EE 411 Midterm # 1 27.11.2002
Solutions Q.1-1

Q.1 :

$$R_{in} = \frac{R_F}{1-A_v} = \frac{R_F}{1-(-7)} = 1000\Omega$$

$$\Rightarrow R_F = 8\times 1000\Omega = 8000\Omega$$

$$R_{in} = \frac{R_E(R_F+R_L)}{R_E+R_L} \Rightarrow$$

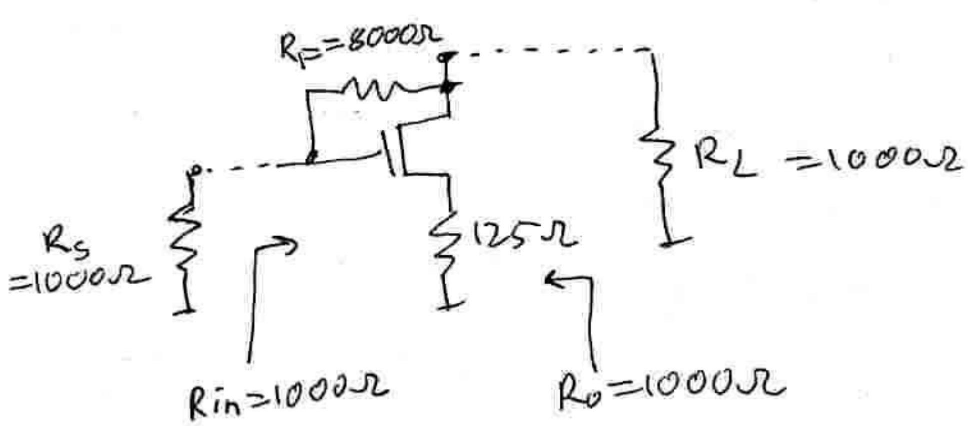
$$1000 = \frac{R_E(8000+1000)}{R_E+1000} \Rightarrow$$

$$1 = \frac{9R_E}{R_E+1000} \Rightarrow R_E + 1000 = 9R_E \Rightarrow$$

$$8R_E = 1000 \Rightarrow$$

$$R_E = \frac{1000}{8} = 125\Omega$$

Therefore the amplifier configuration obtained with the large gm assumption is:



Actually $R_E = \dfrac{1}{g_{m_{eff.}}} \Rightarrow$

$$g_{m_{eff}} = \frac{1}{R_E} = \frac{1}{125\Omega}$$

But $g_{m_{eff}}$ is equal to $\dfrac{g_m}{1+g_m R_1}$

Equating $\dfrac{1}{R_E}$ to $g_{m_{eff..}}$, one obtains:

$$\frac{1}{125\Omega} = \frac{1}{R_E} = \frac{g_m}{1+g_m R_1} \quad \text{or}$$

$$1+g_m R_1 = g_m R_E \Rightarrow g_m R_1 = g_m R_E - 1 \text{ and}$$

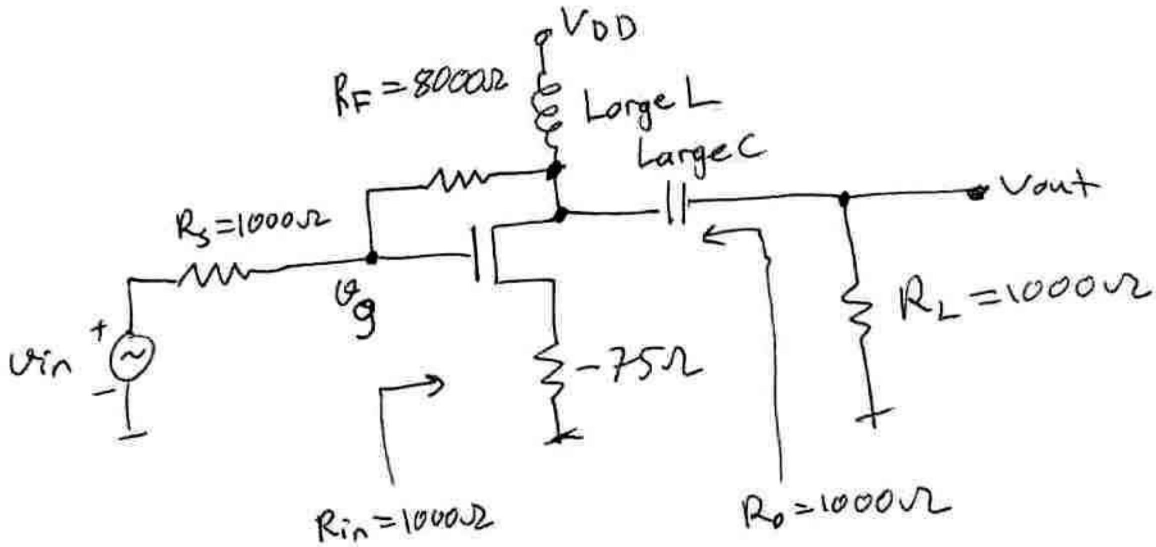
$$R_1 = \frac{g_m R_E}{g_m} = \frac{5\times10^{-3}\times 125 - 1}{5\times10^{-3}} = \boxed{-75\Omega = R_1}$$

