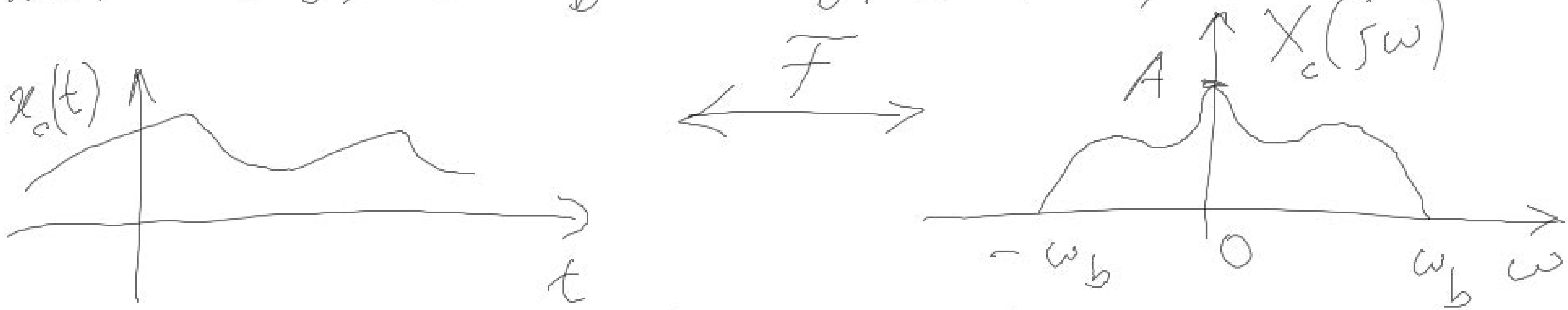
Shannon's Sampling Theorem:

Let $x_c(t)$ be a band-limited signal with highest freq. ω_b. The sampling frequency should be larger than $\omega_s > 2\omega_b$. (Nyquist rate)



Ex: telephone speech has a BW of : 4 KHz.
Sampling freq: 8 KHz. We take 8000 samples per second.

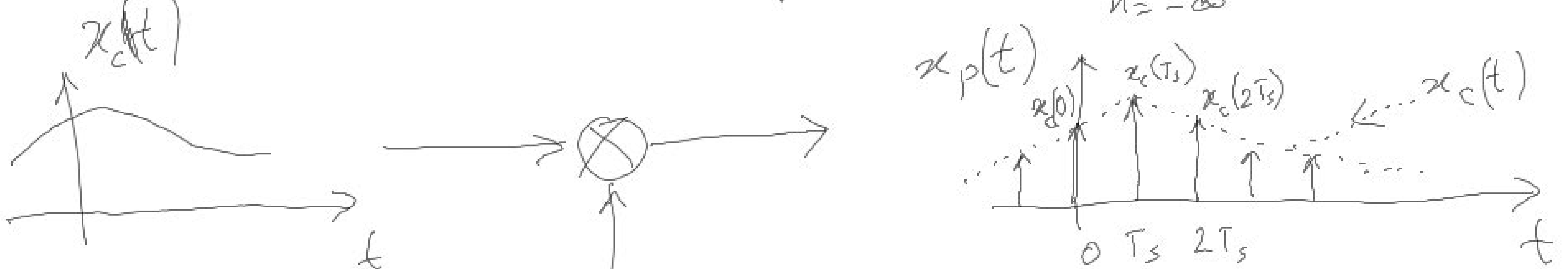
Ex: In CD's: $f_s = 44.1$ KHz.

* If the signal is not band-limited apply a low-pass filter first and then sample the signal.

