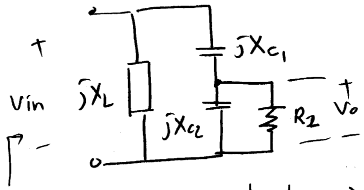
Tapped Impedance match



If the is very high $\Rightarrow R_2 \gg |X_{c2}| \Rightarrow$ this is a simple voltage divider and as the result

$$v_o = n v_{in} = \frac{X_{c2}}{X_{c1}+X_{c2}} v_{in} \Rightarrow n = \frac{X_{c2}}{X_{c1}+X_{c2}} = \frac{\frac{1}{\omega C_2}}{\frac{1}{\omega C_1}+\frac{1}{\omega C_2}}$$

$$\Rightarrow n = \frac{\frac{1}{C_2} C_1 C_2}{\left(\frac{1}{C_1}+\frac{1}{C_2}\right) C_1 C_2} = \frac{C_1}{C_1+C_2}$$

Since dissipation must be the same the equivalent resistance Rin is related R_2 as

$$\frac{\frac{V_{in}^2}{R_{in}}}{\frac{V_o^2}{R_2}} = 1 \Rightarrow \frac{V_{in}^2}{V_o^2} = \frac{R_{in}}{R_2} = n^2$$

$\Rightarrow R_{in} = n^2 R_2$ as in a transformer

Note that the loss is always transformed, but to observe it purely you need to tune out the capacitive part.

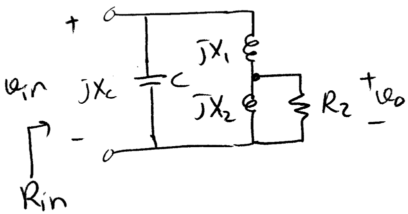
The capacitive part to be tuned out is given by approximately again

$$X_{C_1} + X_{C_2} \quad \text{as} \quad R_2 \ll X_{C_2} \Rightarrow$$

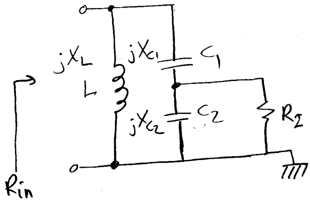
$$C_{eq} = C_1 \text{ in series with } C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

To solve the problem exactly at resonance please refer to "a note on impedance transformer".

The inductive divider acts similarly with only the capacitive and inductive impedances interchanged.



A note on resonant impedence transformer



$X_{C_{2S}}$

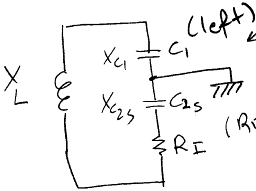
In this circuit we would like to find the impedance seen in parallel with the inductor, namely R_{in}. The problem is similar to Π-match, with the differences:

a-) The ground is at a different position

b-) we are trying to find the equivalent resistance seen in parallel with L instead of in parallel with C_1

The above circuit is equivalent to grounding changed

(left)



$$Q_R = \frac{R_2}{X_{C2}} \Rightarrow X_{C_2} = \frac{R_2}{Q_R}$$

(Right) $R_I = R_L / (Q_R^2 + 1)$

$$X_{C_{2S}} = \frac{Q_R^2}{Q_R^2 + 1} X_{C_2}$$

The circuit resonates at the frequency w_0 defined by X_L & $X_{C_1} + X_{C_{2S}}$

Our aim is to transform the resistance R_L to R_{in} at resonance. If we let

$$K = \frac{R_{P_L}}{R_2} = \frac{R_{in}}{R_2}$$ where R_{P_L} is the equivalent resistance in parallel with L

$$R_{in} = R_{P_L} = R_I(Q^2+1)$$ where $Q = Q_L + Q_R$

$$(Q^2+1)R_I = R_{in} \quad Q$$

$$(Q_R^2+1)R_I = R_2$$

diving the 2 equations

$$\frac{Q^2+1}{Q_R^2+1} = \frac{R_{in}}{R_2} = K \Rightarrow$$

$$Q^2+1 = KQ_R^2 + K \Rightarrow KQ_R^2 = Q^2+1-K \Rightarrow$$

$$Q_R = \sqrt{\frac{Q^2+1-K}{K}}$$

$$Q_L = Q - Q_R = Q - \sqrt{\frac{Q^2+1-K}{K}}$$

After finding Q_L & Q_R for the required Q & K R_I can be found by

$$R_I = \frac{R_2}{Q_R^2+1}$$ and $Q_L = Q - Q_R = \dfrac{X_{C_1}}{R_I}$ and $Q = \dfrac{R_{in}}{X_L}$

$$\Rightarrow X_{C_1} = Q_L R_I \text{ and } X_L = \frac{R_{in}}{Q}$$

or alternatively use the following steps in solving the problem:

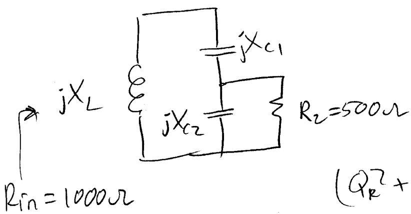
1.) $(Q^2+1)R_I = R_{in} \Rightarrow R_I = \dfrac{R_{in}}{Q^2+1}$ & $X_L = \dfrac{R_{in}}{Q}$

2.) $(Q_R^2+1)R_I = R_2 \Rightarrow Q_R^2 = \dfrac{R_2}{R_I} - 1 \Rightarrow |X_{c2}| = \dfrac{R_2}{Q_R}$

3.) $Q - Q_R = Q_L = \dfrac{|X_{c1}|}{R_I}$

Example:

Find the reactances of the upward transformer which converts 500Ω into 1000Ω employing a $Q = 3$.



$(Q^2+1)R_I = R_{in}$

$(3^2+1)R_I = 1000$

$\Rightarrow R_I = \dfrac{1000}{10} = 100\Omega$

$(Q_R^2+1)R_I = 500\Omega \Rightarrow$

$Q_R^2 + 1 = \dfrac{500}{100} = 5 \Rightarrow Q_R = 2$

$\dfrac{|X_{c1}|}{100} = 3 - 2 = Q_L = 1 \Rightarrow X_{c1} = -100$

$Q_R = 2 = \dfrac{500}{|X_{c1}|} \Rightarrow X_{c1} = \dfrac{-500}{2} = -250j$

$X_L = \dfrac{R_{in}}{Q} = \dfrac{1000}{3} = 333.\overline{33}$