Which means that the transistor gain is actually insufficient to obtain the gain of -7 with 1000Ω i/p and o/p impedances using a shunt-series amplifier. The minimum gain of the transistor, $g_{m_{min}}$, to obtain such gain is given by

$1 + g_{m_{min}} R_1 = g_{m_{min}} R_E$, letting $R_1 = 0$ is

$$g_{m_{min}} = \frac{1}{R_E} = \frac{1}{125\Omega} = 0.008 \text{ siemens} = \boxed{8\times10^{-3} \text{ siemens}}$$

Therefore one possible complete solution of of the question is



$$A_v = \frac{v_{out}}{v_g} = -7$$

$g_m = 5 \times 10^{-3}$ siemens

and $\dfrac{v_{out}}{v_{in}} = -3.5$

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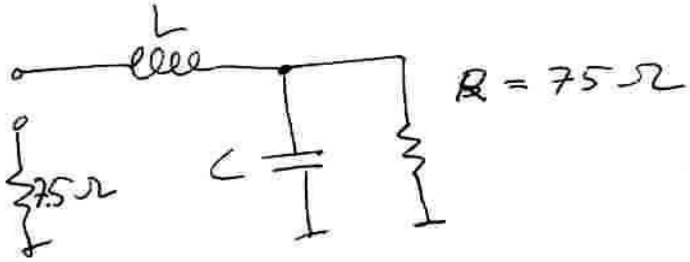
Q-2



Since it should be lowpass the parallel component must be a capacitor to suppress high frequencies and the series component must be open circuit at high frequencies, too. Therefore it must be an inductor.

$$7.5\Omega\,(Q^2+1) = 75\Omega \Rightarrow$$

$$Q^2+1 = 10 \quad \Rightarrow \quad Q = 3$$

$$w \cong 2 \times \pi \times 159 \times 10^6 \simeq 10^9 \text{ rad/sec}$$

$$\frac{wL}{R_s} = Q = 3 \quad \Rightarrow \quad \frac{10^9 L}{7.5\Omega} = 3$$

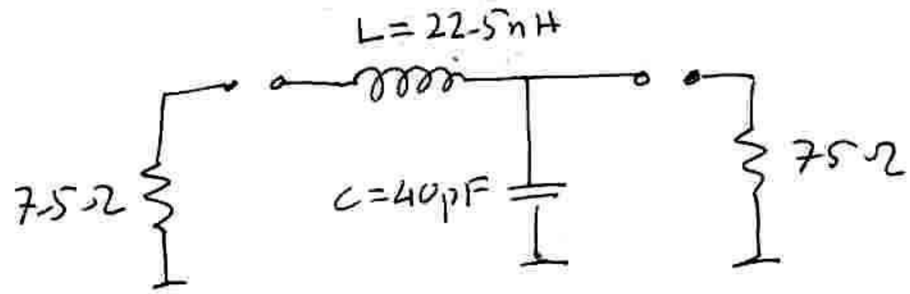
$$L = \frac{22.5}{10^9}\,\Omega = 22.5 \text{ nF}$$

$$wCR_p = Q \quad \Rightarrow \quad 10^9 \times C \times 75 = 3 \Rightarrow$$

$$C = \frac{3}{75 \times 10^9} = \frac{3}{75} \times 10^{-9} = \frac{3000}{75} \times 10^{-12} = 40 \times 10^{-12} F$$

