} - 1}$$

To attack a design problem where typically R_{in}, R_L is known and Q is calculated from the BW of the match, find R_I first. Finding R_I from the equation R_I
re first requires either iteration or high Q assumption.
look,

But actually the problem has an analytical solution. → look at the note on π-Match calculation.

8-10.2003

A-1

Π match design example

$R_{in} = 50\Omega$ $R_S = 10\Omega$ $f_0 = 10MHz$

$Q = 30$

$Q = \frac{X_L}{R_I} \Rightarrow R_I = \frac{X_L}{Q}$ if you find R_I the Q_R & Q_L problem is solved.

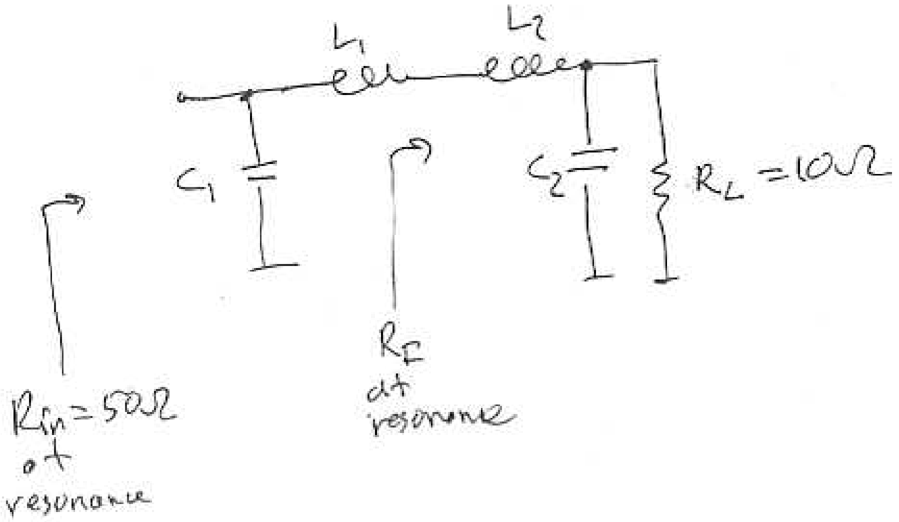
$$Q_R = \frac{[Q^2K + 2K - K^2 - 1]^{1/2} - Q}{K - 1} \quad \text{where } K = \frac{R_{in}}{R_L} = \frac{50}{10} = 5$$

$$Q_R = \frac{[30^2 \times 5 + 10 - 25 - 1]^{1/2} - 30}{4} = \frac{66.96 - 30}{4} = 9.24$$

$Q_L = 30 - 9.24 = 20.76$

$R_I \; (Q_R^2 + 1) = R_L$

$$\frac{R_L}{(Q_R^2+1)} = R_I = \frac{10}{9.24^2+1} = 0.1157\Omega$$



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A-2

$X_L = R_I \times Q = 3.473\ \Omega$
overall

$Q_L = w_o C_1 R_1 = \frac{R_{in}}{-X_{C_1}}$

$X_{C_1} = \frac{R_{in}}{-Q_L} = \frac{50}{-20.76} = -2.41$

$X_{C_2} = \frac{R_L}{-Q_R} = \frac{10}{-9.24} = -1.082$

I have not used the frequency yet. I just found the reactances for a π-match
I now calculate component values by

$wL_{Total} = 3.473\ \Omega$ by letting $w = 2\pi f$
$= 62.8 \times 10^6$ rad/sec

$L = 5.53 \times 10^{-8}$ H $= 55.3$ nH

$\frac{1}{C_1 w} = -X_{C_1}$ $\quad C_1 = \frac{-1}{X_{C_1} w} = 6.61 \times 10^{-9}$ F $= 6.61$ nF

