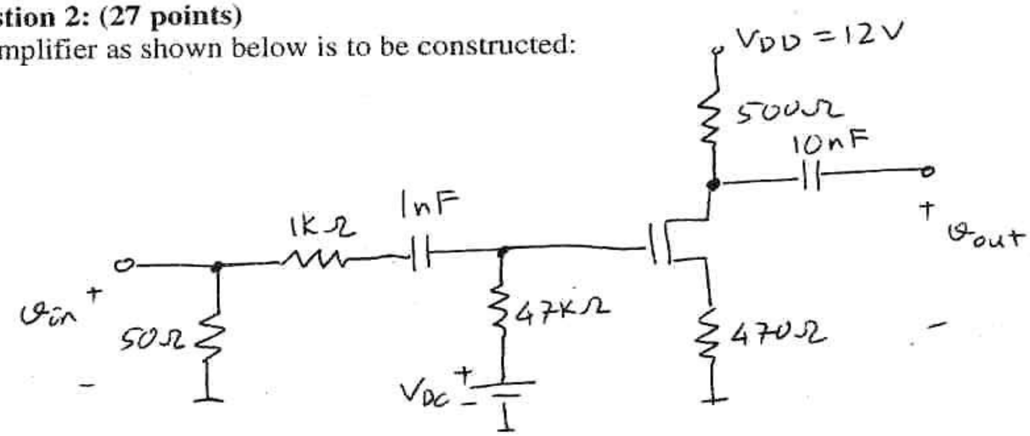
EE411/EEE511 Midterm II
25th December 2002 18.00

Question 1: (16 points)
There is a mathing contest. The winner will be the person who used the minimum number of components to match a 1 Ohm-resistor into 10 Ohms at the frequency of 15.91 MHz. The allowed component types are inductors and capacitors. Do the match, state the component values and draw the circuit.
Note: The matching circuit must reject low frequency components.

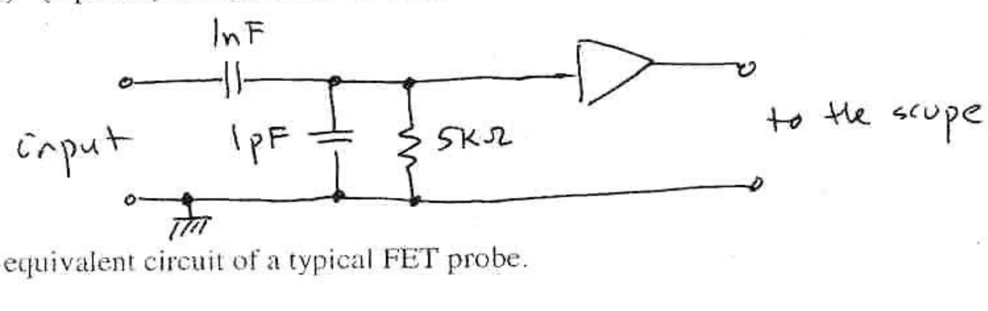
Question 2: (27 points)
An amplifier as shown below is to be constructed:



G_m=8x10^{-3} millisiemens C_{gs}=1pF C_{gd}=0.5pF

The 3dB-bandwidth of the amplifier is not known. The engineer responsible from the design tries to measure the bandwidth of the the amplifier by driving it using a 50 Ohm source and by measuring the output using a FET probe connected to an oscilloscope (The equivalent circuit of the FET probe is given below).

a) (11 points) Estimate the midband gain and the bandwidth of the amplifier without the FET probe.
b) (11 points) Estimate the midband gain and the bandwidth of the amplifier with the FET probe connected.
c) (5 points) Comment on the results.

