$\boxed{\text{Feb 27/1}}$

Design of FIR Filters:

(finite impulse response)

FIR filter:

$$y[n] = \sum_{k=0}^{M-1} h[k]\, x[n-k] \qquad \text{Causal FIR.}$$

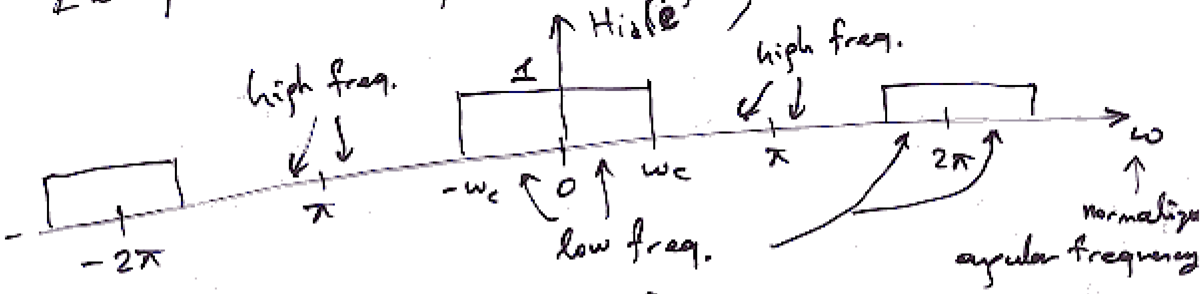
$$y[n] = \sum_{k=0}^{M-1} a_k\, x[n-k] \qquad \text{where } h[k] = a_k.$$

Anticausal FIR:

$$y[n] = \sum_{k=-L}^{L} b_k\, x[n-k] \Rightarrow \text{Impulse resp.: } h[k] = b_k, \quad k = 0, \mp 1, \ldots, \mp L.$$

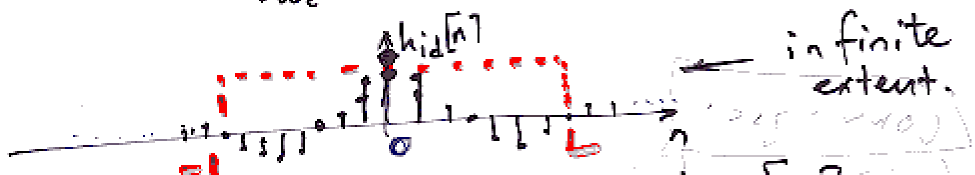
Design problem: Find the impulse response $h[n]$ satisfying some requirements.

Example: Low-pass filter design.



$$h_{id}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{id}(e^{j\omega}) e^{j\omega n} d\omega$$

$$h_{id}[n] = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \frac{\sin(\omega_c n)}{\pi n}, \quad n = 0, \mp 1, \mp 2, \ldots$$



Possible approach: Truncate $h_{id}[n]$

$$h_T[n] = \begin{cases} h_{id}[n], & n = 0, \mp 1, \ldots, \mp L \\ 0, & \text{otherwise} \end{cases} \leftarrow \text{FIR anticausal.}$$

2nd ~~Approach:~~ Least Squares: Approach. [2 | 28 Subat]

$$\min \quad \frac{1}{2\pi}\int_{-\pi}^{\pi} |H_L(e^{j\omega}) - H_{id}(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |h_L[n] - h_{id}[n]|^2$$

freq. resp. of the filter.

$$= \sum_{n=-L}^{L} |h_L[n] - h_{id}[n]|^2 + \sum_{n=L+1}^{\infty} |h_{id}[n]|^2 + \sum_{n=-\infty}^{-L-1} |h_{id}[n]|^2$$

Equivalent minimization problem

$$\min \sum_{n=-L}^{L} \left(\underset{\text{real}}{h_L[n]} - \underset{\text{real}}{\overset{\text{known}}{h_{id}[n]}}\right)^2 = B$$

