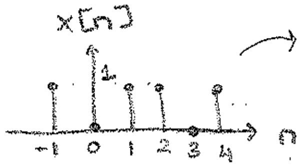
Quiz 2 - Section 2

1) Write down the periodic extension (6 point periodic) $x_p[n]$ of $x[n] = \{\underset{\underset{n=0}{\uparrow}}{a}, b\}$.

$$x_p[n] = \{\ldots, a, b, 0, 0, 0, 0, \underbrace{\underset{\underset{n=0}{\uparrow}}{a}, b, 0, 0, 0, 0}_{\text{6 point periodic}}, a, b, 0, 0, 0, 0, \ldots\}$$

2) Find the 3point DFT of $x[n] = \{1, \underset{\underset{n=0}{\uparrow}}{0}, 1\}$. (k is always positive!)

DFT formula: $\boxed{X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}, \quad k = 0, \ldots N-1.}$ $N = 3$ since 3-point DFT.



Use this 3-point periodic extension of $x[n]$ for computing the 3-point DFT

DFT formula!: $X[k] = \sum_{n=0}^{2} x[n] e^{-j\frac{2\pi}{3}kn} = \underbrace{x[0]}_{0} + \underbrace{x[1]}_{1} e^{-j\frac{2\pi k}{3}} + \underbrace{x[2]}_{1} e^{-j\frac{4\pi k}{3}}$

$$X[k] = e^{-j\frac{2\pi k}{3}} + e^{-j\frac{4\pi k}{3}} \Rightarrow$$

$$X[0] = e^{0} + e^{0} = 2$$

$$X[1] = e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}$$

$$X[2] = e^{-j\frac{4\pi}{3}} + e^{-j\frac{8\pi}{3}}$$

$$X[k] = e^{-j\frac{2\pi k}{3}} + e^{j(\frac{-4\pi}{3}+2\pi)k}$$

$$X[k] = e^{-j\frac{2\pi}{3}k} + e^{j\frac{2\pi}{3}k} = 2\cos\left(\frac{2\pi k}{3}\right)$$

$$X[0] = 2\cos(0) = 2$$

$$X[1] = 2\cos\left(\frac{2\pi}{3}\right) = 2 \times \left(\frac{-1}{2}\right) = -1$$

$$X[2] = 2\cos\left(\frac{4\pi}{3}\right) = 2 \times \left(\frac{-1}{2}\right) = -1$$

↳ Alternative solution: DTFT of $\{1, \underset{\underset{n=0}{\uparrow}}{0}, 1\}$: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \Rightarrow$ DTFT formula!

$$X(e^{j\omega}) = \sum_{n=-1}^{1} x[n] e^{-j\omega n} = \underbrace{x[-1]}_{1} e^{j\omega} + \underbrace{x[0]}_{0} + \underbrace{x[1]}_{1} e^{-j\omega} = e^{j\omega} + e^{-j\omega} = 2\cos(\omega)$$

Sample DTFT at $\omega = \frac{2\pi k}{N} = \frac{2\pi k}{3} \Rightarrow X[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{3}} = 2\cos\left(\frac{2\pi k}{3}\right)$

3) Show that your result of question no. 2 is correct by calculating IDFT?

IDFT : $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}, \quad n = 0, \ldots, N-1$

$$x[n] = \frac{1}{3} \sum_{k=0}^{2} X[k] e^{j\frac{2\pi kn}{3}} = \frac{1}{3}\left(\underbrace{X[0]}_{2} + \underbrace{X[1]}_{-1} e^{j\frac{2\pi n}{3}} + \underbrace{X[2]}_{-1} e^{j\frac{4\pi n}{3}}\right)$$

$$= \frac{1}{3}\left(2 - e^{j\frac{2\pi n}{3}} - e^{j\frac{4\pi n}{3}}\right)$$

$x[0] = \frac{1}{3}(2 - 1 - 1) = 0$; $x[1] = \frac{1}{3}\left(2 - e^{j\frac{2\pi}{3}} - e^{j\frac{4\pi}{3}}\right) = \frac{1}{3}\left(2 - e^{j\frac{2\pi}{3}} - e^{j(\frac{4\pi}{3} - 2\pi)k}\right)$

$$= \frac{1}{3}\left(2 - e^{j\frac{2\pi}{3}} - e^{-j\frac{2\pi}{3}}\right) = \frac{1}{3}\left(2 - 2\cos\left(\frac{2\pi}{3}\right)\right)$$