$$= 40 pF$$

Therefore our matching circuit is



Q-3

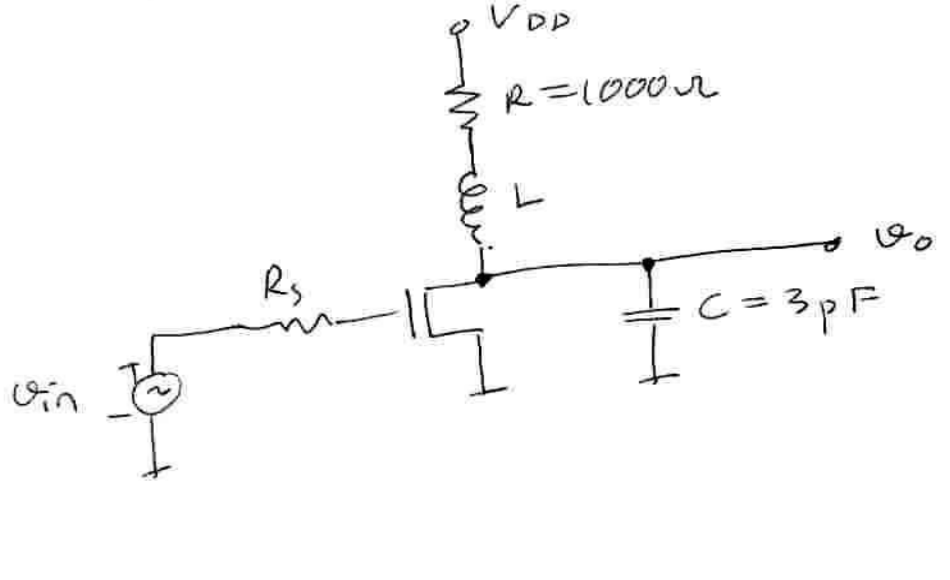
Assuming that the transistor is ideal, the 3dB-cutoff frequency of the amplifier is:

$$f_{cutoff} = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times RC} = \frac{1}{2\pi \times 10^3 \times 3 \times 10^{-12}}$$

$$= \frac{1}{2\pi \times 3 \times 10^{-9}} = \frac{10^9}{2\pi \times 3} = \frac{10^9}{6\pi} \cong 53.05 \times 10^6 \text{ Hertz}$$

$$\frac{97 \text{ MHz}}{53 \text{ MHz}} = 1.83$$

Therefore the 3dB BW of the amplifier must be extended at least by 1.83 times without degrading the other parameters of the amplifier. One of the ways of accomplishing that task is using the shunt-peak amplifier technique. The amplifier configuration, then becomes:



Q3-2

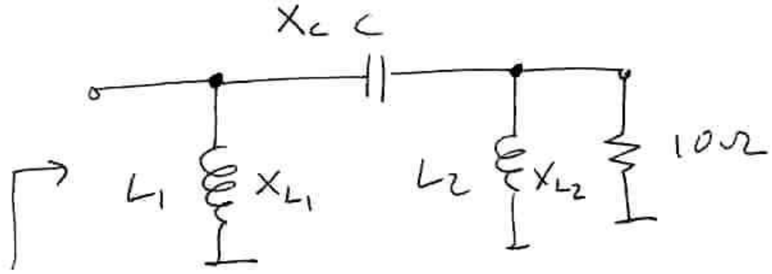
Using the shunt-peaking summary table given during the examination; 1.83 times bandwidth extension can only be provided by $m = 1.41$. Equating m to R^2C/L

$$1.41 = \frac{R^2C}{L} = \frac{10^6 \times 3 \times 10^{-12}}{L} \Rightarrow$$

$$L = \frac{3 \times 10^{-6}}{1.41} = 2.12 \times 10^{-6} H = 2.12 \mu H$$

Q-4

Since Q of the match is explicitly defined, the matching circuit can not be L-match. A Π-match circuit will do the trick. The Π-match, high pass solution is given by



$R_{in} = 50\Omega$

Since the frequency is not given, we can directly use impedances in the equations.

$$\frac{Q_L^2+1}{Q_R^2+1} = K = \frac{R_{in}}{R_L} = 5 \quad \text{and} \quad Q = 10$$