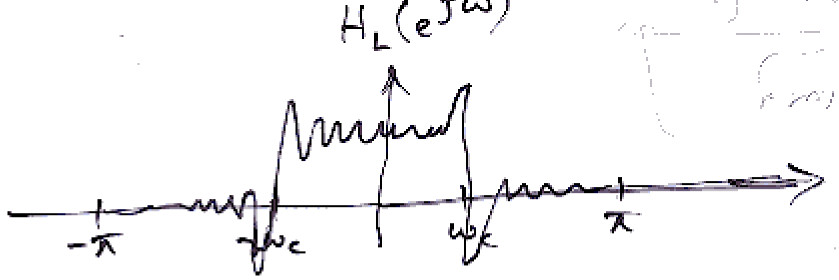
$$\frac{\partial B}{\partial h_L[k]} = 0 \quad , \; k = 0, \mp 1, \mp 2, \dots, \mp L.$$

$$= 2\left(h_L[k] - h_{id}[k]\right) + \cancel{\frac{\partial \sum_{n \neq k} (h_L[n] - h_{id}[n])^2}{\partial h_L[k]}} \rightarrow 0$$

$$\Rightarrow \quad h_L[k] = \begin{cases} h_{id}[k], & k = 0, \mp 1, \mp 2, \dots, \mp L \\ 0, & \text{otherwise.} \end{cases}$$

* This design method produced nothing new! $\Rightarrow$ Rectangular window design. (or truncation) $\Rightarrow$

* Gibbs phenomenon.



Window-based FIR Filte Design:

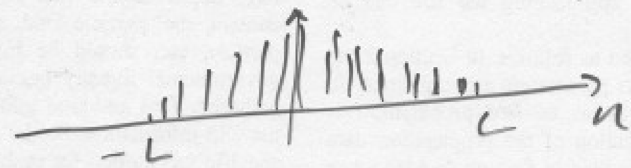
$$h_w[n] = h_{id}[n] \cdot w[n]$$

where $w[n]$ is a window of length $2L+1$ and centered around $n=0$. ($h_{id}[n]$ is symmetric wrt $n=0$ for lowpass, highpass & bandpass filter)

Example: Hanning window:

$$\longrightarrow w_{hn}[n] = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi n}{M-1}\right) \qquad M = 2L+1 \qquad n=-L,\dots 0,\dots L$$

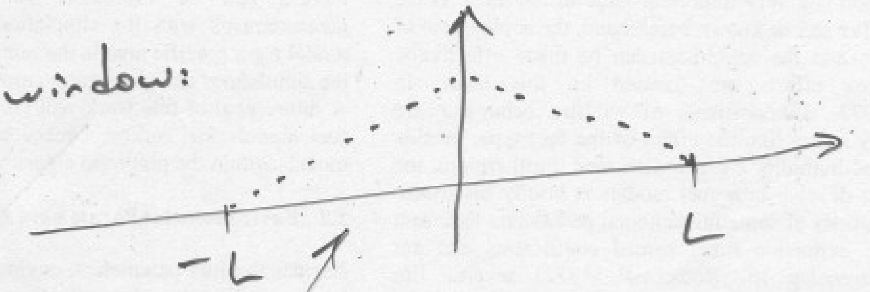
$w_{hn}[0]=1$

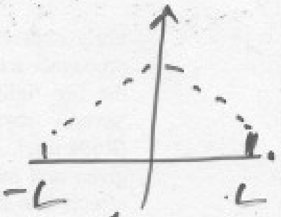


Hamming ~~Blackman~~ window: $w_h[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{M-1}\right)$

$w_h[0] = 1$

Triangula window:





The one with $w_T[\mp L] = 0$

$w_T[-(L+1)] = 0$

$w_T[\mp L] \neq 0$

(Bartlett)

Blackman window: $w_b[n] = 0.42 - 0.5\cos\frac{2\pi n}{M-1} + 0.08\cos\frac{4\pi n}{M-1}$

(Causal)

	transition width of mainlobe	Peak sidelobe is	
rect	$4\pi/2L+1$	-13 dB	below the main lobe
Bartlett	$8\pi/2L+1$	-27 dB	" " "
Hanning	$8\pi/2L+1$	-32 dB	" " "

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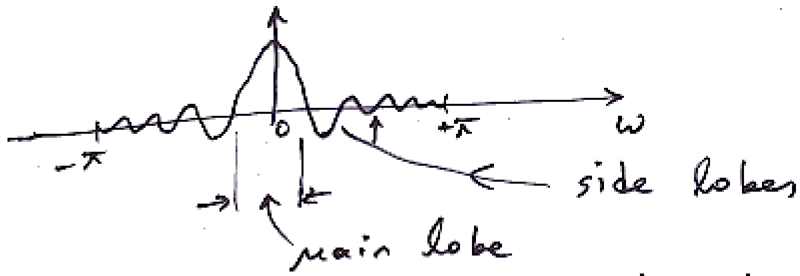
Assume the Window is rectangular.

