EE 424 – Digital Signal Processing

Quiz # 3

Fall 2010

Duration: 30 minutes

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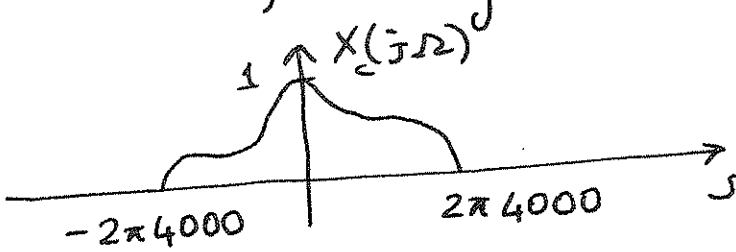
Section: 1

Question-1 (50 pts)	50
Question-2 (50 pts)	50
TOTAL (100 pts)	100

Q1|| Given the following CTFT:



$T_s = \frac{1}{8000}$ s

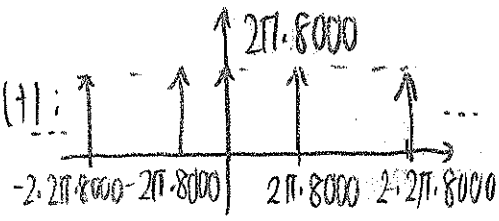
This signal is sampled with $f_s = 8000$ Hz.

$x[n] = x_c(nT_s)$, $n = 0, \mp 1, \mp 2, \ldots$

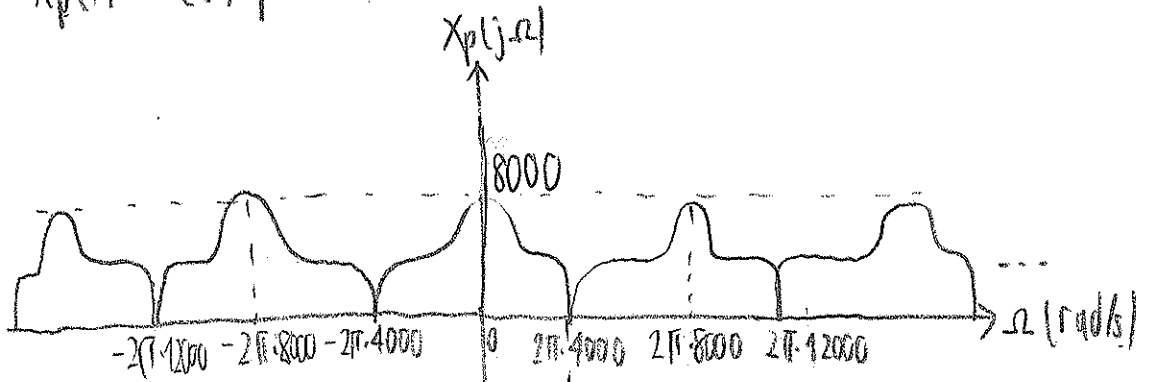
a) Plot $X(e^{j\omega})$ (D.T.F.T. of $x[n]$).

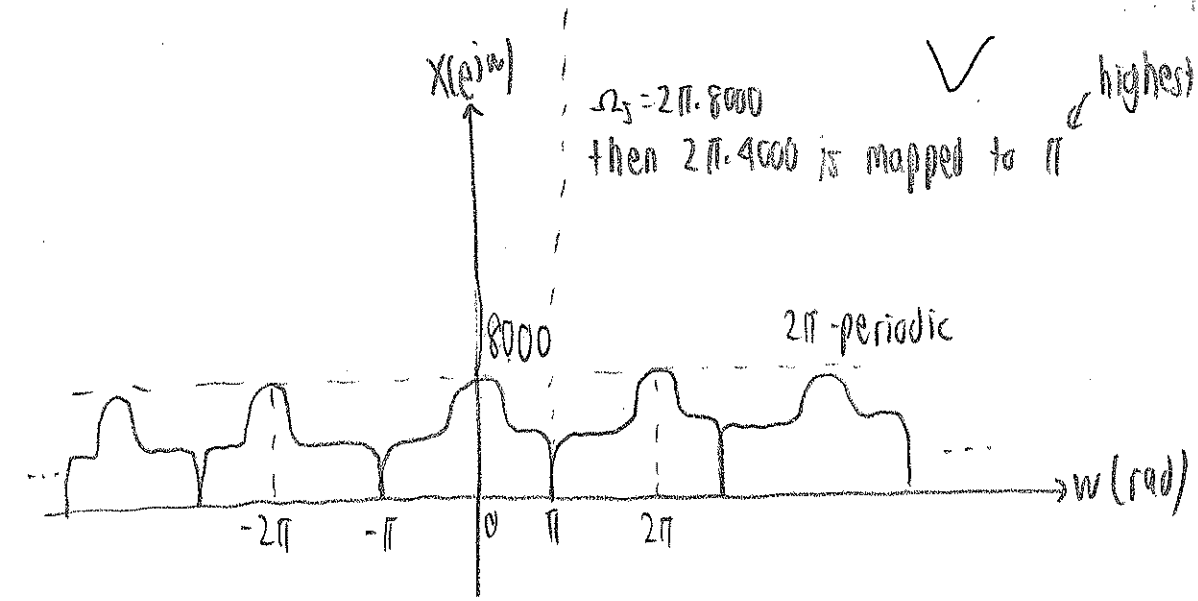
b) We have $x[n]$, $n = 0, +1, +2, \ldots, +2048$. Approximately plot $|X[k]|$ (DFT of the data).

