Q1) Given the signal;

$x[n]=\{\underset{\uparrow\, n=0}{1},\ 2,\ 3,\ 4,\ 3,\ 2,\ 1\}$

a)

i) Interpolate this signal by 2 using the filter $h[n]=\left\{\frac{1}{4},\ \underset{\uparrow\, n=0}{\frac{1}{2}},\ \frac{1}{4}\right\}$. Assume $x[-1]=x[7]=0$.

ii) You should use $2h[n]$. Why?

b) Decimate $x[n]$ by 2, using the same filter: $h[n]=\left\{\frac{1}{4},\ \underset{\uparrow\, n=0}{\frac{1}{2}},\ \frac{1}{4}\right\}$

a) i) First we use upsampling;

$a[n] = x[2n]$

$a(n)$	0	1	0	2	0	3	0	4	0	3	0	2	0	1	0
	$n=-1$	$n=0$	$n=1$		$n=3$		$n=5$		$n=7$		$n=9$		$n=11$		$n=13$

$a(n) = x(n) * 2h(n)$

$a(n) = \sum_{k=-\infty}^{\infty} x(n-k) \cdot 2h(k) = x[n+1] \cdot 2 \cdot h(-1) + x[n] \cdot 2 \cdot h(0) + x(n-1) \cdot 2 \cdot h(1)$

$a[n] = \frac{1}{2} x[n+1] + x[n] + \frac{1}{2} x[n-1]$

$a[-1] = \frac{1}{2} \cdot 1 + 0 + 0 = 0,5$

$a(1) = \frac{1}{2} \cdot 2 + 0 + \frac{1}{2} \cdot 1 = 1,5$

$a(3) = \frac{1}{2} \cdot 3 + 0 + \frac{1}{2} \cdot 2 = 2,5$

$a[5] = \frac{1}{2}\ 4 + 0 + \frac{1}{2} \cdot 3 = 3,5$

$a[7] = \frac{1}{2} \cdot 3 + 0 + \frac{1}{2} \cdot 4 = 3,5$

$a[9] = \frac{1}{2}\ 2 + 0 + \frac{1}{2}\ 3 = 2,5$

$a(11) = \frac{1}{2}\ 1 + 0 + \frac{1}{2}\ 2 = 1,5$

$a(13) = \frac{1}{2}\ 0 + 0 + \frac{1}{2}\ 1 = 0,5$

so; $\boxed{a(n) = \{0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3,\ 3.5,\ 4,\ 3.5,\ 3,\ 2.5,\ 2,\ 1.5,\ 1,\ 0.5\}}$

Interpolated version of $x[n]$

ii) We should use $2h[n]$ to avoid amplitude change of the system due to coefficients of low-pass filter.

By using $2h[n]$ we obtain a moving average filter which gives us first order interpolation.

b)

let $b[n] = x[n] * h[n] = \sum_{n=-\infty}^{\infty} x[n-k]\, h[k] = x[n+1]h[-1] + x[n] + h[0] + x[n-1]\, h[1]$

$b[-1] = \frac{1}{4} \cdot 1 + \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot 0 = 0,25$

$b[7] = \frac{1}{4} \cdot 0 + \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot 1 = 0,25$

$b[n] = \{0.25, \underset{\substack{\uparrow \\ n=0}}{1}, 2, 3, 4, 3, 2, 1, 0,25\}$

$a[n] = b[\frac{n}{2}]$

$\Rightarrow \boxed{a[n] = \{1, 3, 3, 1\} \text{ decimated version of } x[n]}$

Q2) Let $x_c(t) = \cos(2\pi 1000t) + \cos(2\pi 333t)$

a) This signal is sampled with $f_s = 8000\,\text{Hz}$ and $x[n] = x_c(nT_s)$ is obtained where $n = 0, \mp 1, \mp 2 \ldots$, $T_s = 1/8000\,\text{sec}$. Plot $X(e^{j\omega})$.

