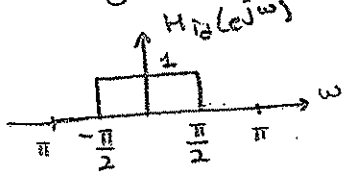
QUIZ 3 - Section 1 Solutions

1) Design a LPF with these requirements,



$h_{id}[n] = ?$

$h_{id}[n] = \text{IDTFT}\{H_{id}(e^{j\omega})\}$

$$h_{id}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{id}(e^{j\omega})e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\pi/2} \overbrace{1}^{H_{id}(e^{j\omega})}\cdot e^{j\omega n}\,d\omega \Rightarrow H_{id}(e^{j\omega}) = \begin{cases} 1, & -\frac{\pi}{2} < \omega < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\pi/2} e^{j\omega n}\,d\omega = \frac{1}{2\pi}\left(\frac{e^{j\omega n}}{jn}\Bigg|_{\omega=-\frac{\pi}{2}}^{\pi/2}\right) = \frac{1}{2\pi n}\left(\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{j}\right)$$

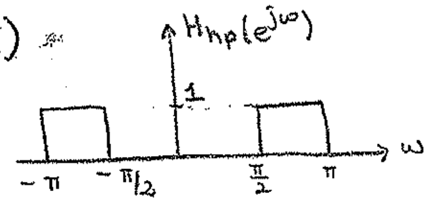
$$= \frac{1}{\pi n}\underbrace{\left(\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{2j}\right)}_{\sin\left(\frac{\pi}{2}n\right)} \quad ; \quad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

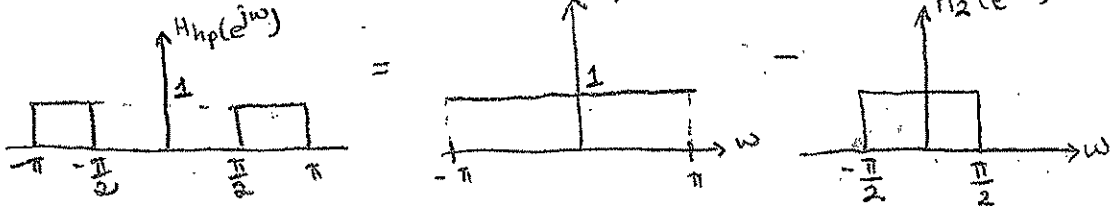
$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

sinc function

$$h_{id}[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} = \frac{\sin\left(\frac{\pi}{2}n\right)}{2\times\frac{\pi}{2}\times n} = \frac{\text{sinc}(n\pi/2)}{2}$$

2) $h_{hp}[n] = ?$





IDTFT

$$h_{hp}[n] = h_1[n] - h_2[n] \quad ; \quad h_1[n] = \text{IDTFT}\{1\} = \delta[0]$$

$$h_2[n] = \text{IDTFT}\{H_2(e^{j\omega})\} = \frac{\text{sinc}\left(\frac{n\pi}{2}\right)}{2}$$

from Q-1

$$= \delta[0] - \left(\frac{\text{sinc}\left(\frac{n\pi}{2}\right)}{2}\right)$$

3) $h[n] = g[n-n_0]$, $H(e^{j\omega}) = ?$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} g[\underbrace{n-n_0}_{k}]e^{-j\omega n} \quad \rightarrow \text{put } n-n_0 = k \Rightarrow n = k+n_0$$

$$H(e^{j\omega}) = \underbrace{\sum_{k=-\infty}^{\infty} g[k]e^{-j\omega k}}_{G(e^{j\omega})} e^{-j\omega n_0} = G(e^{j\omega})e^{-j\omega n_0}$$

$$g[n-n_0] \underset{\text{IDTFT}}{\overset{\text{DTFT}}{\rightleftarrows}} G(e^{j\omega})e^{-j\omega n_0} \Rightarrow$$

A shift by n_0 in time domain is multiplication by $e^{-j\omega n_0}$ in frequency domain.