Using the formula derived before or by iteration

Q_R is found to be

$$Q_R = \frac{\left[Q^2K + 2K - K^2 - 1\right]^{1/2} - Q}{K-1} = \frac{[500+10-25-1]-10}{5-1}$$

$$= \frac{484^{1/2} - 10}{4} = \frac{22-10}{4} = \frac{12}{4} = 3 \Rightarrow$$

$Q_L = 10 - 3 = 7$

$$Q_L = \frac{R_{in}}{X_{L_1}} \Rightarrow 3 = \frac{50}{X_{L_1}}$$

$$X_{L_1} = \frac{50}{7}\Omega = 7.14\,\Omega$$

$$Q_R = \frac{R_L}{X_{L2}} = \frac{10\Omega}{X_{L2}} \Rightarrow$$

$$X_{L2} = \frac{10}{3}\Omega = 3.33\,\Omega$$

$$Q = Q_R + Q_L = 10 = \frac{X_C}{R_I} \quad \text{and} \quad R_I\,(Q_R^2+1) = R_L$$

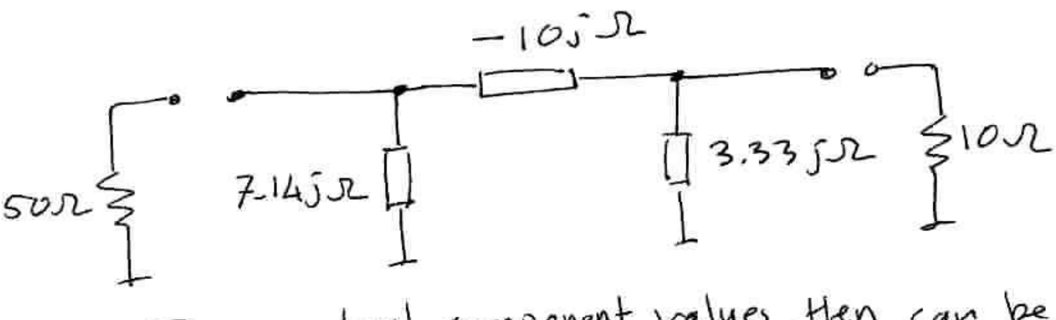
$$R_I\,(Q_L^2+1) = R_{in}$$

$$\Rightarrow R_I = \frac{R_L}{Q_R^2+1} = \frac{10}{3^2+1} = 1\,\Omega \quad \text{or}$$

$$R_I = \frac{R_{in}}{Q_L^2+1} = \frac{50}{49+1} = 1\,\Omega \ \checkmark$$

$$Q = \frac{X_C}{R_I} \Rightarrow 10 = \frac{X_C}{1\Omega} \Rightarrow$$

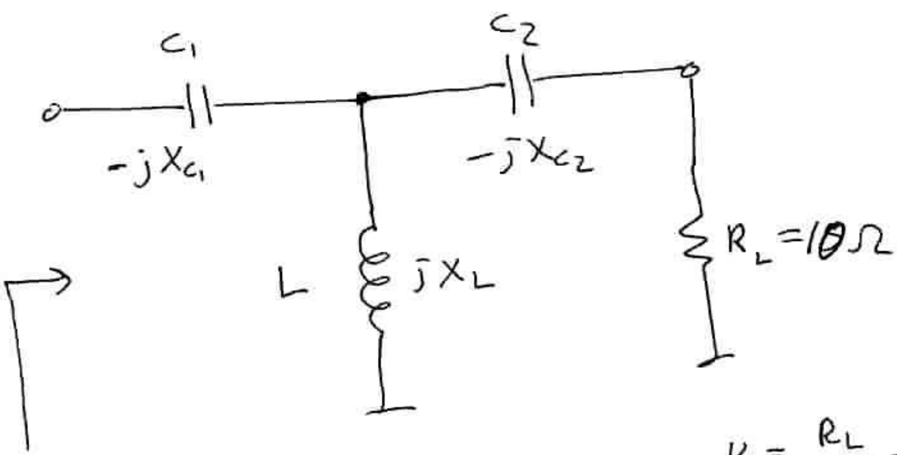
$X_C = 10 \times 1\Omega = 10\Omega$. Therefore our solution is



The actual component values, then can be determined if the operating frequency is given.

