EEE 411/511 Midterm solutions
Dec 2, 2003

Q.1

a-) The output tank is a parallel resonant circuit composed of 200nH inductor, 12.7pF capacitor and 5K resistor. The transistor is an ideal transistor, therefore it does not load the tank.
First calculate the resonant frequency and the Q of the tank

$$w_0 = \frac{1}{\sqrt{LC}} = 627\ \text{Mrad/sec}$$

$$Q = \frac{R}{wL} = \frac{5000}{0.627\times10^{9}\times0.2\times10^{-6}} = 39.843 \approx 40$$

Now as a check (not part of the solution) let us find the series Q defined by the 1Ω power supply resistance.

$$Q = \frac{wL}{r} = 125.49$$ (Our assumption is somewhat valid)

The voltage created at D1 by T1 is

$$1mV_{rms} \times g_m \times R_{L_{T_1}} = 1mV \times 25\times10^{-3} \times 1000$$
$$= 25mV_{rms}$$

The current of the reactive elements of a parallel tank circuit is Q times the drive current. Therefore the current on the inductor is $25mV_{rms} \times g_m \times Q = 25\times10^{-3}\times25\times10^{-3}\times40 = 25mA_{rms}$

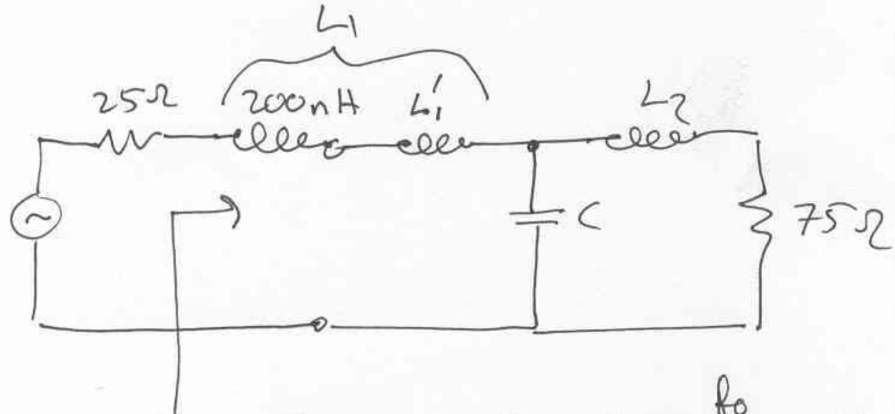
Therefore the voltage on 1Ω resistor and at D1 induced by the current on the inductor is $25mA \times 1\Omega = 25mV_{rms}$
and it is equal to the drive by T1. Therefore the power supply output impedance must be improved for proper operation and to prevent oscillation.

b-) Again the voltage on D1 driven by T1 is $25 mV_{rms}$. The drain current of T2 is $g_m \times 25mV_{rms} = 25 \times 25 \times 10^{-3} = 625$ micro amps.

So the voltage induced on D1 by the drain current of T2 is $\underline{0.625 mV_{rms}}$
(Much smaller compared to the actual drive voltage on that point)

Q.2

The questron boils down to designing a low-pass matching circuit to match 25Ω into 75Ω. The important point is to take the Q of the match twice the one required by the BW because 25Ω of the sourse also loads the tank. It is easier to choose T-match since 200nH of the source can easily be absorbed into the matching circuit inductor.



$R_{in} = 25\,\Omega \qquad Q = 2 \times \frac{f_o}{BW} = 2 \times \frac{10\,MHz}{2\,MHz} = 10$

$$K = \frac{R_L}{R_{in}} = \frac{75\,\Omega}{25\,\Omega} = 3$$

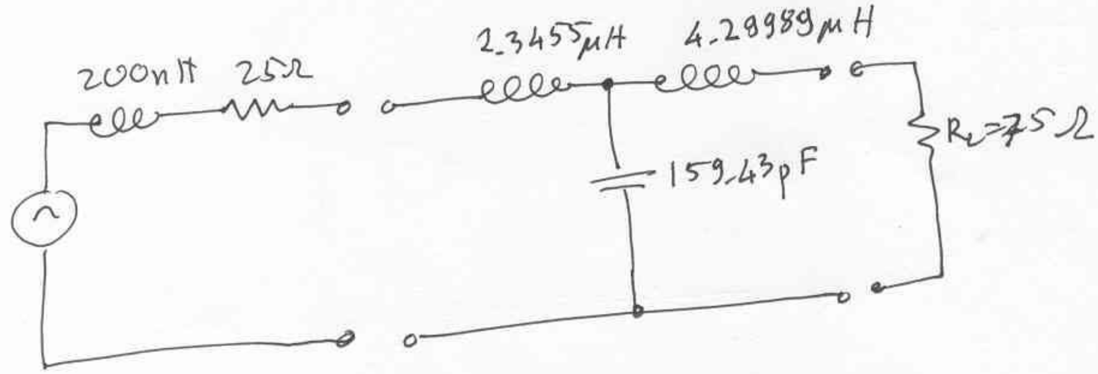
$$Q_R = \frac{\left[Q^2K + 2K - K^2 - 1\right]^{1/2} - Q}{K-1} = 3.602$$

$$Q_L = Q - Q_R = 6.3977$$

$$Q_R = \frac{X_{L2}}{R_L} \Rightarrow X_{L2} = Q_R \times R_L = 270.17\,\Omega$$
$$X_{L1} = Q_L \times R_{in} = 159.94\,\Omega$$

$$R_I = (Q_L^2+1)R_{in} = 998.26\,\Omega$$
$$Q = \frac{R_I}{X_C} \Rightarrow X_C = \frac{R_I}{Q} = 99.825\,\Omega$$
$$L_1 = \frac{X_{L1}}{w} = \frac{159.94}{w} = 2.5455 \times 10^{-6}\,H$$
$$\Rightarrow L_1' = 2.5455\,\mu H - 0.2\,\mu H = 2.3455\,\mu H$$
$$L_2 = \frac{X_{L2}}{w} = 4.29989\,\mu H$$
$$\frac{1}{wC} = X_C \Rightarrow C = \frac{1}{wX_C} = 1.5943 \times 10^{-10}\,F = 159.43\,pF$$

