# Differentiator:

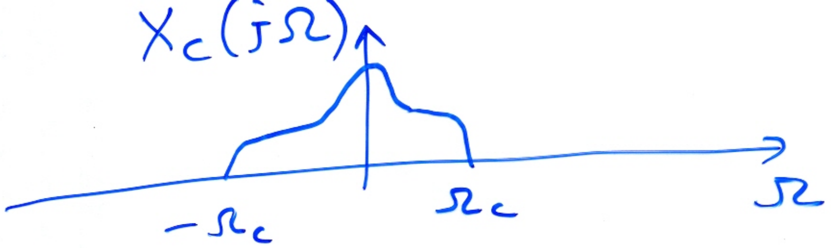
$$y_c(t) = \frac{d\, x_c(t)}{dt} \qquad \text{LTI system.}$$

$$Y_c(j\Omega) = j\Omega\, X_c(j\Omega)$$

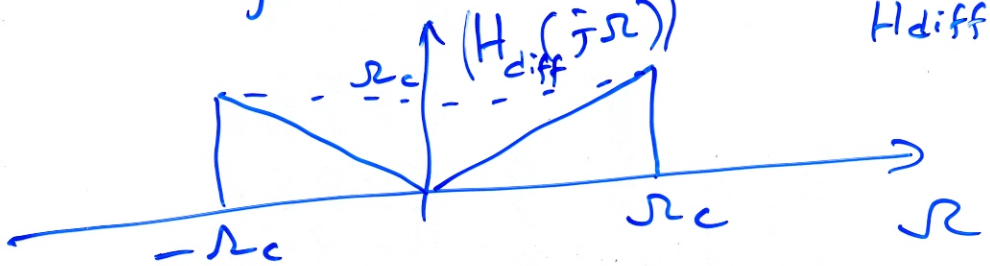
So $\quad H(j\Omega) = j\Omega$


unstable

We assume that the input $x_c(t)$ is band limited:




Consider the system: with freq. response:



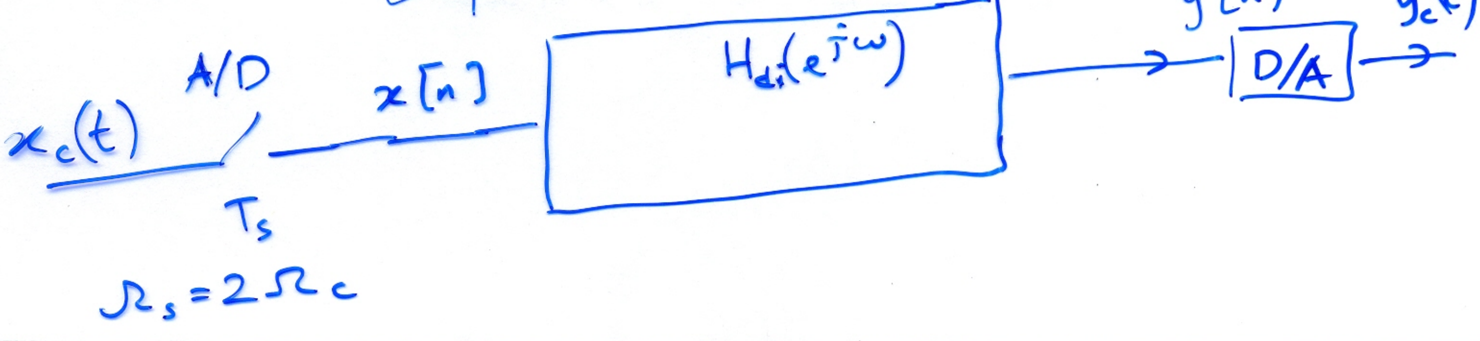

$$H_{diff}(j\Omega) = \begin{cases} j\Omega & \text{for } |\Omega| < \Omega_c \\ 0 & \text{other} \end{cases}$$

stable system

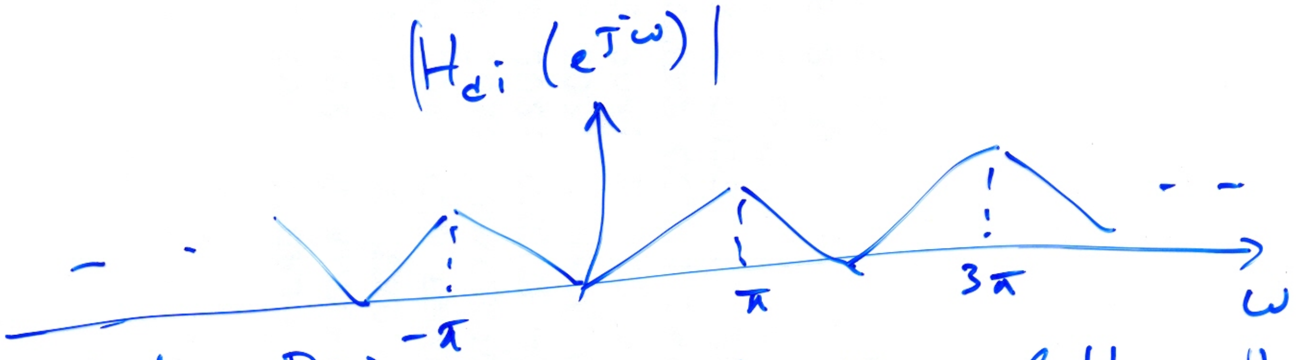
$H_{diff}(j\Omega)$ will produce the same output as the ideal diffrentiator which is unstable.

