Name: **ID:** **Section:**

Score: / **100**

EEE-424 Digital Signal Processing: Mid-Term Exam 2009

Duration: 2 hours
Instructions: No calculators, book or notes allowed. SHOW YOUR WORK! No credit for results without explanations or steps!!

Q.1

Consider the continuous-time signal $x(t) = \sin(2\pi at) + \sin(2\pi bt)$, where $b > a$.

Q.1a Plot the continous-time Fourier-transform $X(j\Omega)$ of $x(t)$.

Q.1b What is the lower bound for the sampling frequency so that $x(t)$ can be theoretically reconstructed from its samples?

Q.1c Plot the block-diagram of the system which samples $x(t)$ to yield the discrete-time signal $x[n]$ without aliasing. Specify all components. Hint: use impulse-train.

Q.1d Plot the block-diagram of the system which reconstructs $x(t)$ from $x[n]$. Specify all components.

Q.2

Given $x[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$ and $y[n] = 3\delta[n-2] - 2\delta[n-3]$.

Q.2a Compute $z[n] = x[n] * y[n]$.

Q.2b Compute the 4-point DFT $X[n]$ of $x[n]$.

Q.2c Compute the 4-point DFT $Y[n]$ of $y[n]$.

Q.2d Compute $z[n]$ by DFT and compare your result with **Q.2a**.

Q.3

Given a discrete-time sequence $x[n]$, $n = 0, ..., 8$, the 9-point-DFT shall be computed.

Q.3a Derive a decimation-in-time-FFT method for this task.

Q.3b Plot the corresponding FFT-flowgraph.

Q.3c Compare the computational complexity of calculating the FFT method developed in **Q.3a** with regular 9-point DFT.

Q.4

Given a lowpass filter $H_{\text{id}}(e^{j\omega})$ with cutoff frequency ω_c.

Q.4a Plot $H_{\text{id}}(e^{j\omega})$ in the range $[-4\pi, 4\pi]$.

Q.4b Compute the corresponding impulse response $h_{\text{id}}[n]$.

Q.4c Determine an FIR-filter $h_a[n]$ of order $2L + 1$, $L = 3$ as an approximation of $h_{\text{id}}[n]$ (rectangular windowing method). Specify the rectangular windowing function $w[n]$.

Q.4d Plot roughly the frequency response $H_a(e^{j\omega})$ of $h_a[n]$. Explain the differences between $H_{\text{id}}(e^{j\omega})$ and $H_a(e^{j\omega})$ and the **cause** of these differences as detailed as possible. Hint: use time-domain and frequency-domain plots or equations of the windowing function $w[n]$.

Cheat Sheet

- Discrete-time convolution:

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

- Continous-Time Fourier Transform (CTFT):

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}d\Omega$$

- Discrete-Time Fourier Transform (DTFT):

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
$$x[n] = \frac{1}{2\pi}\int_{0}^{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

- N-point Discrete Fourier Transform (DFT):

$$X[n] = \sum_{k=0}^{N-1} x[k]e^{-j\frac{2\pi}{N}kn}$$
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn}$$