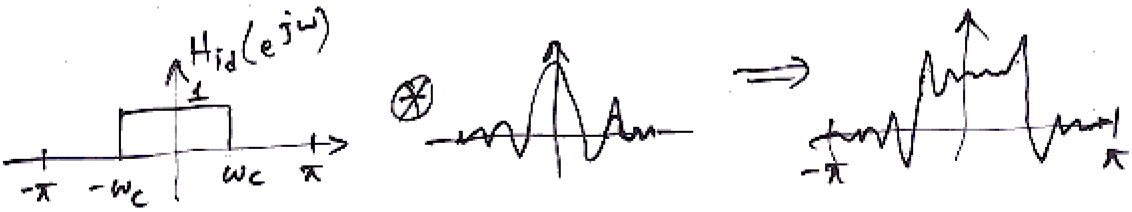
$$W_r(e^{j\omega}) = \sum_{n=-L}^{L} 1 \cdot e^{-j\omega n}$$

$$= 1 + e^{j\omega} + e^{-j\omega} + e^{j\omega 2} + e^{-j\omega 2} \cdots + e^{-j\omega L} + e^{j\omega L}$$

$$W_r(e^{j\omega}) = 1 + 2(\cos\omega + \cos 2\omega + \cdots + \cos L\omega)$$



As $L \nearrow$ width of main lobe decreases.



	Type of window	Approximate transition width of main lobe	peak sidelobe (dB)
	Rectangular	$4\pi/(2L+1)$	−13
(triangle)	Bartlett.	$8\pi/(2L+1)$	−27
	Hanning	$8\pi/(2L+1)$	−32
	Hamming	8 "	−43
	Blackman	$12\pi/(2L+1)$	−58

Wider main lobe $\Rightarrow$ slow transition from passband to stopband!

(If $W(e^{j\omega}) = \delta(\omega)$ $\Rightarrow$ instant transition.)

lower peak side lobe $\Rightarrow$ Reduced Gibbs effect!

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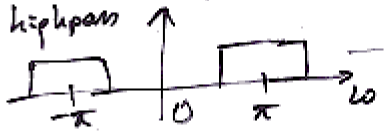
* Highpass, bandpass, Notch, bandstop ....

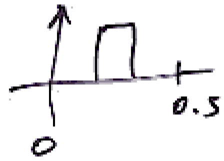
highpass



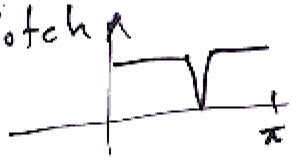
$\xrightarrow{\mathcal{F}_{DT}^{-1}} h_{hp.}[n] \rightarrow w[n] h_{hp}[n] = h_d[n]$

Bandpass

in some books



Notch



bandstop

* ~~Anti~~ Causal filter design.

$\Longrightarrow h_{id}[n]$ is (infinite extent. anticausal, symmetric wrt. 0) in all of the above filters.

$h_d[n] = w[n] \cdot h_{id}[n] \Rightarrow$ Anticausal.
Filter Order $2L+1$.

Causal

$h_c[n] = h_d[n-L]$

$h_d[n] = \{ h_d[-L], \ldots, \underset{\substack{\uparrow \\ n=0}}{h[0]}, \ldots, h_d[L] \}$

$h_c[n] = \{ \underset{\substack{\uparrow \\ n=0}}{h_d[-L]}, \ldots, h[0], \ldots, \underset{\substack{\uparrow \\ n=2L+1}}{h_d[L]} \}$

Filte order $M = 2L+1$ odd!