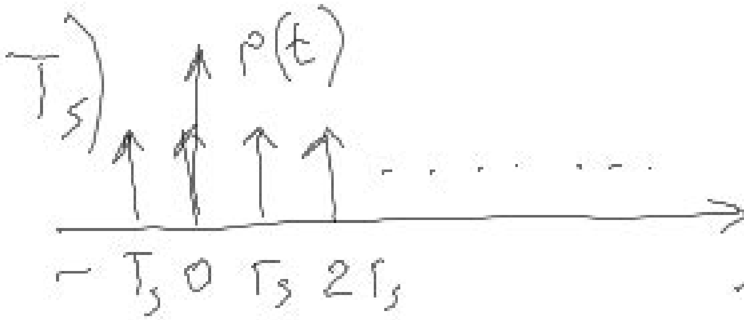
Discrete-time signal: $x[n] = x_c(nT_s)$, $n = 0, \mp 1, \mp 2, \mp 3, \ldots$

Sampling period $T_s = \dfrac{1}{f_s} = \dfrac{2\pi}{\omega_s}$, $\omega_s = 2\pi f_s$

Equivalent cont-time signal: $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$



$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$



$x_p(t) = x_c(t)p(t)$

$x_p(t) \equiv x[n]$

$x_p(t) \not\equiv x[n]$

cont-time discrete

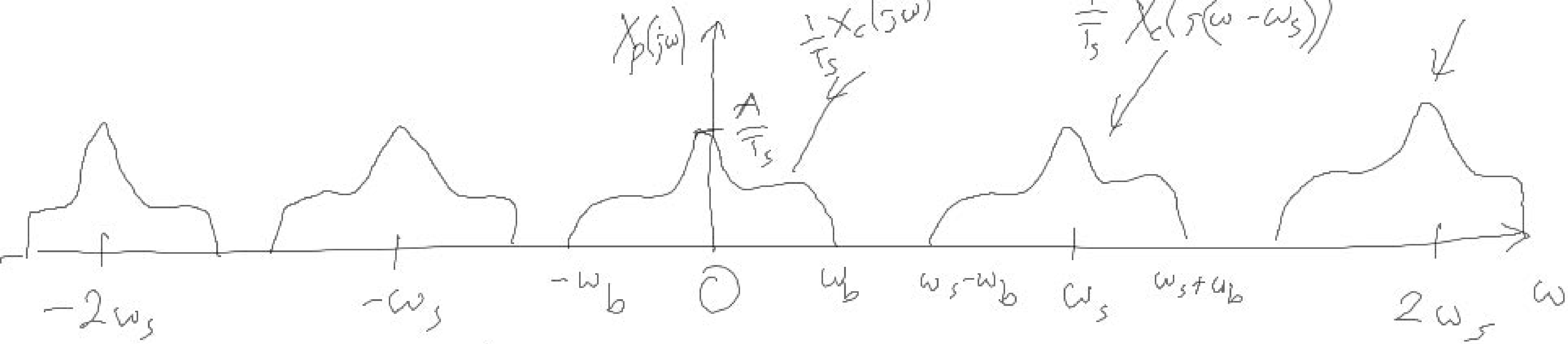
$\mathcal{F}$

$X_p(j\omega) = \dfrac{1}{2\pi} P(j\omega) * X_c(j\omega)$

$P(j\omega) = \dfrac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$

$$X_c(j\omega) * \delta(\omega-\omega_s) = X_c(j(\omega-\omega_s))$$
$$X_c(j\omega) * \delta(\omega-k\omega_s) = X_c(j(\omega-k\omega_s)) \Longrightarrow$$

$$X_p(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_c(j(\omega-k\omega_s))$$



with the assumption that $\omega_s-\omega_b > \omega_b \Rightarrow \underline{\underline{\omega_s > 2\omega_b}}$

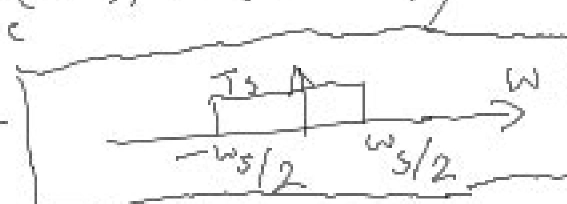
Nyquist rate

$x[n] = x_c(nT_s)$

$x(nT_s)$ samples go to the receiver instead of $x_c(t)$

$$x_p(t) = \sum_n x_c(nT_s)\,\delta(t-nT_s)$$

At the receiver $x_p(t) \rightarrow$



$\rightarrow x_c(t)$ L.P. Filter

Summary:

$n = 0, \mp 1, \mp 2, \ldots$

$x_c(t)$ —— / —— $x[n] = x(nT_s)$

T_s, f_s

$x_c(t) \overset{F}{\longleftrightarrow} X_c(j\omega)$

$x[n] \equiv x_p(t) \overset{F}{\longleftrightarrow} X_p(j\omega)$

Send samples to the receiver

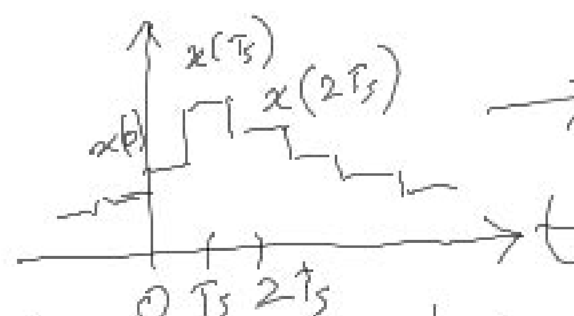
Low-pass filter

$x_p(t) \rightarrow$  $\rightarrow x_c(t)$

We don't compute them inside telephones, computers etc.

We do not $X_c(j\omega) = H(j\omega) X_p(j\omega)$

* In practice $x_p(t)$  is not used but we use a staircase signal

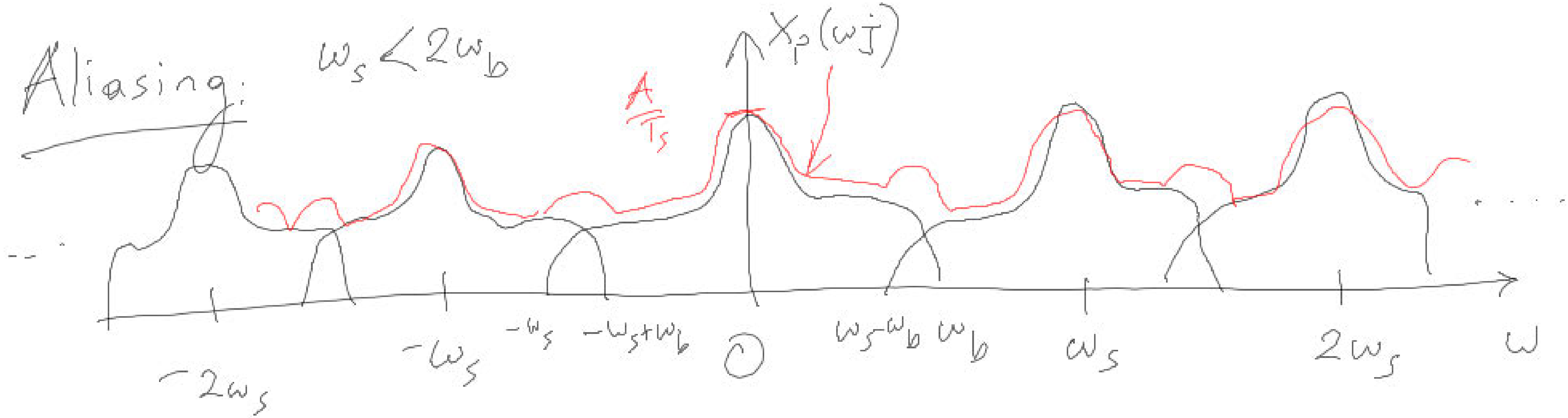
 $\rightarrow$ $\boxed{L.P.F}$ $\rightarrow \tilde{x}_c(t)$

$\tilde{x}_c(t) \neq x_c(t)$ but it is very close. (provided that $\omega_s > 2\omega_b$)

* In $\boxed{A/D}$ converters there is built in low-pass filter with cut-off freq. $\frac{f_s}{2}$ $\left(\frac{\omega_s}{2}\right)$!