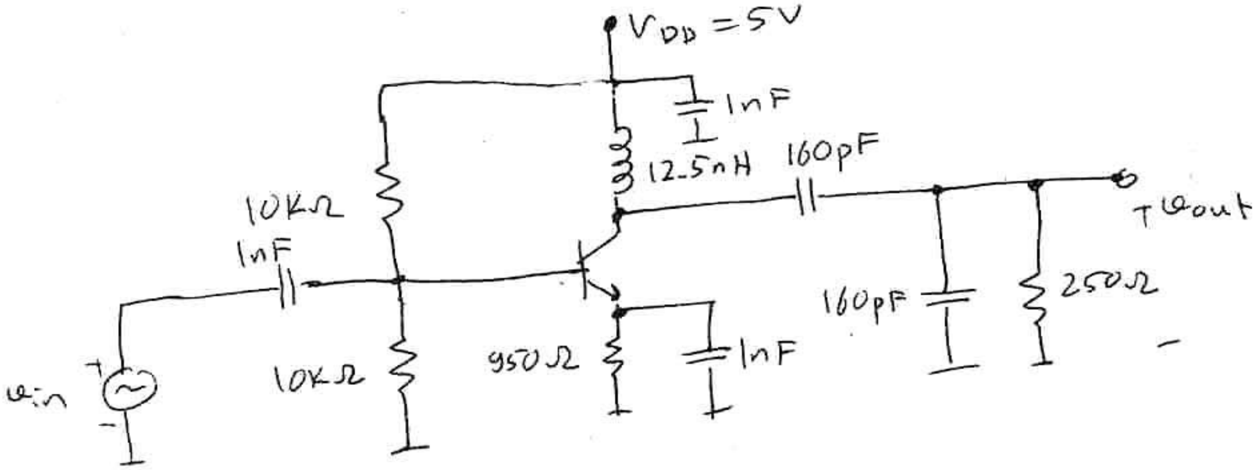


The equivalent circuit of a typical FET probe.

Question 3: (30 points)

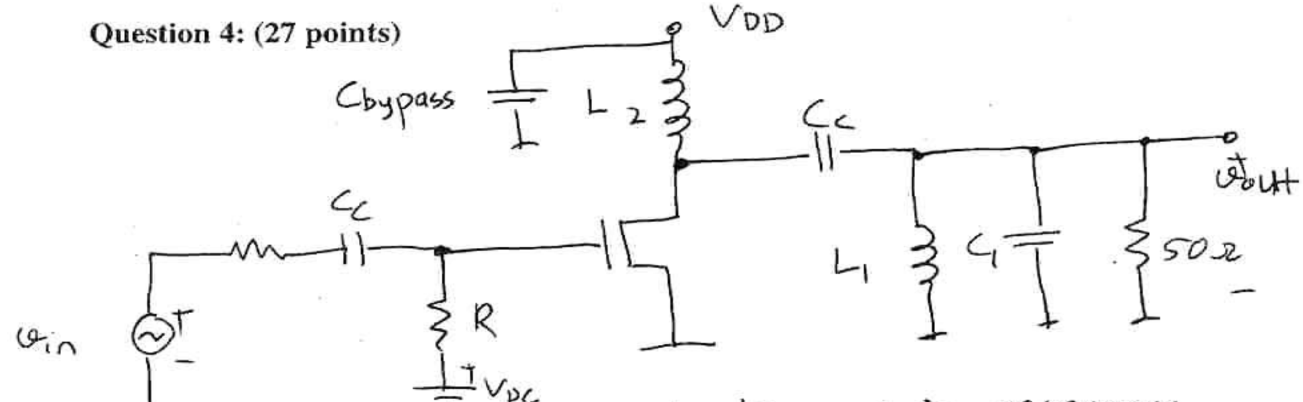
Given the circuit below:

a) (10 points) Calculate the resonance frequency of the output tank
b) (10 points) Calculate the Q of the output tank
c) (10 points) If $v_{in} = 10^{-3}\cos 10^{9}t$, what is the rms voltage at the output?



The transistor is an ideal BJT with V_{BE}=0.6 Volts.

Question 4: (27 points)



L_1 and C_1 are in resonance.
L_2 is large. C_c's are coupling capacitors.

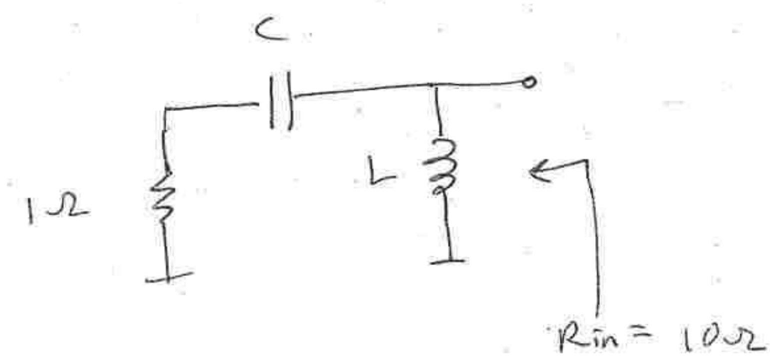
There is an RF power amplifier which was designed to operate either in class-A or class-B modes depending on the application. The delivered power is 1 Watts.

a) (9 points) What must be the power supply voltage V_{DD} so that the amplifer operates at maximum efficiency at both class-A and class-B modes?
b) (9 points) In class-A operation, find the power supply current, transistor dissipation and the peak transistor current.
c) (9 points) In class-B operation, find the power supply current, transistor dissipation and the peak transistor current.