T-match solution for question 4:

Since we need a high-pass math with $Q = 10$ a T-match circuit shown as below will also serve as a solution.



$$K = \frac{R_L}{R_{in}} = \frac{10}{50} = 0.2$$

$R_{in} = 50\Omega$

$$Q_R = \frac{\left[Q^2 K + 2K - K^2 - 1\right]^{1/2} - Q}{K - 1}$$

$$= 7$$

$$Q_L = 10 - 7 = 3$$

$$Q_R = 7 = \frac{X_{C_2}}{R_L} \Rightarrow X_{C_2} = 7 \times 10\Omega = 70\Omega$$

$$Q_L = 3 = \frac{X_{C_1}}{R_{in}} \Rightarrow X_{C_1} = 3 \times 50 = 150\Omega$$

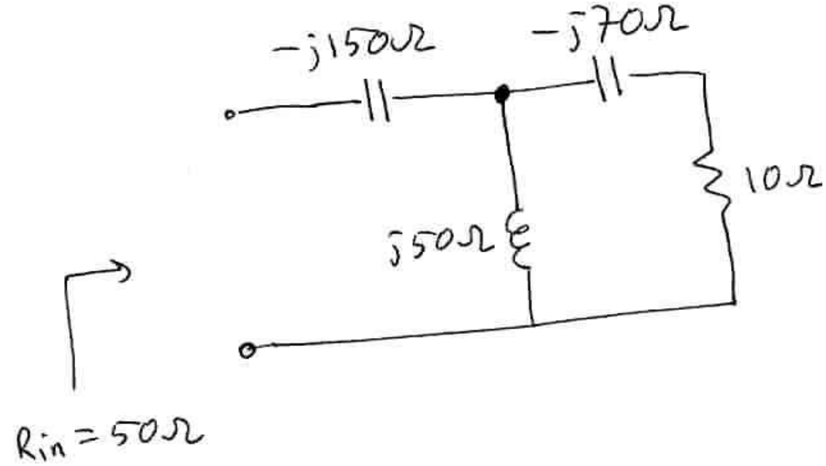
$$Q_T = 10 = Q_L + Q_R = \frac{R_I}{X_L}$$ and $$R_I = (Q_L^2 + 1) R_{in} = (9+1) \times 50 = 500\Omega$$

or

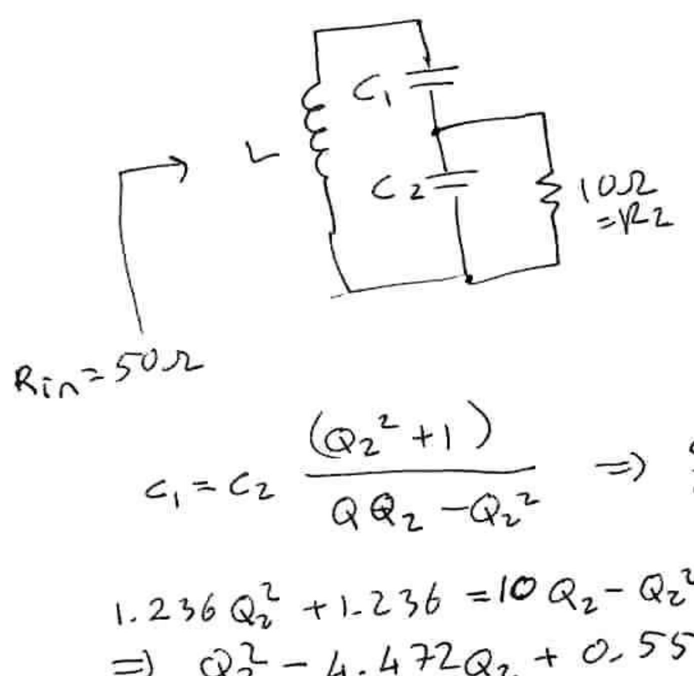
$$R_I = (Q_R^2 + 1) R_L = (49 + 1) \times 10 = 500\Omega$$

$$\Rightarrow X_L = \frac{R_I}{Q_T} = \frac{500}{10} = 50\Omega$$

Therefore our complete solution is



Tapped capacitor solution



$R_2 = 10\Omega$

$R_{in} = 50\Omega$

$Q = 10$

$\boxed{j\omega_o L = j\frac{R_{in}}{Q} = j\frac{50}{10} = j5\,\Omega}$

$n^2 = \left(\frac{C_1 + C_2}{C_1}\right)^2 = \frac{50}{10}$

$\frac{C_1 + C_2}{C_1} = \sqrt{5} = 2.236$

$\Rightarrow C_1 + C_2 = 2.236\, C_1$

$C_2 = 1.236\, C_1$

$$C_1 = C_2 \frac{(Q_2^2 + 1)}{Q Q_2 - Q_2^2} \Rightarrow \frac{C_2}{C_1} = 1.236 = \frac{Q Q_2 - Q_2^2}{Q_2^2 + 1} \Rightarrow$$