f_s

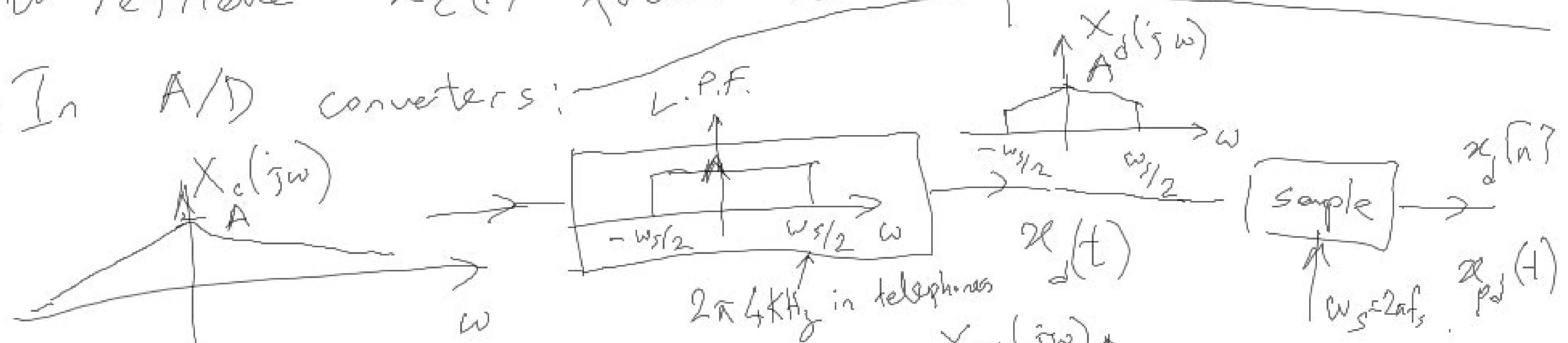
Aliasing: $\omega_s < 2\omega_b$

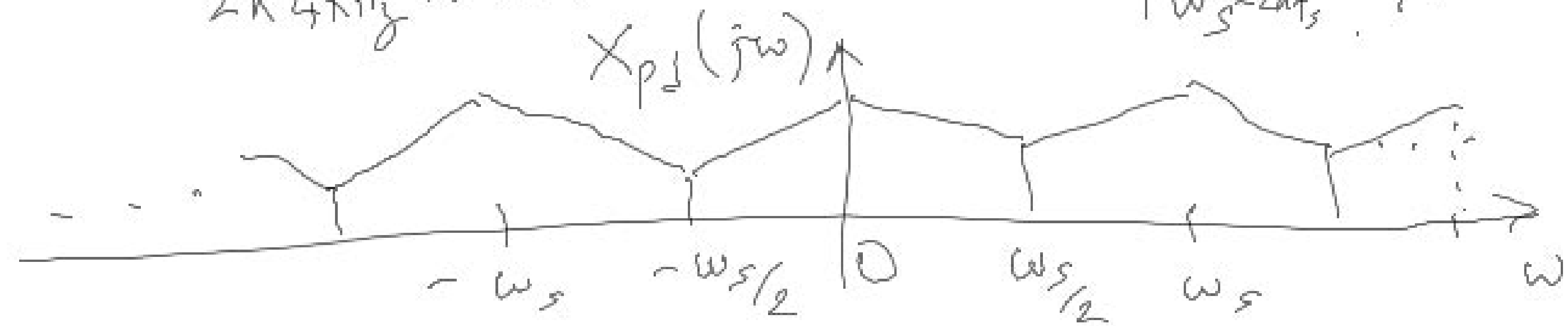


$$\omega_s - \omega_b < \omega_b \Rightarrow \omega_s < 2\omega_b$$

High-freq. components are corrupted and it is impossible to retrieve $x_c(t)$ from its samples $x[n] = x_c(nT_s)$.