$C_2 = \frac{-1}{X_{C_2} w} = 1.47 \times 10^{-8}$ F $= 14.7$ nF

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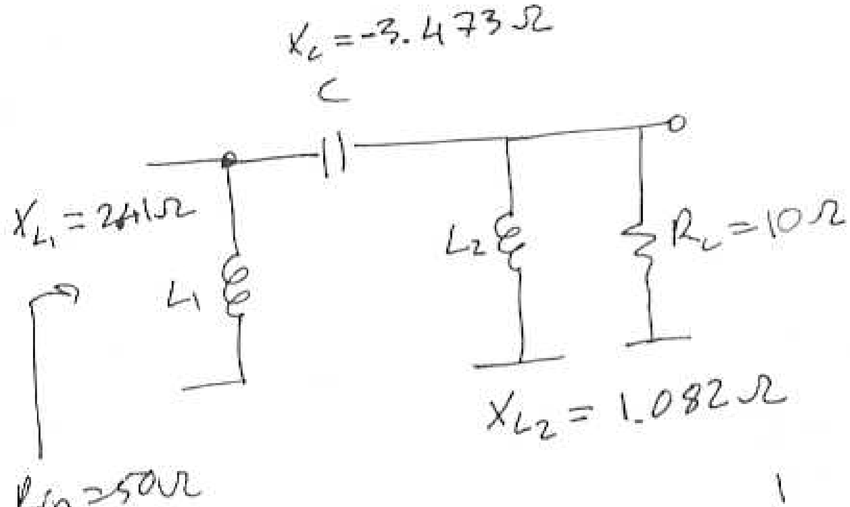
A-3

Assume we need a high pass Π-match



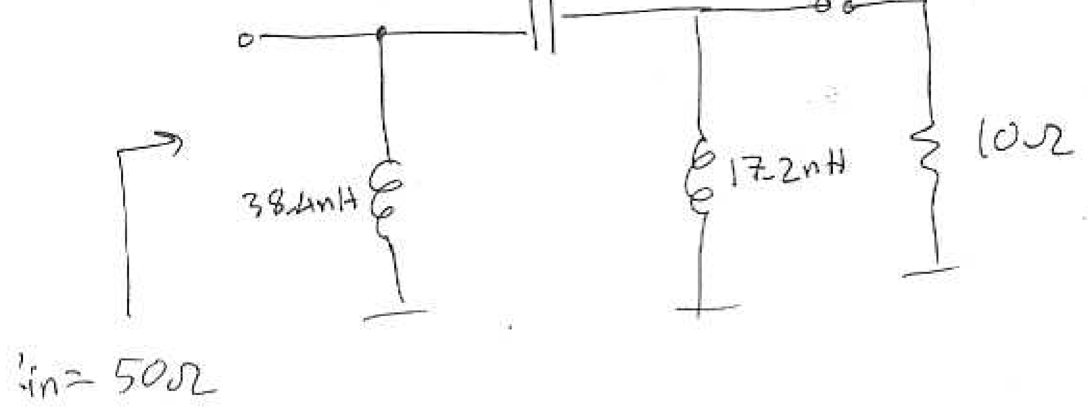
$$\frac{1}{wC} = -3.473\,\Omega \Rightarrow C = \frac{1}{3.473 \times w} = 4.58 \times 10^{-9}\,F$$

$C = 4.58\,nF$

$wL_1 = 2.41\,\Omega \Rightarrow L_1 = \frac{2.41}{w} = 3.84 \times 10^{-8}\,H = 38.4\,nH$

$wL_2 = 1.028\,\Omega \Rightarrow L_2 = \frac{1.082}{w} = 1.72 \times 10^{-8}\,H = 17.2\,nH$

Therefore the mathing circuit is



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A-4

Here we must note that:

1. C_1 & L_1 are in resonance at w_0

$$\frac{1}{(C_1 L_1)^{1/2}} =$$

$$\frac{w_0 L_1}{R_1} = Q_L \rightarrow L = \frac{Q_L R_I}{w_0} = \frac{20.76 \times 0.1157}{62.8 \times 10^6}$$

$$L_1 = 3.825 \times 10^{-8} H = 38.25 nH$$

$$w_0 = \frac{1}{(C_1 L_1)^{1/2}} = 62.9 \text{ Mrad/sec}$$

$$L_2 = L_T - L_1 = 55.3 - 38.25 = 17.05 nH$$

2. L_2 and C_2 series is in resonance

$$C_{series} = C_p \frac{Q^2+1}{Q}$$

$$C_{2series} = 14.7 \times \frac{Q_R^2+1}{Q_R^2} = 14.7 \times \frac{9.25^2+1}{9.25^2}$$

$$= 14.87 nF$$

$$w_0 = 62.8 \text{ Mrad/sec}$$

3. Note that C_1 series $C_{2series}$ and L_T is in resonance:

$$C_{eq} = \frac{C_1 \times C_{2ser}}{C_1 + C_{2ser}} = 4.576 nF$$

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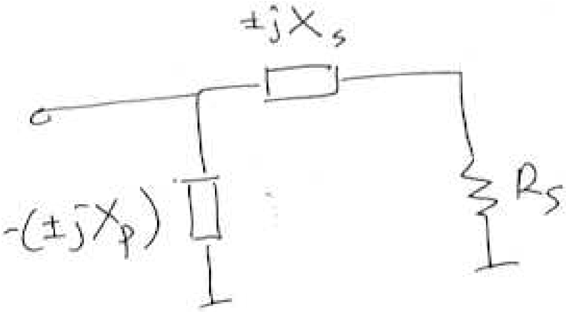
A-5

$$\omega_0 = \frac{1}{C_{eq} \times L_T} = \frac{1}{(4.576 \times 10^{-9} \times 55.3 \times 10^{-9})^{1/2}}$$

