Q1

Given the two-dimensional signal

$x[n_1, n_2]$:

1	1	1	1
1	1	1	1
2	2	1	1
2	2	1	1

$n_2 = 0 \longrightarrow$ (bottom row)

$\uparrow$ $n_1 = 0$ (left column)

a) Decimate this signal by a factor of two. You can use the filter that you used in the HW # 2.

b) Briefly describe "Interpolation by $\underline{3}$". Draw the block diagram of your method. Draw the block diagram of your interpolation scheme.

ⓐ Let $h[n] = \left\{ \frac{1}{4}, \frac{1}{2}, \frac{1}{4} \right\}$

1	1	1	1	1	1
1	1	1	1	1	1
2	2	2	1	1	1
2	2	2	1	1	1

$\xrightarrow[\text{row-wise}]{* \; h[n]}$

1	1	1	1
1	1	1	1
1	1	1	1
2	7/4	5/4	1
2	7/4	5/4	1
2	7/4	5/4	1

Choose 0th and 2nd columns

$\downarrow * \; h[n]$

1	1
5/4	17/16
7/4	19/16
2	5/4

Choose 0th and 2nd rows

$\longleftarrow$

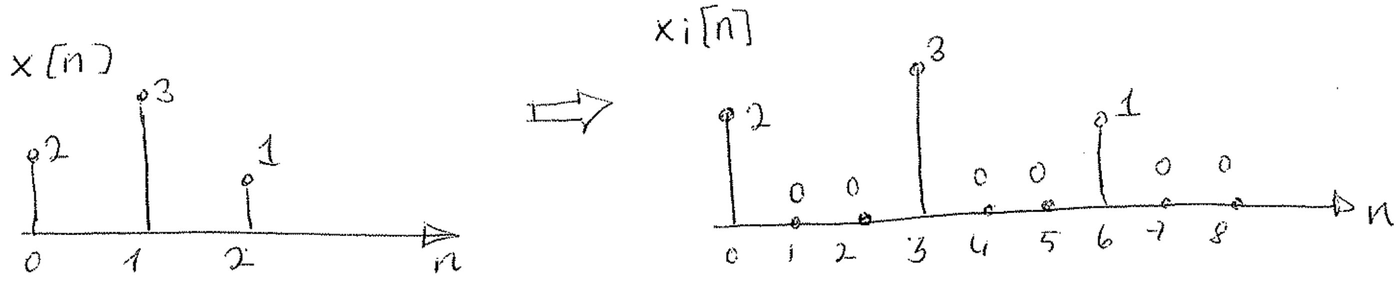
$y[n_1, n_2]$

5/4	17/16
2	5/4

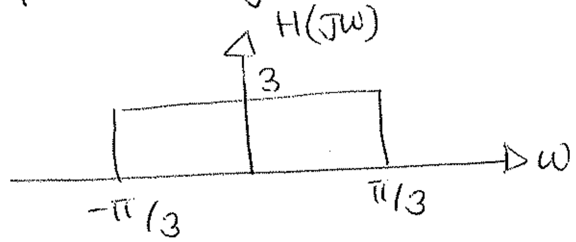
$n_2 = 0 \longrightarrow$ (bottom row)

$\uparrow$ $n_1 = 0$ (left column)

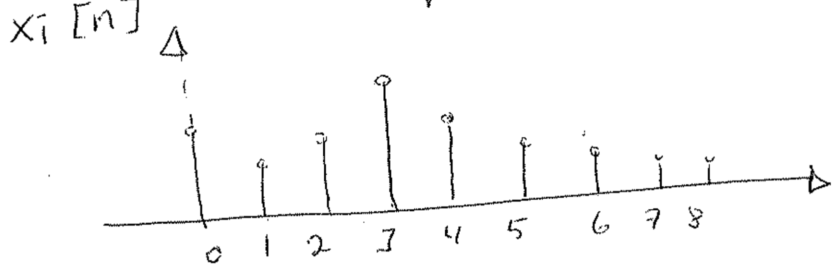
(b) (1) For "interpolation by 3" we should add 2 more samples for each existing sample. Assume we add zero's.



(2) Then we should low-pass filter $x_i[n]$ with a filter of gain 3. Ideally:



Resulting signal will be something like:



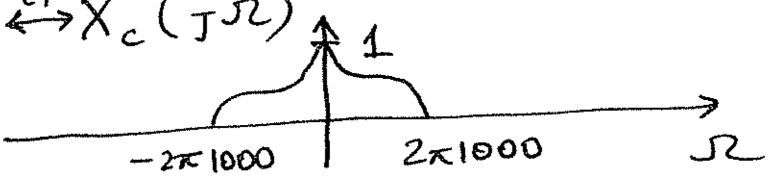
Q2

a) Let $x_c(t) = \cos(2\pi 250 t)$. This signal is sampled with $f_s = 8000$ Hz:

$x[n] = x_c(nT_s)$, $n = 0, 1, 2, \ldots, 512$.