* In A/D converters:





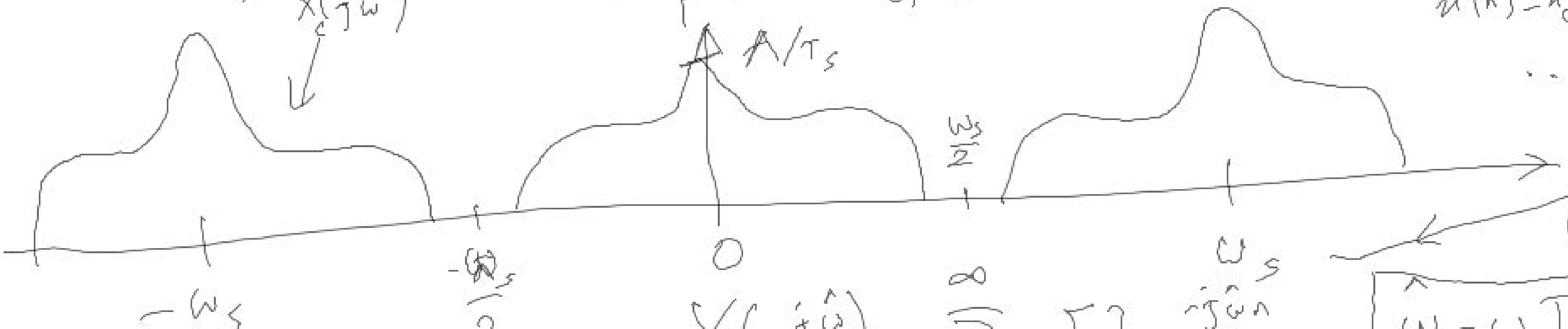
In telephone speech $f_s = 8\ KHz$ that is why we hear "mechanical" sound.

$$x[n] \xleftrightarrow{F_{DT}} X(e^{j\hat{\omega}}) \equiv X_p(j\omega)$$

equivalent to but not equal.

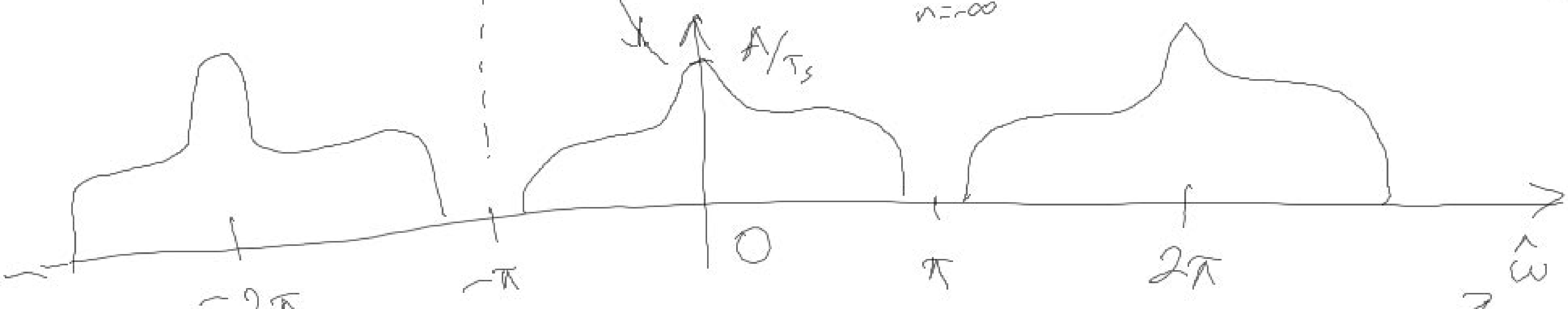
$$X_p(j\omega) = \mathcal{F}_{CT}[x_p(t)] = \sum_{n=-\infty}^{\infty} x_c(nT_s)\, e^{-j\omega T_s n}$$

$x[n] = x_c(nT_s)$



$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\hat{\omega} n}$$

$$\boxed{\hat{\omega} = \omega T_s}$$



Normalized freq.

* We cannot capture freq. above $\frac{\omega_s}{2}$ with ω_s as our sampling freq.

* $\hat{\omega}_s = \pi$ is the highest freq in $\hat{\omega}$ domain because it corresponds to $\frac{\omega_s}{2}$.

$$\hat{\omega}_s = \frac{\omega_s}{2} \cdot T_s = \frac{1}{2}\left(\frac{2\pi}{T_s}\right) \cdot T_s = \pi \, .$$