EE411 / EE511
Midterm #2 / 25/12/2002
Solutions:

Q. 1

Matching can be done in numerous ways; L- Match, π-Match, and inductor and capacitive transformers and transformers can be used. Since no other components than capacitors and inductors are allowed, transformers are out. L- Match requires 2 components. π -Match and T-match require 3 components. Inductive and capacitive transformers require 3 components since they need to be resonated. Therefore the correct answer is L- Match. since it must reject low-frequency components it must be in high-pass configuration.



In this configuration 1Ω in series with a capacitor is resonated by a parallel inductor.

$R_s(Q^2+1) = R_p \Rightarrow 1\times(Q^2+1)=10$

$\Rightarrow Q = 3$

$w = 15.91\times10^6\times2\pi$ rad/sec $= 99.96$ Mrad/sec $= 100$ M rad/sec

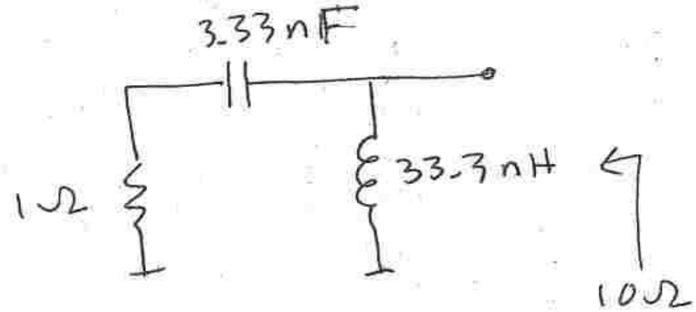
$$Q = \frac{1}{R_s w C} \Rightarrow 3 = \frac{1}{1\times10^8 C} \Rightarrow$$

$$C = \frac{1}{3\times10^8} = 3.33\,nF$$

$$Q = \frac{R_p}{wL} = \frac{10\,\Omega}{10^8 L} = 3 \Rightarrow$$

$$L = \frac{1}{10^7 \times 3} = 33.3\,nH$$

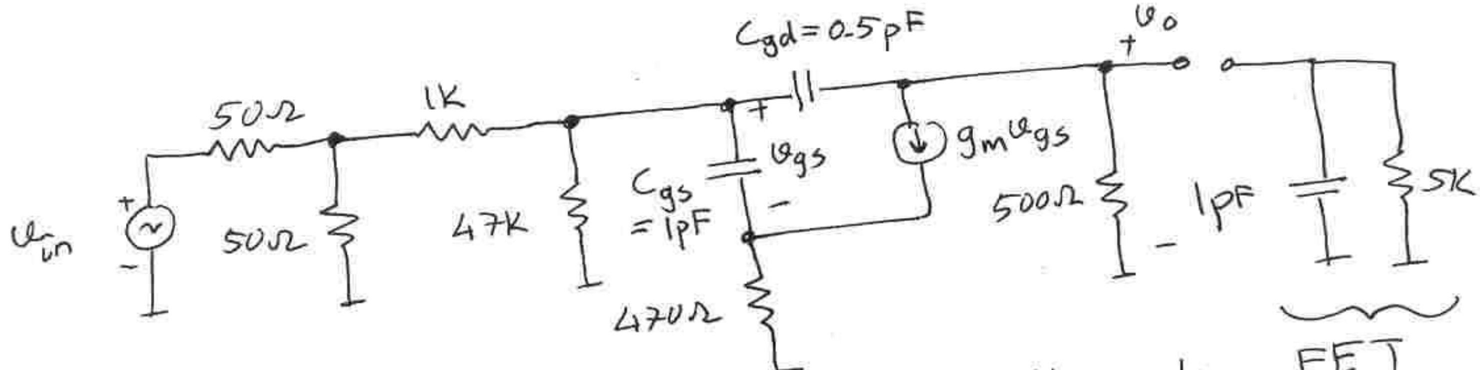
Therefore our matching circuit is:



EE 411/EEE 511
Midterm #2 Solutions
25/12/2002

Question 2:

The AC equivalent circuit is:



The 1nF and 10nF capacitors and the gate bias DC sources are shorted.

a-) Without the FET probe the output is not loaded by 1pF capacitor and 5KΩ resistor. Therefore in the open-circuited time constant method can be applied only to C_{gs} and C_{gd} capacitors.