Approximately plot $|X[k]|$ where $X[k]$ is the 512-point DFT of $x[n]$. Clearly identify the peak locations.

b) Given $x_c(t) \overset{\mathcal{F}_{CT}}{\longleftrightarrow} X_c(j\Omega)$



$x_c(t)$ is sampled with $f_s = 8000$ Hz:

and $x[n] = x_c(nT_s)$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

Plot $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$.

(a) $$X[k] = \sum_{n=0}^{511} x[n]\, e^{-j 2\pi \frac{n}{512} k}$$

$$= \sum_{n=0}^{511} \cos\left(2\pi \frac{\cancel{250}\, n}{\cancel{8000}\ 32}\right) e^{-j 2\pi \frac{n}{512} k}$$

$$= \frac{1}{2} \sum_{n=0}^{511} \left(e^{-j 2\pi \frac{n}{512}(k-16)} + e^{-j 2\pi \frac{n}{512}(k+16)} \right.$$

$|X[k]|$

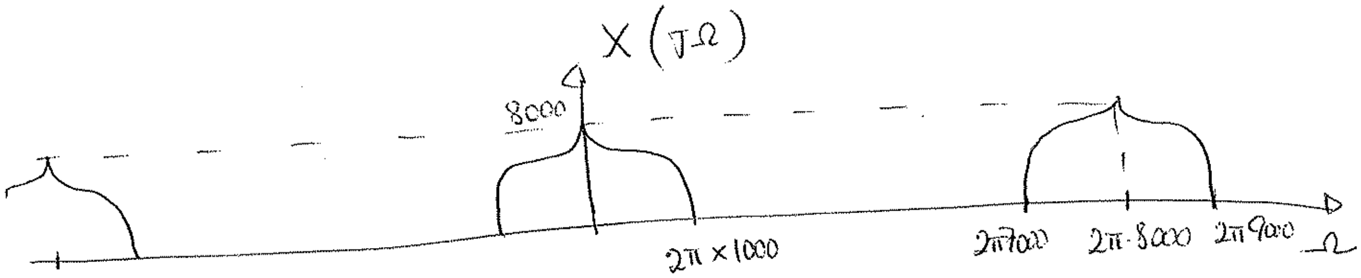
(b) First find CTFT, then transform to DTFT

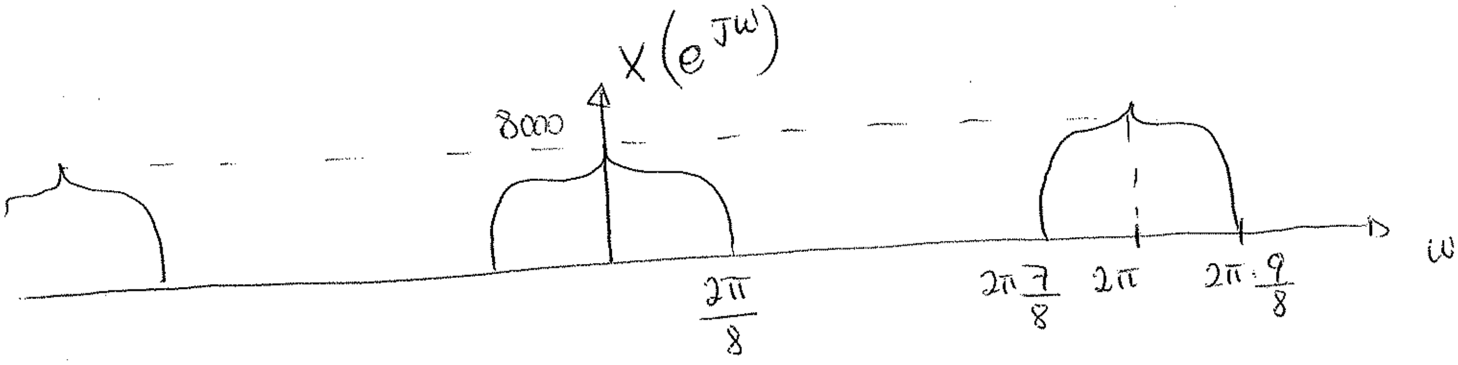
with
$$\boxed{\underset{(\text{DTFT})}{X(e^{j\omega})} = \underset{(\text{CTFT})}{X(j\Omega)}\Big|_{\Omega = \frac{\omega}{T_s}}}$$

Sampled continuous time signal ⇒ $x_s(t) = x_c(t) \cdot \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{n}{8000}\right)$

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{T_s}\right)$$

$$= 8000 \; X_c(j\Omega) * \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k . 8000)$$





as, $\omega = \dfrac{2\pi \times 1000}{8000}$ @ $\Omega = 2\pi \times 1000$