$H_c(e^{j\omega}) = \underset{\substack{\uparrow \\ \text{real}}}{H_d(e^{j\omega})}\, e^{-j\omega L}$ ← linear phase term.

$|H_c(e^{j\omega})| = |H_d(e^{j\omega})|$, $|e^{j\omega L}| = 1$

$\phi_c(e^{j\omega}) = -\omega L$

Zerophase design : "Anticausal design"

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In Oppenheim Windows are defined for Causal filters: e.g.

Blackman : $w_{bl}[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\frac{4\pi n}{M-1}$

$n = 0, 1, \ldots, M-1$

Hanning : $w_{hn}[n] = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M-1}\right)\right)$

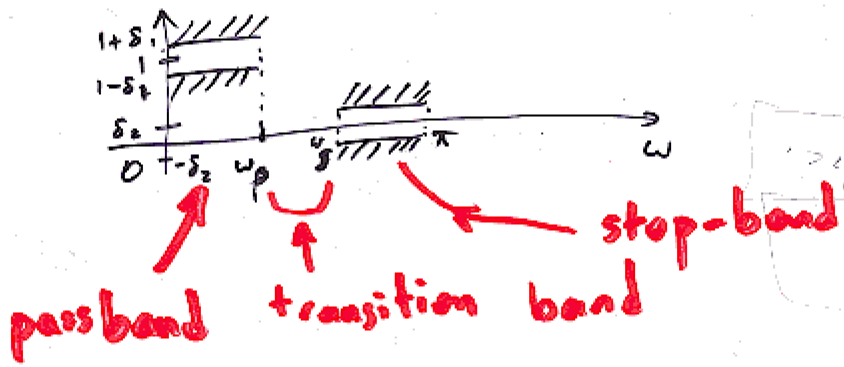
Filter order: M? ($M = 2L+1$)

higher $M \uparrow$ → better approximation.
→ computational cost increases

* One may need to design filters with several M values and check the frequency response until it is satisfactory!

* Kaiser's formula for estimating M:

$$\hat{M} = \frac{-20\log_{10}\left(\sqrt{\delta_1\delta_2}\right) - 13}{14.6\,(\omega_s - \omega_p)/2\pi} + 1$$



transition band: don't care region.

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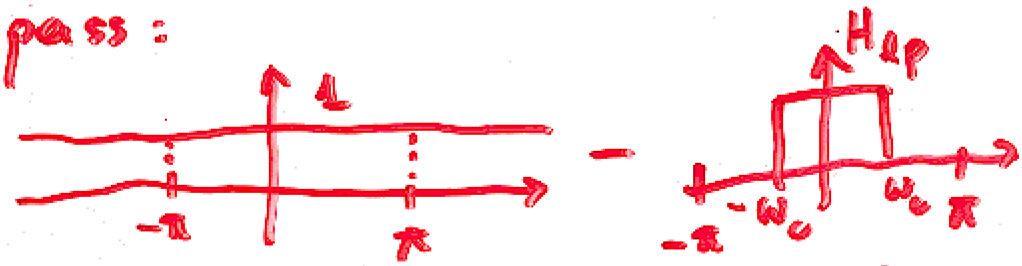
General rules:

* Narrow transition $(\omega_s-\omega_p)\downarrow \Rightarrow M\uparrow$
* $\delta_1, \delta_2 \downarrow \Rightarrow M\uparrow$
* $M\uparrow \Rightarrow$ high computational load.
* Ex Design a highpass from a lowpass:



= highpass filter with cut off ω_c

$$H_{hp}(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$$

$$h_{hp}[n] = \delta[n] - h_{lp}[n]$$

Eg $h_{lp}[n] = \{\frac{1}{4}, \underset{\substack{\uparrow \\ n=0}}{\frac{1}{2}}, \frac{1}{4}\}$

$$h_{hp}[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}\} - \{\frac{1}{4}, \underset{\substack{\uparrow \\ n=0}}{\frac{1}{2}}, \frac{1}{4}\} = \{-\frac{1}{4}, \underset{\substack{\uparrow \\ n=0}}{\frac{1}{2}}, -\frac{1}{4}\}$$