From the book:

$$R_{gs} = \frac{R_s + R_E}{1 + g_m R_E}$$

$$R_s = (50 \parallel 50\Omega + 1K) \parallel 47K = \frac{1025\Omega \times 47K\Omega}{1025 + 47000} = 1003\Omega \simeq 1000\Omega$$

$$R_E = 470\Omega$$

$\Rightarrow$

$$R_{gs} = \frac{1003 + 470}{1 + 8\times10^{-3}\times 470} = \frac{1473}{1 + 3.76} = 309\Omega \Rightarrow$$

$$\tau_{gs} = 309\Omega \times 10^{-12}F = 0.309\ nsec$$

$R_{gd} = r_L + r_R + r_L r_R \, g_{m\,eff}$

$$g_{m_{eff}} = \frac{g_m}{1 + R_E g_m} = \frac{8\times10^{-3}}{1 + 3.76} = \frac{8\times10^{-3}}{4.76} = 1.68\times10^{-3} \text{ siemens}$$

=

$R_{gd} = 500\Omega + 1003\Omega + 1003 \times 500 \times 1.68\times10^{-3}$

$= 1503\Omega + 843\Omega = 2346\Omega$

$\tau_{gd} = 2346 \times 0.5\times10^{-12} = 1.173 \text{ nsec}$

$\tau_{total} = 1.482 \text{ nsec}$

$w_{est} = 674.8$ Mrad/se $f_{est} = $ **107.4 MHz**

$$G_{midband} = \frac{47K}{1 + 47K} \cdot g_{m_{eff}} \cdot 500\Omega$$

$G_{midband} = 0.8225$

b-) with the FET probe connected:
The τ_{gs} is not affected ⇒

$\tau_{gs} = 0.309$ nsec

In the τ_{gd} calculation, R_{Right} must be replaced by $500\Omega // 5K\Omega = 454.5\Omega$ ⇒

$R_{gd} = 1003\Omega + 454.5\Omega + 454.5\Omega \times 1003\Omega \times 1.68\times10^{-3}$

$= 1457.5 + 765.5\Omega = 2223\Omega$ ⇒

$\tau_{gd} = 2223\Omega \times 0.5\times10^{-12} = 1.111$ nsec

$$\tau_L = (5K\Omega \parallel 500\Omega) \times C_L = 454.3\Omega \times 10^{-12} F =$$

$$= 0.4543 \text{ nsec}$$

$$\sum \tau_n = 0.309 + 1.111 + 0.4543 = 1.8743 \text{ nsec}$$

$w_{est} = 533.5$ Mrad/sec   **$f_{est} = 84.9$ MHz**

$$G_{midband} = \frac{47}{47+1} \cdot g_{meff} \times 454.3\Omega$$

$G_{midband} = 0.747$

c-)

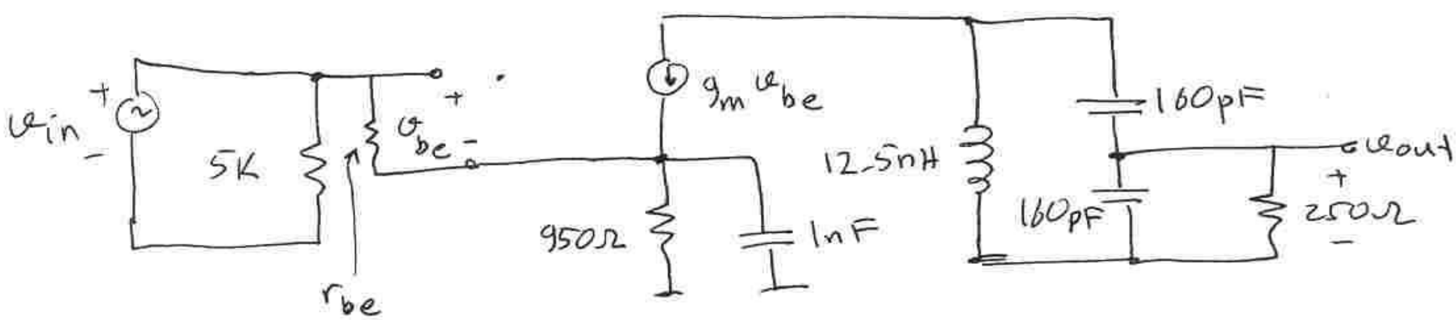
	Without FET probe	With FET probe
Gmidband	0.8225	0.747
BW	107.4 MHz	84.9 MHz