In Discrete-time domain:

Equivalent system:



$$H_{di}(e^{jw}) = \begin{cases} \dfrac{jw}{T_s} \leftarrow \text{scaling factor.} & |w| < \pi \\ \text{periodic extension} & \text{otherwise} \end{cases}$$



**Differentiator Design:**

Consider the design of the filter $H_{di}(e^{jw})$:

Ideal impulse response:

$$h_{id}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} jw\, e^{+jwn}\, dw$$

for $n=0$ $\quad h_{id}[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} jw\, dw = 0$

for $n \neq 0$ $\quad h_{id}[n] = \frac{\cos \pi n}{n}$, $\leftarrow$ infinite-extent impulse resp.

$$\Downarrow$$

$$h_{id}[n] = -h_{id}[-n]$$

We need to design FIR filters to implement!

**Equiripple Design:**

Time domain constraint: for a filter of order $M = 2L+1$

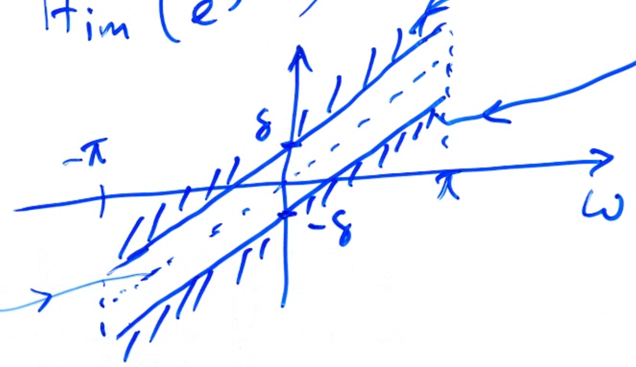
(i) $h[n] = -h[-n]$

(ii) $h[n] = 0$ for $|n| > L$,

Freq. domain constraints:

(iii) $H_{real}(e^{jw}) = 0$ for all $w$!

(iv) $H_{im}(e^{j\omega})$ should satisfy the bounds




$H_{im}^{ideal}(e^{j\omega}) = \omega$

for $-\pi < \omega < \pi$

upper bound $\quad \omega + \delta = H_{im}^{up}$

lower " $\quad \omega - \delta = H_{im}^{down}$

*Develop the iterative algorithm based on these four constraints.

* You can also use the "firpm" in MATLAB!

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Hilbert Transformer: $\quad -\pi < \omega < \pi$

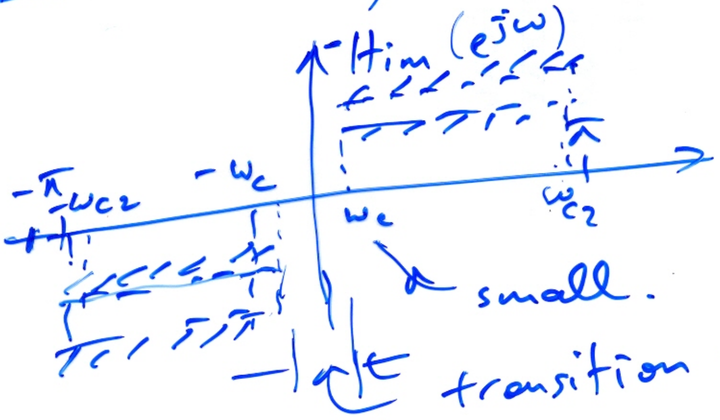
purely imaginary freq. respose

$$\rightarrow H_{h,id}(e^{j\omega}) = \begin{cases} e^{-j\pi/2} = -j & \omega > 0 \\ e^{+j\pi/2} = j & \omega < 0 \end{cases}$$

$$h_{h,id}[n] = \begin{cases} \frac{2}{\pi} \frac{\sin^2(\pi n/2)}{n}, & n \neq 0 \\ 0 & \text{for } n = 0 \end{cases}$$

$$h_{h,id}[n] = -h_{h,id}[n]$$

Freq. Domain specs:




small.

transition region!