2030243 — Quiz #4 — 100

(1) Given $x[n] = \{1, 0, -1, 2, 2, 0, 2, 1\}$ (↓ $n=0$)

a) Determine the PMF and PDF.

b) Compute $\hat{m}_x$, σ_x^2

c) Compute $\hat{r}_x[k]$, $k=0,1,2$. d) Plot histogram of $x[n]$

ANSWER

(1) (a) PMF ⇒ $n=-1 \Rightarrow P_X[-1] = 1/8$

$n=0 \Rightarrow P_X[0] = 2/8 = 1/4$

$n=1 \Rightarrow P_X[1] = 2/8 = 1/4$

$n=2 \Rightarrow P_X[2] = 3/8$

(b) $\hat{m}_x = -1 \cdot 1/8 + 0 \cdot 1/4 + 1 \cdot 1/4 + 2 \cdot 3/8 = -1/8 + 0 + 1/4 + 6/8 = \frac{5}{8} + 1/4 = \boxed{7/8}$

$\sigma_x^2 = E[(X-\mu)^2] = E[X^2] - (E[X])^2 =$ (↓)

$(1 \cdot 1/8 + 0 \cdot 1/4 + 1 \cdot 1/4 + 4 \cdot 3/8) - (7/8)^2$

$(1/8 + 0 + 1/4 + 12/8) - \frac{49}{64} = \frac{120}{64} - \frac{49}{64} = \boxed{\frac{71}{64}}$

(c) $\hat{r}_x[k] = \frac{1}{N} \sum_{n=0}^{N-1-k} x[n]\, x[n+k]$, $k=0,1,2$

$\hat{r}_x[0] = \frac{1}{8} \sum_{n=0}^{8-1-0} x[n]\, x[n+0] = \frac{1}{8} \sum_{n=0}^{7} x[n] \cdot x[n] = 15/8$

$\hat{r}_x[1] = \frac{1}{8} \sum_{n=0}^{8-1-1} x[n]\, x[n+1] = 1/8 \sum_{n=0}^{6} x[n]\, x[n+1] = 1/2$

$\hat{r}_x[2] = 1/8 \sum_{n=0}^{8-1-2} x[n]\, x[n+2] = 1/8 \sum_{n=0}^{5} x[n]\, x[n+2] = 1/8$

(d)