Comment: Although FET probes are one of the best probe tehnologies created not to load RF circuits, one still must be very quotious when using them. The probe still decreases both the midband gain and the BW. (Note: writing the last sentence in the exam is sufficient)

EE 411 / EEE 511
Midterm #2 solutions
25.12.2002

Question 3:

a-) The equivalent circuit is:



assuming that the Q of tank is high enough

$$w = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{12.5nH \times 80pF}} = 10^9 \text{ rad/sec}$$

$$f_{res} = \frac{10^9}{2\pi} = 159.2 \text{ MHz}$$

b-) Again assuming high Q 250Ω is reflected to the terminals of the inductor by being multiplied by $2^2 = 4$

$\Rightarrow$ $R_{parallel} \cong 1000\Omega$

$$Q = \frac{R_p}{wL} = \frac{1000}{10^9 \times 12.5 \times 10^{-9}} = 80 \Rightarrow \text{high } Q$$

assumption is right.

c.-) The gain of the circuit is

$G = g_m R_{LOAD} \times$ transformer ratio

$R_{LOAD} = 1K\Omega$

transformer ratio is $\frac{1}{2}$

$$g_m = \frac{I_{DC}}{26mV} = \frac{\left(5V \times \frac{10K}{20K} - 0.6V\right) / 950\Omega}{26mV}$$

$$= \frac{(2.5V - 0.6) / 950\Omega}{26mV} = \frac{1.9 / 950\Omega}{26mV}$$

$$= \frac{2mA}{26mV} = 0.077 \text{ siemens} \Rightarrow$$

$$G = 0.077 \times 1000 \times \frac{1}{2} = 38.4$$

$$v_{out} = v_{in} \times G = 1mV_{peak} \times 38.4 = 38.4mV_{peak}$$

$$v_{out} = \frac{38.4}{\sqrt{2}} = 27.28mV_{rms} = 0.02728\ V_{rms}$$

EE 411 / EEE 511
Midterm #2 Solutions
25-12-2002

a-) At both Class-A and Class-B operations the peak output swing must be equal to V_{DD}. ⇒

$$\frac{V_p^2}{2R} = 1 \text{ Watts} \Rightarrow V_p^2 = 1 \text{ Watts} \times 2 \times 50 = 100$$

$$\boxed{V_{DD} = V_p = 10\,V}$$

b-) The maximum efficiency in Class-A operation is 0.5. Therefore the total power input to the circuit is 2W. Dividing it by the voltage yields the power supply current.

$$I_{ps} = \frac{2W}{10V} = 0.2A$$

The transistor dissipation is half the power supply power. Therefore it is

$$P_{trans} = 1 \text{ Watts}$$

The peak current $= I_{DC} + I_p = 0.2 + 0.2 = 0.4A$

$$I_{peak} = \frac{V_P}{50\Omega} = \frac{10V}{50\Omega} = 0.2A$$ (anyway it is also equal to I_{DC})

c-) In Class-B case, the maximum efficiency is 0.785 $\Rightarrow$

$$P_{power\ supply} = \frac{1W}{0.785} = 1.274\ W$$

$$I_{ps} = I_{power\ supply} = \frac{1.274W}{10V} = 0.1274\ A$$

$$I_{ps} = \frac{1}{T}\int_0^T i_{trans}(t)\,dt = \frac{I_{peak}}{T}\int_0^T \sin\frac{2\pi}{T}t\,dt \quad \text{if } \sin\frac{2\pi}{T}t \geq 0$$

otherwise $i_{trans}(t) = 0$

$$= \frac{I_{peak}}{\pi}$$

$$I_{ps} = 0.1274\,A = \frac{I_{peak}}{\pi} \Rightarrow$$

$$I_{peak} = \pi \times 0.1274\,A$$

$$\boxed{I_{peak} = 0.4002\,A}$$

(Anyway since the normalized power output capabilities of class-A and Class-B amplifiers are the same, The peak transistor currents must be the same for the same output voltage and output power.)