Name: **ID:** **Section:**

Score:	/ 100

EEE-424 Digital Signal Processing: Grand Quiz Spring 2011

Duration: 70 minutes
Instructions: No calculators, book or notes allowed. SHOW YOUR WORK! No credit for results without explanations or steps!!

Q.1 Overlap-and-Add

Given $x[n] = \{1, 2, 0, 1, 0, 2, -2, -1\}$ and $h[n] = \{1, -1\}$, both starting at $n = 0$.

Q.1a (10 pts) Compute $y[n] = h[n] * x[n]$ using regular convolution.

Solution : $y[n] = \{1, 1, -2, 1, -1, 2, -4, 1, 1\}$

Q.1b (23 pts) Compute $y[n] = h[n] * x[n]$ using the overlap-and-add method by breaking $x[n]$ into two parts, where

$$x[n] = x_1[n] + x_2[n],\ \ x_1[n] = \{x[0], ..., x[3]\}, x_2[n] = \{0, 0, 0, 0, x[4], ..., x[7]\} \tag{1}$$

How many samples do overlap?

Solution : $x_1[n] = \{1, 2, 0, 1\}$, $x_2[n] = \{0, 0, 0, 0, 0, 2, -2, -1\}$, both starting at $n = 0$:

$$h[n] * x[n] = h[n] * (x_1[n] + x_2[n]) = h[n] * x_1[n] + h[n] * x_2[n] \tag{2}$$

$$= \{1, 1, -2, 1, -1\} + \{0, 0, 0, 0, 0, 2, -4, 1, 1\} = \{1, 1, -2, 1, -1, 2, -4, 1, 1\} \tag{3}$$

Q.2 Accumulator

Consider the so called accumulator: $y[n] = \sum_{k=-\infty}^{n} x[k]$.

Q.2a (11 pts) Is the accumulator a linear system? **Prove** your answer.

Solution : Let $y_1[n] = \sum_{k=-\infty}^{n} x_1[k]$ and $y_2[n] = \sum_{k=-\infty}^{n} x_2[k]$.
Define $y_3[n] = \sum_{k=-\infty}^{n}(ax_1[k] + bx_2[k]) = ay_1[n] + by_2[n]$.

Q.2b (11 pts) Is the accumulator a time-invariant system? **Prove** your answer.

Solution : Let $x_1[n] = x[n - n_0]$. To show: $y_1[n] = \sum_{k=-\infty}^{n} x_1[k] = y[n - n_0]$

$$y_1[n] = \sum_{k=-\infty}^{n} x_1[k] = \sum_{k=-\infty}^{n} x[k - n_0] = \sum_{k=-\infty}^{n-n_0} x[k] = y[n - n_0]. \tag{4}$$

Q.2c (11 pts) Determine the impulse response $h[n]$ of the accumulator system, e.g., $y[n] = \sum_{k=-\infty}^{n} x[k] = h[n] * x[n]$. Prove your answer,e.g., show that your solution for the impulse response corresponds to the accumulator system.

Solution :

$$h[n] = u[n] \Rightarrow \sum_{k} h[k]x[n - k] = \sum_{k=0}^{\infty} x[n - k] = x[n] + x[n - 1] + ... = \sum_{k=-\infty}^{n} x[k]$$

Q.3 Random Signal

Consider the system

$$\frac{1}{2}y[n] - \frac{1}{4}y[n-1] + x[n] + w[n] = 0,$$

where $x[n]$ is W.S.S. zero-mean white noise with variance σ_x^2 and $w[n]$ is W.S.S. zero-mean white noise with variance σ_w^2.

Q.3a (24 pts) Determine $r_y[k],\ k = 0, 1, 2$. Simplify terms as far as possible.

Solution : $r_y[0] = \frac{16}{3}(\sigma_x^2 + \sigma_w^2), r_y[1] = 1/2 r_y[0] = \frac{8}{3}(\sigma_x^2 + \sigma_w^2), r_y[2] = 1/2 r_y[1] = \frac{4}{3}(\sigma_x^2 + \sigma_w^2)$

Q.3b (10pts) Determine the expectation m_y of $y[n]$, defined by $m_y = \mathrm{E}[y[n]]$.

Solution : $\mathrm{E}[y[n]] = \mathrm{E}[1/2 y[n] - 2x[n] - 2w[n]] = 1/2 \mathrm{E}[y[n-1]] \Rightarrow m_y = 0$

Formulas

- Discrete-time convolution:

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

- Autocorrelation (W.S.S.):

$$r_x[k] = \mathrm{E}[x[n]x[n+k]]$$