Quiz ID:

① Calculate 10 elements of $y[n] = x_1[n] \circledS x_2[n]$ where $x_1[n] = \{\underset{\uparrow n=0}{1}, 2\}$ and $x_2[n] = \{\underset{\uparrow n=0}{-1}, 0, 3\}$

$$\left(y[n] = \sum_{m=0}^{4} x_1[m] \cdot x_2[(n-m)_5] \right)$$

② Compute the DFT of $x[n] = \{\frac{1}{2}, 0, \frac{1}{2}\}$ by first calculating the DTFT and sampling it ($N=10$).

$X[0] = \ldots \big|_{(k=0)}$

$X[1] = \ldots \big|_{(k=1)}$

Solutions:

①

$$x_1[n] = \{\overset{0}{1}, \overset{1}{2}, \overset{2}{0}, \overset{3}{0}, \overset{4}{0}, 1, 2, 0, 0, 0, 1, \ldots\}$$

$$x_2[n] = \{-1, 0, 3, 0, 0, -1, 0, 3, 0, 0, -1, \ldots\}$$

use (mod 5)

$x_2[(-1)_5] = x_2[4]$
$x_2[(-2)_5] = x_2[3]$
$x_2[(-3)_5] = x_2[2]$
$x_2[(-4)_5] = x_2[1]$

$$y[n] = x_1[n] \circledS x_2[n]$$

$$= \sum_{m=0}^{4} x_1[m] \cdot x_2[(n-m)_5]$$

$$y[0] = \sum_{m=0}^{4} x_1[m] \cdot x_2[(-m)_5]$$

$$= \overbrace{x_1[0] \cdot x_2[0]}^{m=0} + \overbrace{x_1[1] \cdot x_2[4]}^{m=1} + \overbrace{x_1[2] x_2[3]}^{m=2} + \overbrace{x_1[3] x_2[2]}^{m=3} + \overbrace{x_1[4] x_2[1]}^{m=4}$$

$$= 1 \cdot (-1) + 2 \cdot 0 + 0 \cdot 0 + 0 \cdot 3 + 0 \cdot (-1) = -1$$

$$y[1] = \sum_{m=0}^{4} x_1[m] \cdot x_2[(1-m)_5]$$

$$= x_1[0] \cdot x_2[1] + x_1[1] \cdot x_2[0] + x_1[2] \cdot x_2[4] + x_1[3] \cdot x_2[3] + x_1[4] x_2[2]$$

$$= 1 \cdot 0 + 2 \cdot (-1) + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = -2 \quad ✓$$

$$y[2] = \sum_{m=0}^{4} x_1[m] \cdot x_2[(2-m)_5]$$

$$= x_1[0] \cdot x_2[2] + x_1[1] \cdot x_2[1] + x_1[2] \cdot x_2[0] + x_1[3] \cdot x_2[4] + x_1[4] x_2[3]$$

$$= 1 \cdot 3 + 2 \cdot 0 + 0 \cdot (-1) + 0 \cdot 0 + 0 \cdot 0 = 3$$

$$y[3] = \sum_{m=0}^{4} x_1[m] \cdot x_2[(3-m)_5]$$

$$= x_1[0] \cdot x_2[3] + x_1[1] \cdot x_2[2] + x_1[2] \cdot x_2[1] + x_1[3] \cdot x_2[0] + x_1[4] x_2[4]$$

$$= 1 \cdot 0 + 2 \cdot 3 + 0 \cdot 0 + 0 \cdot (-1) + 0 \cdot 0 = 6$$

$$y[4] = \sum_{m=0}^{4} x_1[m] \cdot x_2[(4-m)_5]$$

$$= x_1[0] \cdot x_2[4] + x_1[1] x_2[3] + x_1[2] x_2[2] + x_1[3] x_2[1] + x_1[4] x_2[0]$$

$$= 1 \cdot 0 + 2 \cdot 0 + 0 \cdot 3 + 0 \cdot 0 + 0 \cdot (-1) = 0$$

since $x_1[n]$ and $x_2[n]$ repeats themselves for each (periodic) 5 points:

$$y[5] = y[0] = -1$$
$$y[6] = y[1] = -2$$
$$y[7] = y[2] = 3$$
$$y[8] = y[3] = 6$$
$$y[9] = y[4] = 0$$

✓

2) DTFT: $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j \cdot wn}$

$$X(e^{jw}) = \sum_{n=-1}^{1} x[n] \cdot e^{-jwn}$$

$$= x[-1] \cdot e^{jw} + x[0] \cdot 1 + x[1] \cdot e^{-jw} = \frac{1}{2} e^{jw} + 0 + \frac{1}{2} e^{-jw}$$

$$= \frac{e^{jw} + e^{-jw}}{2} = \cos w$$

DFT : DTFT $\Big|_{w = \frac{2\pi k}{10}}$

to find DFT, we can sample the DTFT at $\boxed{w = \frac{2\pi k}{N}}$

$$X[k] = \cos\left(\frac{2\pi k}{10}\right)$$

✓

$X[k]$ is the DFT.

$$\begin{cases} X[0] = \cos(0) = 1 \\ X[1] = \cos\left(\frac{2\pi}{10}\right) \\ X[2] = \cos\left(\frac{4\pi}{10}\right) \\ \vdots \end{cases}$$

$$X[k] = X(e^{jw})\Big|_{w = \frac{2\pi k}{N}} = \cos(w)\Big|_{w = \frac{2\pi k}{10}} = \cos\left(\frac{2\pi k}{10}\right)$$