$$1.236\, Q_2^2 + 1.236 = 10\, Q_2 - Q_2^2 \Rightarrow 2.236\, Q_2^2 - 10 Q_2 + 1.236 = 0$$

$$\Rightarrow Q_2^2 - 4.472 Q_2 + 0.553 = 0 \Rightarrow$$

$$Q_2 = \frac{4.472 \pm \sqrt{4.472^2 - 4 \times 0.553}}{2} = \frac{4.472 \pm 4.217}{2} = \begin{matrix} 4.345 \\ 0.1275 \end{matrix}$$

using $Q_2 = 4.345$

$\boxed{Z_{C_2} = \frac{-j R_2}{Q_2} = \frac{-10j}{4.345} = -2.3j}$ $Z_{C_1} = Z_{C_2} \cdot \frac{C_2}{C_1} = -2.3j \times 1.236 = \boxed{-2.84j = Z_{C_1}}$

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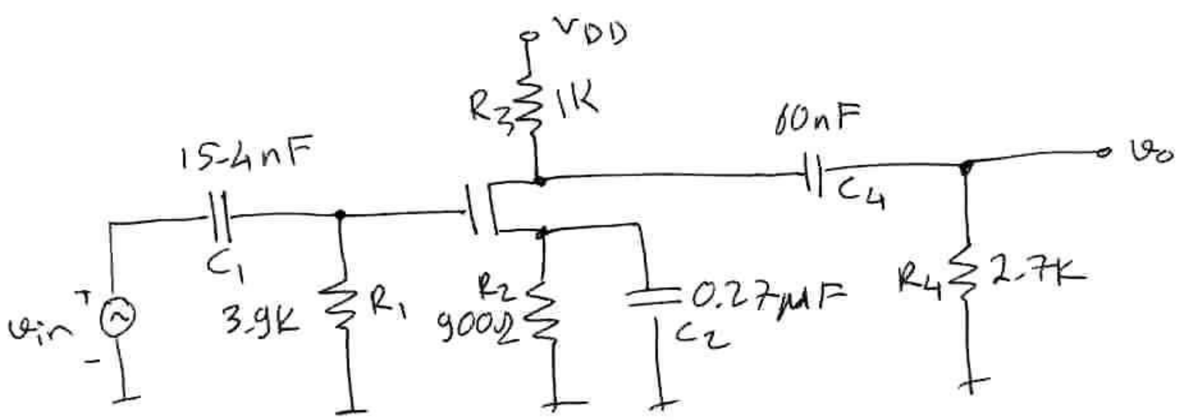
Q. 5

Since the lower cutoff frequency of the amplifier is asked, the high frequency parameters of the transistor are not important. In any case it was expressed orally during the examination that the transistor can be assumed to be ideal. Under the circumstances, it looks reasonable to use the short-circuited time constants method.



With the other capacitors short-circuited and of course vin is short-circuited too, the resistance seen by C_1 is R_1. Therefore:

$$\tau_1 = R_1 C_1 = 15\text{-}4 \times 10^{-9} \times 3.9 \times 10^{3} = 60 \times 10^{-6}$$

$$w_1 = \frac{1}{\tau_1} = 1.66 \times 10^{4} \text{ rad/sec}$$

Again with the other capacitors short-circuited, i.e. the gate is grounded, the resistance seen by C_2 is $R_2 // R_{in}$ of the transistor. Since the gate

Q5-2

is grounded $R_{in} = \frac{1}{g_m} = \frac{1}{10\times10^{-3}} = 10^2 \Omega$

$\Rightarrow R_{in} // R_2 = 100//900 = \frac{100\times900}{100+900} = 90\Omega$

$\Rightarrow \tau_2 = 90\Omega \times C_2 = 90\Omega\times0.27\times10^{-6} = 24.3\times10^{-6}$

$w_2 = \frac{1}{\tau_2} = \frac{1}{24.3\times10^{-6}} = 4.12\times10^4 \text{ rad/sec}$

Since the output is driven by a current source with amplitude equal to $g_m v_{gs}$ which has naturally infinite o/p resistance, the resistance seen by C_4 is $R_3 + R_4$, whether or not the rest of the capacitors are short-circuited. Therefore the time constant τ_3 is given by