$$= \frac{1}{3}\left(2 - 2\left(\frac{-1}{2}\right)\right) = 1$$

$$x[2] = \frac{1}{3}\Big(2 - \underset{\underset{e^{-j\frac{2\pi}{3}}}{\|}}{e^{j\frac{4\pi}{3}}} - \underset{\underset{e^{j\frac{2\pi}{3}}}{\|}}{e^{j\frac{8\pi}{3}}}\Big) = \frac{1}{3}\left(2 - e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}}\right) = \frac{1}{3}\left(2 - 2\cos\left(\frac{2\pi}{3}\right)\right)$$

$$= 1$$

$$\begin{Bmatrix} x[0] = 0 \\ x[1] = 1 \\ x[2] = 1 \end{Bmatrix} \underset{\xleftarrow{\text{IDFT}}}{\xrightarrow{\text{DFT}}} \begin{Bmatrix} X[0] = 2 \\ X[1] = -1 \\ X[2] = -1 \end{Bmatrix}$$

4) convolve $x_1[n] = \{\underset{\underset{n=0}{\uparrow}}{-1}, 2, 1\}$ and $x_2[n] = \{\underset{\underset{n=0}{\uparrow}}{0}, 1, 3\}$

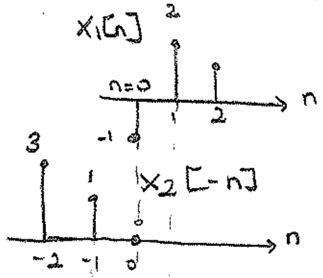
$$y[k] = \sum_{n=0}^{4} x_1[n]\, x_2[k-n] = x_1[n] * x_2[n]$$

↳ Convolution length $= \underset{\text{length of } x_1[n]}{\overset{\uparrow}{3}} + \underset{\text{length of } x_2[n]}{\overset{\uparrow}{3}} - 1 = 5$

$$y[0] = \sum_{n=0}^{4} x_1[n]\, x_2[-n]$$

$$= x_1[0]\, x_2[0]$$

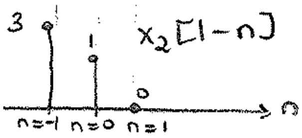
$$= (-1) \times 0 = 0$$



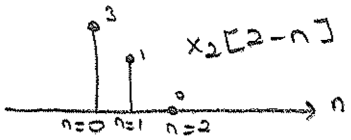
$$y[1] = \sum_{n=0}^{4} x_1[n]\, x_2[1-n]$$

$$= \underbrace{x_1[0]\, x_2[1]}_{n=0} + \underbrace{x_1[1]\, x_2[0]}_{n=1}$$

$$= (-1) \times 1 + 2 \times 0 = -1$$



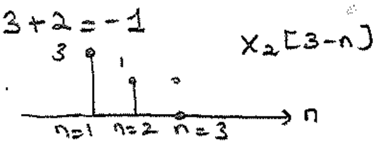
$$y[2] = \sum_{n=0}^{4} x_1[n]\, x_2[2-n]$$



$$= \overbrace{x_1[0]\, x_2[2]}^{n=0} + \overbrace{x_1[1]\, x_2[1]}^{n=1} + \overbrace{x_1[2]\, x_2[0]}^{n=2}$$

$$= (-1) \times 3 + 2 \times 1 + 1 \times 0 = -3 + 2 = -1$$

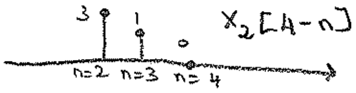
$$y[3] = \sum_{n=0}^{4} x_1[n]\, x_2[3-n]$$



$$= \overbrace{x_1[0]\, x_2[3]}^{n=0} + \overbrace{x_1[1]\, x_2[2]}^{n=1} + \underbrace{x_1[2]\, x_2[1]}_{n=2} + \underbrace{x_1[3]\, x_2[0]}_{n=3}$$

$$= (-1) \times 0 + (2) \times 3 + 1 \times 1 + 0 \times 0 = 7$$

$$y[4] = \sum_{n=0}^{4} x_1[n]\, x_2[4-n]$$



$$= x_1[0]\, x_2[4] + x_1[1]\, x_2[3] + x_1[2]\, x_2[2] + x_1[3]\, x_2[1] + x_1[4]\, x_2[0]$$

$$= \cancel{(-1) \times 0} + \cancel{2 \times 0} + 1 \times 3 + 0 \times 1 + 0 \times 0 = 3$$

$$y[k] = \{\underset{\underset{k=0}{\uparrow}}{0}, -1, -1, 7, \underset{\underset{k=4}{\uparrow}}{3}\}$$