a) So we are multiplying with the impulse train $p(t)$:



$x_p(t) \triangleq x_c(t) \cdot p(t)$ then:



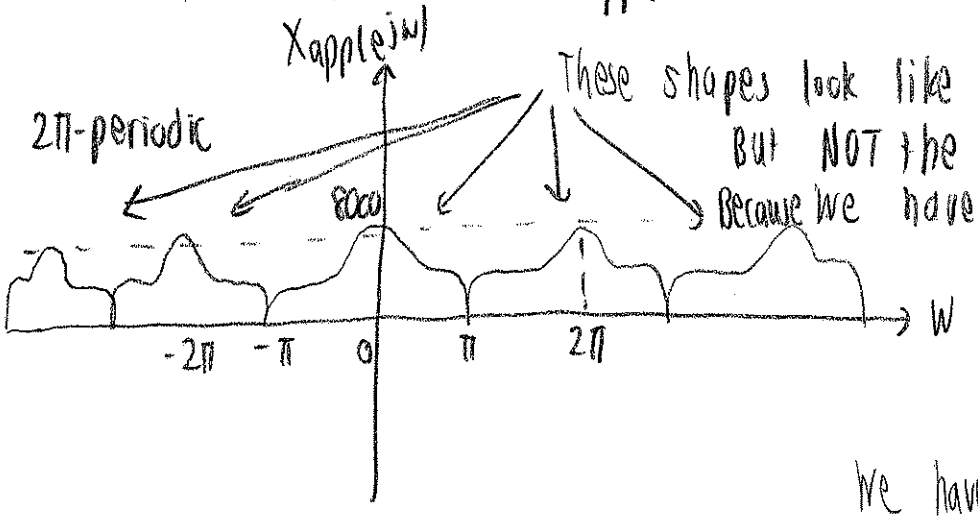


$\Downarrow$ highest normalized frequency

Continues →

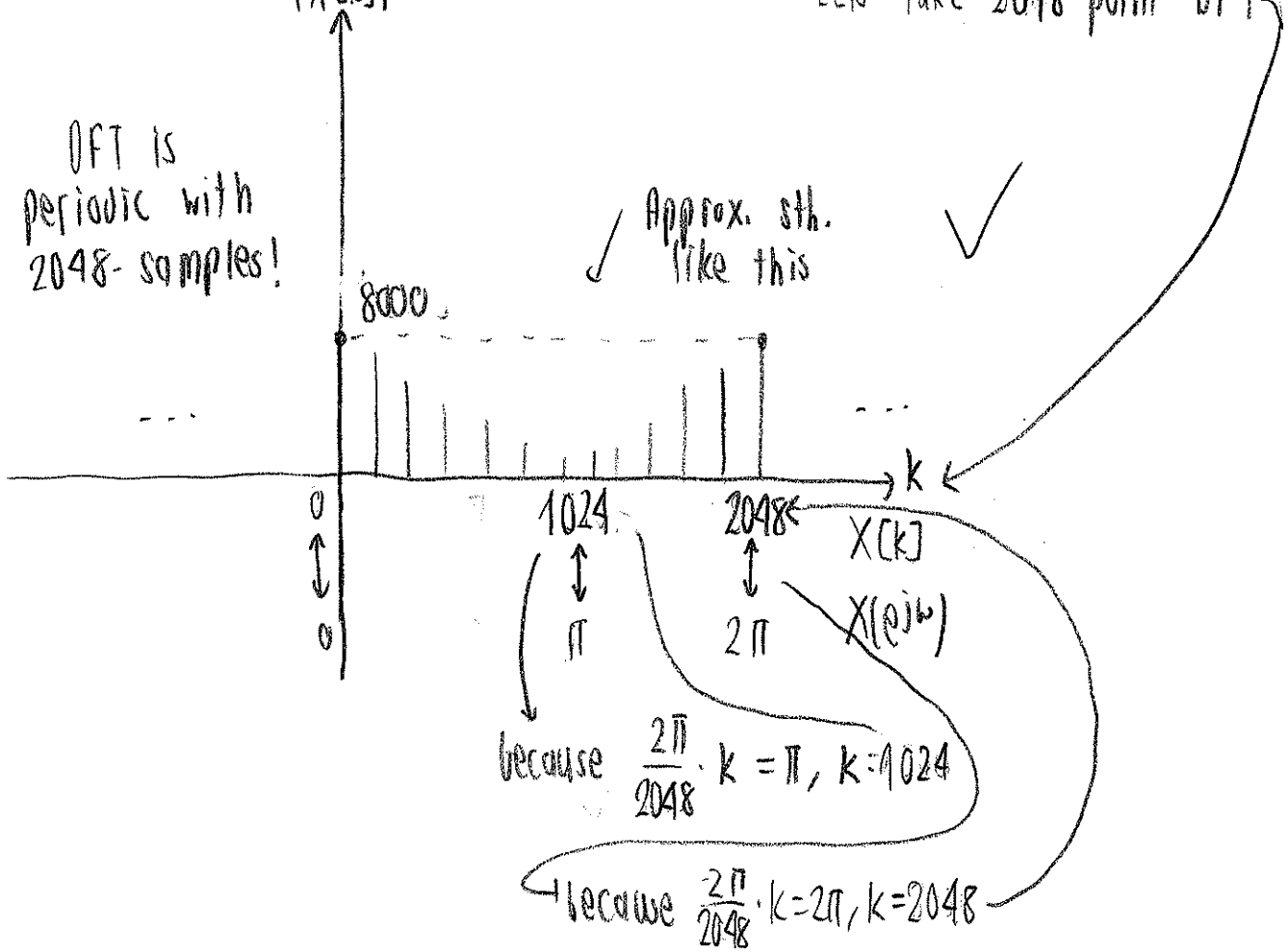
b) Lets denote the approximate DTFT by $X_{app}(e^{j\omega})$

Instead of $X(e^{j\omega})$ we have $X_{app}(e^{j\omega})$



These shapes look like the ones in $X(e^{j\omega})$.
But NOT the same!
Because we have finite number of samples!

We have 2048 samples.
Lets take 2048 point DFT



because $\frac{2\pi}{2048} \cdot k = \pi, \; k = 1024$

because $\frac{2\pi}{2048} \cdot k = 2\pi, \; k = 2048$

Q2// For the signal given in (Q1):

a) We have the $X[128] = 5 + j5$.
What does the index $k=128$ correspond to?
i.e., find the actual freq. and its value.

b) What is the value of $X[2048 - 128] = ?$
assuming that $x[n]$ is real.

a) We took 2048-point DFT in Q1. We will solve this part with respect to this fact:

$X[k] = \sum_{n=0}^{2047} x[n] e^{-j\frac{2\pi k}{N}n}$ and $X(e^{jw}) = \sum_{n=0}^{2047} x[n] e^{-jwn}$, so

to find w: $\frac{2\pi k}{N} \xrightarrow{2048\text{-point}} \frac{2\pi}{2048} \cdot k \xrightarrow{k=128} \frac{2\pi}{2048} \cdot 128 = \frac{2\pi}{16} = \frac{\pi}{8}$ ✓

$k=128$ corresponds to $w = \frac{\pi}{8}$ rad, since $w = \Omega T_s$, $\Omega = \frac{w}{T_s} = \frac{\pi}{8} \cdot 8000 = 1000\pi$

$\Omega = 2\pi \cdot 500$ rad/s, $f = \frac{\Omega}{2\pi} = 500$ Hz ← Actual frequency and its value

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b) In this 2048-point DFT, since $X[n]$ is real we have the following property: (2048, complex conjugation operator)

$X[k] = X^*[N-k]$

Then: $X[2048-128] = X^*[128] = (5+j5)^* = 5-j5$ ✓

So that:

$\boxed{X[2048-128] = 5-j5}$