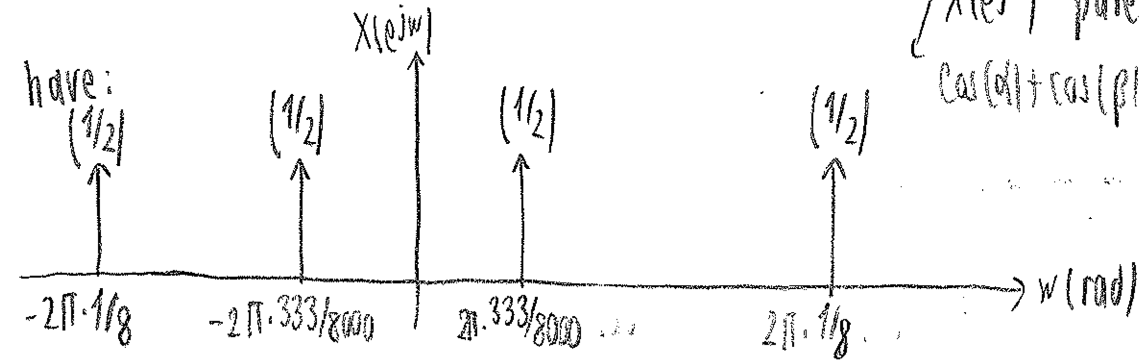
b) Given $x[n] = x_c(nT_s)$, $n = 0, 1, 2, 3, \ldots, 999$ and DFT$_{1000}$ $X[k]$ of $x[n]$ is computed. Approximately plot $|X[k]|$. Clearly indicate peak locations in your $|X[k]|$ plot.

a) Here since it says $n = 0, \mp 1, \mp 2, \ldots$ we have infinite samples.

Then: $x[n] = \cos\left(2\pi \cdot 1000 \cdot \frac{1}{8000} \cdot n\right) + \cos\left(2\pi \cdot 333 \cdot \frac{1}{8000} \cdot n\right)$

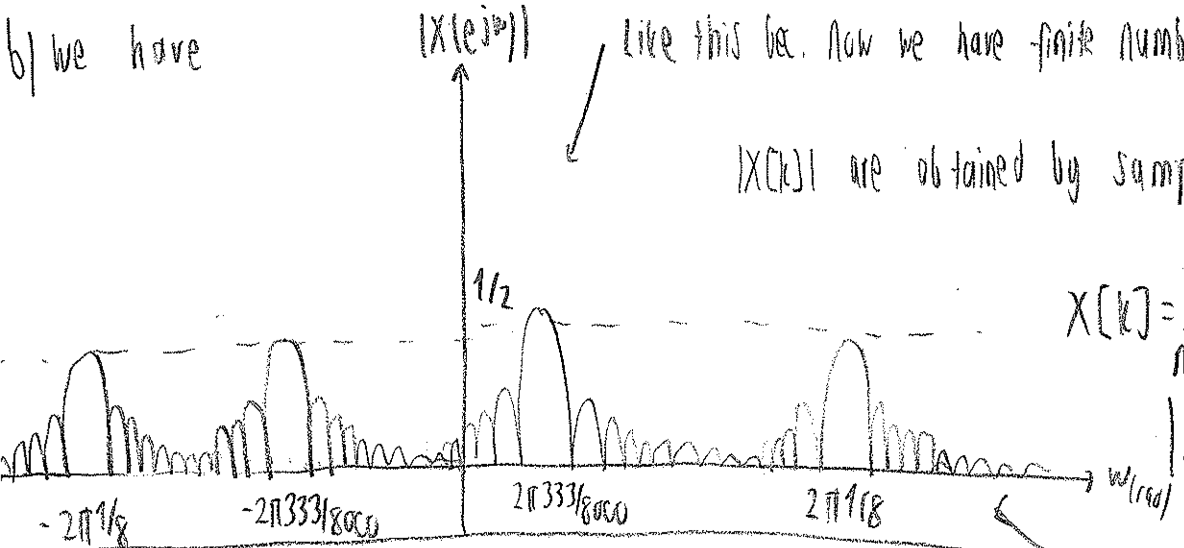
$= \cos\left(2\pi \cdot \frac{1}{8} n\right) + \cos\left(2\pi \cdot \frac{333}{8000} n\right)$

we have:



$X(e^{jw})$ purely real since $\cos(\alpha) + \cos(\beta)$ is real and even symmetric

b) we have



Like this be. Now we have finite number of samples.

$|X[k]|$ are obtained by sampling this.

$$X[k] = \sum_{n=0}^{999} x[n] e^{-j\frac{2\pi k n}{1000}}$$

$$|X[k]| = |X(e^{jw})|_{w = \frac{2\pi k}{1000}}$$

So sample this with intervals of $2\pi/1000$ from 0 to $2\pi\, 999/1000$

So we don't sample at peaks exactly.

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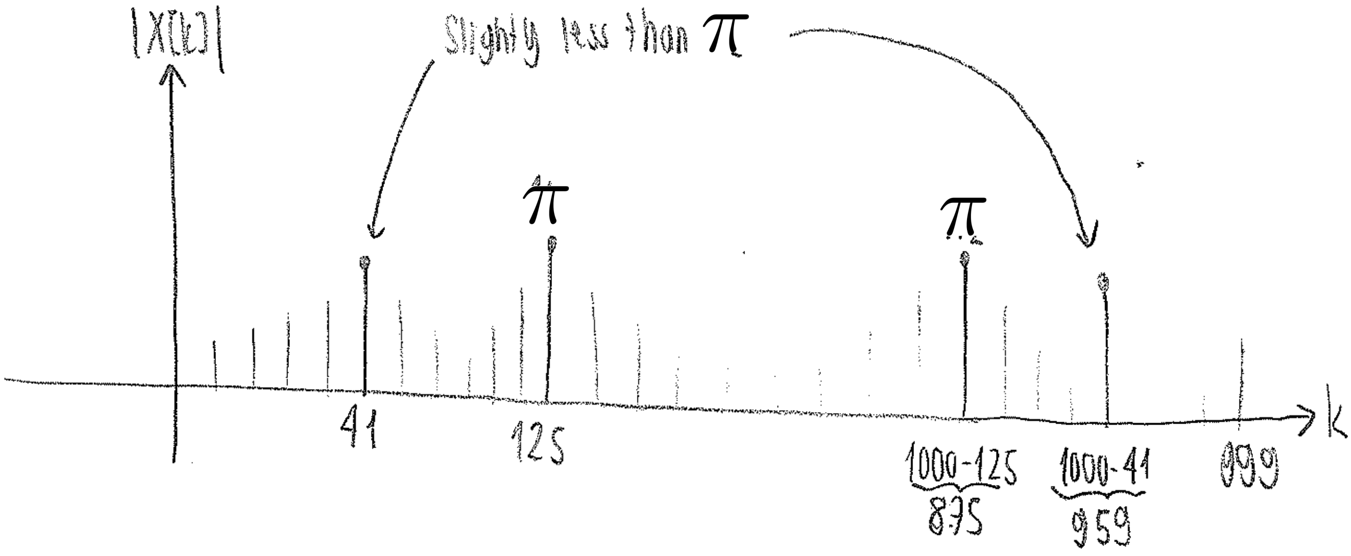
Approximately: $\frac{2\pi k}{1000} = \frac{2\pi 333}{8000}$, $k = \frac{333}{8} \approx 41$ and $\frac{2\pi k}{1000} = \frac{2\pi \cdot 1000}{8000}$, $k = \frac{1000}{8} = 125$

Continues! $\longrightarrow$

So that we have approximately:



Approximately sth. like this.