Name: **ID:** **Section:**

Score:	/ 100

EEE-424 Digital Signal Processing: Grand Quiz 2009

Duration: 80 minutes
Instructions: No calculators, book or notes allowed. SHOW YOUR WORK! No credit for results without explanations or steps!!

Q.1

Consider the FIR filter $y[n] = h[n] * x[n]$, where $h[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$.

Q.1a (7 pts) Compute output of $x[n] = \delta[n] - 3\delta[n-1] + 2\delta[n-2] + 5\delta[n-3]$

Q.1b (7 pts) Calculate the frequency response $H(e^{j\omega})$ of $h[n]$.

Q.1c (19 pts) Determine a second FIR filter $g[n]$ so that the combined filter $c[n] = g[n] * h[n]$ is causal. Calculate $c[n]$.

Q.1d (7 pts) Compute the output of the input sequence given in [Q.1a] using the filter $c[n]$. Compare with the result of [Q.1a].

Q.1e (10 pts) Calculate the frequency response $H_c(e^{j\omega})$ of the filter $c[n]$. Compare with $H(e^{j\omega})$ from [Q.1b].

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a) $x[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, -3, 2, 5\}$ $\quad h[n] = \{1, \underset{\substack{\uparrow \\ n=0}}{-2}, 1\}$

$$\begin{array}{rrrrrr}
 & 1 & -3 & 2 & 5 & \\
 & 1 & -2 & 1 & & \\
\hline
1 & -3 & 2 & 5 & & \\
 & -2 & 6 & -4 & -10 & \\
 & & 1 & -3 & 2 & 5 \\
\hline
\end{array}$$

$\{\underset{\substack{\uparrow \\ n=0}}{1}\ -5\ 9\ -2\ -8\ 5\} = y[n]$

b) $H(e^{j\omega}) = \sum_{n=-1}^{1} h[k] e^{-j\omega k} = e^{j\omega} - 2 + e^{-j\omega}$

$= -2 + 2\cos(\omega) = 2(\cos(\omega) - 1)$

c) $g[n]$ should be an one unit delay $\Rightarrow g[n] = \delta[n-a]$ where $a \geq 1$

$\Rightarrow c[n] = \delta[n+1-a] + \delta[n-a] + \delta[n-1-a]$

d) $y_2[n] = c[n] * x[n] \Rightarrow$ Since conv. is for LTI systems $\Rightarrow y_2[n] = \delta[n-1] * y[n]$

$y_2[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, -5, 9, -2, -8, 5\}$

e) $c[n] = \delta[n-1] * h[n] \Rightarrow C(\omega) = e^{-j\omega} \cdot H(\omega)$

Q.2

Consider the IIR filter $y[n] = x[n] + y[n-1] - y[n-2]$.

Q.2a (7 pts) Compute output $y[n], n = 0, ..., 8$ of this filter for $x[n] = 4\delta[n] - 3\delta[n-1] + \delta[n-2]$. Assume $y[n] = 0$ for $n < 0$.

Q.2b (19 pts) Determine $y[k + 1 + 6n]$ for $k = 0, ..., 5,\ n \geq 0$. E.g. $y[1 + 6n] = ..., y[2 + 6n] = ..., ..., y[6 + 6n] = ...$

Q.2c (10 pts) Compute the z-transform $H(z)$ of the IIR filter.

Q.2d (7 pts) Compute the corresponding frequency response $H(e^{j\omega})$.

Q.2e (7 pts) Plot the flow diagram of the filter.

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a) $x[n] = \{4, -3, 1\}$

$\Rightarrow y[0] = x[0] + \cancel{y[-1]} - \cancel{y[-2]} = 4$

$y[1] = x[1] + y[0] - y[-1] = -3 + 4 = 1$

$y[2] = x[2] + y[1] - y[0] = 1 + 1 - 4 = -2$

$\Rightarrow$ for $n > 2$ $x[n] = 0$, therefore it does not effect the output

$y[3] = y[2] - y[1] = -3$

$y[4] = y[3] - y[2] = -3 + 2 = -1$

$y[5] = y[4] - y[3] = 2$

$y[6] = y[5] - y[4] = 3$

$y[7] = y[6] - y[5] = 1$

$y[8] = y[7] - y[6] = -2$

$\Rightarrow$ Current output depends on only the last two output instances. Therefore from time instance $n = 7$ the system recurses itself. (Period = 6)

$\boxed{\Rightarrow y[n] = y[n+6]}$

b) $y[1+k+6n] = y[1+k] \Rightarrow y[1+k] = \{1, -2, -3, -1, 2, 3\}$

c) $Y(z) = X(z) + z^{-1}Y(z) - z^{-2}Y(z) \Rightarrow Y(z)(1 - z^{-1} + z^{-2}) = X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1} + z^{-2}}$

d) $H(e^{jw}) = H(z)\Big|_{z = e^{jw}} = \frac{1}{1 - e^{-jw} + e^{-2jw}}$

e)