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Section: 1

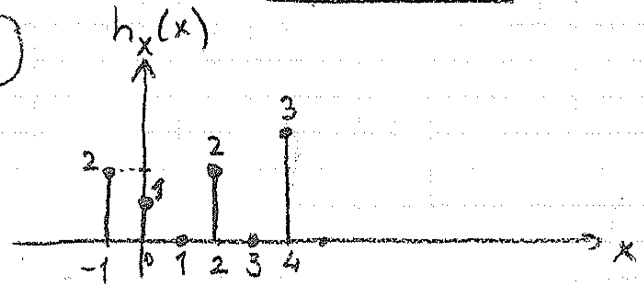
Quiz

$x[n] = \{-1, 2, 4, 0, 2, -1, 4, 4\}$

1. Plot the histogram of $x[n]$
2. Determine P.M.F and PDF of $x[n]$
3. Calc. estimated mean m_x and variance $\hat{\sigma}_x^2$ of $x[n]$.
4. Calc. $r_x[k]$ for $k = 0, 1, 2$.

Answers

(1)



(2) PMF

$$P_x(x) = \frac{h_x(x)}{N} \quad N = 8 \quad \Rightarrow$$

PMF of $x[n]$

$$P_x(x) = \begin{cases} \frac{2}{8} = \frac{1}{4} & , x = -1 \\ \frac{1}{8} & , x = 0 \\ \frac{1}{4} & , x = 2 \\ \frac{3}{8} & , x = 4 \\ 0 & , \text{otherwise} \end{cases}$$

So, PDF of $x[n]$:

$$f_x(x) = \frac{1}{4}\delta[x+1] + \frac{1}{8}\delta[x] + \frac{1}{4}\delta[x-2] + \frac{3}{8}\delta[x-4]$$

↓ continued

(3) $$\hat{m}_x = \frac{-1+2+4+2-1+4+4}{8} = \frac{14}{8} = \frac{7}{4}$$

estimated mean

$$\hat{\sigma}_x^2 = E[(X-\hat{m}_x)^2]$$

$$\hat{\sigma}_x^2 = E[X^2] - (E[X])^2 = E[X^2] - (\hat{m}_x)^2 = \frac{29}{4} - \left(\frac{7}{4}\right)^2 = \frac{29}{\underset{(4)}{4}} - \frac{49}{16} = \frac{116-49}{16}$$

$$E[X^2] = \frac{1}{4}\cdot(-1)^2 + \frac{1}{4}\cdot(2)^2 + \frac{3}{8}\cdot 4^2 = \frac{1}{4} + 1 + 6 = \frac{29}{4}$$

$$\Downarrow$$

$$\boxed{\hat{\sigma}_x^2 = \frac{67}{16}}$$

(4) $$r_x[k] = \frac{1}{N}\sum_{n=0}^{N-k-1} x[n]\,x[n+k]$$

$$k=0 \Rightarrow r_x[0] = \frac{1}{8}\sum_{n=0}^{7} x[n]\cdot x[n] = \frac{1}{8}\left[(-1)^2+2^2+4^2+2^2+(-1)^2+4^2+4^2\right]$$

$$= \frac{1}{8}\left[1+4+16+4+1+16+16\right] = \frac{26+32}{8} = \frac{58}{8} = \frac{29}{4}$$

$$k=1 \Rightarrow r_x[1] = \frac{1}{8}\sum_{n=0}^{6} x[n]\cdot x[n+1] = \frac{1}{8}\left[(-1)2+2\cdot4+4\cdot0+0\cdot2+2\cdot(-1)+(-1)\cdot4+4\cdot4\right]$$

$$= \frac{1}{8}\left[-2+8-2-4+16\right] = 2$$

$$k=2 \Rightarrow r_x[2] = \frac{1}{8}\sum_{n=0}^{5} x[n]\,x[n+2] = \frac{1}{8}\left[(-1)\cdot4+2\cdot0+4\cdot2+0\cdot(-1)+2\cdot4+(-1)\cdot4\right]$$

$$= \frac{1}{8}\left[-4+8+8-4\right] = 1$$