$\tau_3 = 60\,nF \times (2.7+1)\times10^3 = 60\times10^{-9}\times3.7\times10^3$
$= 222\times10^{-6}$ sec

$w_3 = \frac{1}{\tau_3} = 0.45\times10^4 \text{ rad/sec}$

$w_{cutoff} = w_1 + w_2 + w_3 = 1.66\times10^4 + 4.12\times10^4 + 0.45\times10^4$
$= 6.23\times10^4 \text{ rad/sec}$

or

$f_{cutoff} = \frac{w_{cutoff}}{2\pi} = 0.991\times10^4 \text{ Hertz} \cong 10 \text{ kHz}$

Q.5-3

In the circuit in question, the cut off frequencies of different circuit elements have actually been designed to be independent. Therefore they can be calculated independently.

The output voltage of the circuit is given by

$$v_o = v_{in} \cdot \frac{3.9K}{3.9K + \frac{1}{j\omega C_1}} \cdot \frac{g_m}{1 + g_m (R_2 // \frac{1}{j\omega C_2})} \cdot \frac{R_3 R_4}{\frac{1}{j\omega C_4} + R_3 + R_4}$$

Therefore

$$\frac{v_o}{v_{in}} = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} \cdot \frac{g_m}{1 + g_m \frac{R_2 \cdot \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}} \cdot \frac{R_3 R_4 j\omega C_4}{1 + (R_3 + R_4) j\omega C_4}$$

$$G = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} \cdot \frac{g_m}{1 + g_m \frac{R_2}{1 + j\omega C_2 R_2}} \cdot \frac{R_3 R_4 j\omega C_4}{1 + (R_3 + R_4) j\omega C_4}$$

$$G = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} \cdot \frac{g_m}{1 + g_m R_2} \cdot \frac{1 + j\omega C_2 R_2}{1 + j\omega C_2 \frac{R_2}{1 + g_m R_2}} \cdot \frac{R_3 R_4}{1 + (R_3 + R_4) j\omega C_4}$$

The time constanst of the gain function are

$$\tau_1 = C_1 R_1, \quad \tau_2 = \frac{C_2 R_2}{1 + g_m R_2} = C_2 \frac{R_2 \cdot \frac{1}{g_m}}{\frac{1}{g_m} + R_2} = C_2 \cdot (R_2 // \frac{1}{g_m})$$

$$\tau_3 = C_4 (R_3 + R_4)$$

So in independent cases the method of short-circuited time constants give the exact time constants. But the estimate of BW is still an approximation.

Also notice that the low-frequency gain of the transistor circuit is $\frac{gm}{1+R_2 gm} = \frac{10^{-2}}{1+900\times10^{-2}} = \frac{10^{-2}}{10} = 10^{-3}$

is chosen to be much lower than the the high frequency gain of the transistor circuit which is $gm = 10^{-2}$, so that the breakpoint at $w = \frac{1}{C_2 R_2}$ has a meaning. Otherwise the breakpoint there would not have a physical meaning although we would still have estimated it to be $w_{cutoff} = \frac{1}{C_2 (R_2 // \frac{1}{gm})}$

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solutions

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Q. 6 – 1

Q. 6

a–) Since the maximum efficiency of a class-A amplifier is 50%, the efficiency is taken as 50%.
Assuming that the tuning and matching circuits which consists of the transformer and the capacitor are lossless, the power input to the transistor must be

$$\frac{6W}{0.5} = 12 \text{ Watts}$$

$\Rightarrow$ the quiescent current of the transistor should be

$$\frac{12W}{7V} = 1.714 \text{ Amps}$$

The load resistance which should be seen by the transistor for maximum efficiency must be

$$R_{Load} = \frac{V_p}{I_p} = \frac{V_{DD}}{I_{DC}} = \frac{7V}{1.714A} = 4.083\,\Omega$$

Therefore the transformer ratio, n is defined by

$$n^2 = \frac{50\Omega}{4.083\Omega} = 12.24 \Rightarrow$$

$n = 3.49 \simeq 3.5$ or

2 : 7 turns

Q.6-2

b-) The transistor dissipation at the maximum output power is

Input power to the trans. – Output power from the trans.

$= 12W - 6W = 6W$

c-) At no drive level the transistor power is

$= 12W - 0W = 12W$