$= 62.86$ M rad/sec

L- Match

Match a load into the source (both resistive using a tank circuit.
Using the parallel-to-series transformation



R_p always higher than R_s for $Q > 1$

$$Q_s = \frac{X_s}{R_s} = \frac{R_p}{X_p} = Q_p = Q$$

$$R_p = (Q^2 + 1) R_s \Rightarrow \frac{R_p}{R_s} - 1 = Q^2$$

$\Rightarrow$ Q is determined by the transformation ratio.

26.9.2005

L-match -2

$$\frac{X_s}{R_s} = Q \Rightarrow X_s = Q \cdot R_s$$
$$\frac{R_p}{X_p} = Q \Rightarrow X_p = \frac{R_p}{Q}$$
$$\Rightarrow X_s \cdot X_p = \frac{\cancel{Q} \cdot R_s \cdot R_p}{\cancel{Q}}$$

or

$$\Rightarrow \frac{X_s}{R_s} = Q = Q = \frac{R_p}{X_p} \Rightarrow$$ all is determined

only one series reactance & one parallel reactance for a given set of R_p, R_s & Q

Example: a-) Match 75Ω into a 15Ω driver

b-) Find component values for a highpass circuit with $f = 1\,MHz$

c-) Find component values for a low pass circuit with $f = 20\,GHz$

d-) Find the loaded Q when driven by a 75Ω source and a 15Ω load.

solution

a-) $$Q^2 + 1 = \frac{75}{15} = 5 \Rightarrow Q = 2$$

$$X_s = 2 \times R_s = 2 \times 15 = 30\,\Omega$$
$$X_p = \frac{R_p}{Q} = \frac{75}{2} = 37.5\,\Omega$$

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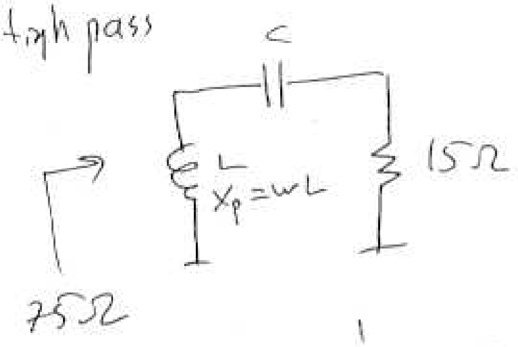
L-match -3

$X_s = \frac{1}{wC}$

b-) High pass



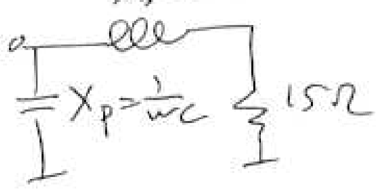
$w = 6.28 \times 10^{6}$

$X_s = 30 = \frac{1}{wC} \Rightarrow \quad C = \frac{1}{X_s w} = 5.307 \times 10^{-9}\,F$

$X_p = 37.5 = wL \Rightarrow L = \frac{X_p}{w} = \frac{37.5}{w} = 5.97 \times 10^{-6}\,H$

c-) Low pass

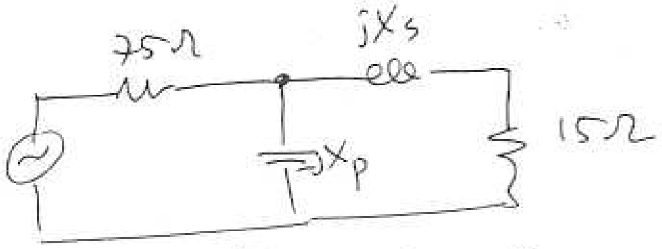
$X_s = wL$



$X_s = 30 = wL \Rightarrow \quad L = \frac{30}{w} = 2.388 \times 10^{-10}\,H$

$X_p = \frac{1}{wC} \Rightarrow \quad C = \frac{1}{X_p w} = \frac{1}{37.5 w} = 2.12 \times 10^{-13}\,F$

d-)



$Q_{Loaded} = \frac{Q_{match}}{2} = \frac{2}{1} = 1 = \frac{75 // 75}{37.5} = \frac{R_p}{X_p}$