Therefore our solution is



Q-3

$\tau_1 = 100nF \times 2225\Omega = 2.225 \times 10^{-4}$ sec

$\frac{1}{\tau_1} = 4494$ rad/sec

$\tau_2 = \left(220 \,//\, \frac{1}{g_m}\right) \times 1\mu F = 33.846 \times 10^{-6} = 3.384 \times 10^{-5}$ sec

$\frac{1}{\tau_2} = 29545$ rad/sec

$\tau_3 = (1+1)k\Omega \times 100nF = 2 \times 10^{-4}$ sec

$\frac{1}{\tau_3} = 5000$ rad/sec

$w_{3db} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} = 39039.83$ rad/sec

$f_{3dB} = 6213.38\ Hz = \frac{w_{3dB}}{2\pi}$

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Q-4

a-) Let us start with high-Q assumption

$$C_{eq} = \frac{15 \times 30}{15+30} + C_{gd}(1 + g_m(R_S + R_g))$$

$$= 10pF + 2pF(1 + 12 \times 10^{-3} \times 55)$$

$$= 13.32pF$$

$$\Rightarrow w_0 = \frac{1}{\sqrt{LC}} = 2.826 \times 10^9 \text{ rad/sec}$$

$$\underline{f_0 = 0.44978 \text{ GHz}}$$

to check the high-Q assumption

$$Q = \frac{Rn^2}{wL} = \frac{R \times 3^2}{wL} = 50.8 \checkmark$$

b-) The voltage gain is

$$-\frac{g_m R_L}{n} = -\frac{12 \times 10^{-3} \times 150 \times 9}{3} = -5.4$$

At question 4-b Y_{in} was assumed to be zero. Let us see what happens if we do not neglegt it.

Y_{in} is given by

$$Y_{in} = \frac{Y_F + Y_F Z_L g_m}{1 + Y_F Z_L} + j\omega C_{gs} Y_F$$

where Y_F (the feedback conductance) is equal to $j\omega C_{gd}$

and $Z_L = n^2 150\Omega = 9 \times 150 = 1350\Omega$ at resonance.

First let us calculate $Y_F Z_L$

$$Y_F Z_L = j \times 2.826 \times 10^{9} \times 2 \times 10^{-12} \times 1350 = j\,7.63$$

$$Y_{in} = \frac{j\omega C_{gd}(1 + Z_L g_m)}{1 + j\,7.63} + j\omega C_{gs}$$

$$= j\omega C_{gd} \frac{17.2}{1 + j\,7.63} + j\omega C_{gs}$$

$$= j\,\frac{5.65 \times 10^{-3} \times 17.2}{1 + j\,7.63} + j\,1.13 \times 10^{-2} \text{ mhos}$$

$$= j\,0.0972 \left[\frac{1 - j\,7.63}{57.16}\right] + j\,0.0113 \text{ mhos}$$

$$= \underbrace{+0.01297 + 0.0017}\,j + 0.0113\,j \text{ mhos}$$

$$= 0.01297 + 0.013\,j = 0.013 + 0.013\,j \text{ mhos}$$

Notice that the feedback impedance is almost real !!!
Also notice that C_{gs} is significant.

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6B

$$Z_{in} = \frac{1}{1+j} \cdot \frac{1}{0.013}$$

$$= 76.9\left[\frac{1-j}{2}\right] = 38.46 - 38.46j$$

In order to calculate the gain of the circuit, we must calculate v_{gs} in terms of v_{in}

$$v_{gs} = \frac{38.46 - 38.46j}{50 + 38.46 - 38.46j} \times v_{in}$$

$$= \frac{38.46 - 38.46j}{88.46 - 38.46j}$$

$$|G_1| = \left|\frac{v_{gs}}{v_{in}}\right| = \left(\frac{38.46^2 + 38.46^2}{88.46^2 + 38.46^2}\right)^{1/2} = 0.564$$

$$|G| = |G_1| * g_m \times 1350 \times \frac{1}{3} = 3.05$$

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Q.5

The overall noise factor is given by

$$F = F_1 + \frac{F_2 - 1}{G_1}$$ where all F's an G's are in ratios

$F_1 = 4$ (3 dB is twice the power)

$G_1 = (10-3)\overset{dB}{=} \frac{10 \text{ times}}{2} = 5$

$F_2 = 10^{10.4/10} = 10.945 \simeq 11$

$G_2 = 10$ (10 dB is 10 times the power)

$$\Rightarrow \quad F = 4 + \frac{11-1}{5} = 4 + 2 = 6$$

$N_o = F \times G \times N_i = F \times G_1 G_2 k T_o B$ with 75 Ω term.
$= 6 \times G_1 G_2 k T_o B$

$N_{o_a} = 4 \times F \times G \times N_i = 4 \times 6 \times G_1 G_2 k T_o B$

$$\frac{N_{o_a} - N_o}{G_1 G_2} = \text{excess antenna noise}$$

$$= \frac{4 \times 6 \times G_1 G_2 k T_o B - 6 \times G_1 G_2 k T_o B}{G_1 G_2}$$

$= 3 \times 6\ k T_o B = 18\ k T_o B$

or

12.55 dB above $k T_o B$