Name: **ID:** **Section:**

Score:	/ 100

EEE-424 Digital Signal Processing: Final-Term Exam 2010-05

Duration: 2 hours
Instructions: No calculators, book or notes allowed. SHOW YOUR WORK! No credit for results without explanations or steps!!

Q.1

Let $h_I[n]$ be the identity filter, i.e., $h_I[n] * x[n] = x[n]$. Furthermore, let

$$g[n] = a\delta[n-1] + \delta[n] + a\delta[n+1],\ \ a \neq 0.$$

Q.1a (6 pts) Determine $h_I[n]$. Prove your result by testing it: calculate $h_I[n] * x[n]$.

Solution : $h_I[n] = \delta[n]$.

Q.1b (6 pts) Calculate the DTFT $G(e^{j\omega})$ of $g[n]$ and plot it for $a = 0.5$ and $\omega \in [-\pi, \pi]$.

Solution : $G(e^{j\omega}) = 1 + 2a\cos(\omega)$.

Q.1c (13 pts) We would like to find a filter $v[n]$ which neutralizes the filter $g[n]$, i.e., $v[n] * g[n] = h_I[n]$. Do NOT try to determine $v[n]$, but find its DTFT $V(e^{j\omega})$. What can you say about the role of a?

Solution : $V(e^{j\omega}) = 1/(1 + 2a\cos(\omega))$, a should be chosen sucht that the denominator is not 0 for $\omega \in [-\pi, \pi]$.

Q.2

Consider the LPC-10 speech vocoder.

Q.2a (5 pts) Describe the important properties of speech signals which enable their effective coding.

Solution : Speech is quasi periodic.

Q.2b (5 pts) What is the main idea to reduce the data amount of the encoded speech? Is the encoding lossless or lossy? Explain!

Solution : To use prediction. In LPC-10, encoding is lossy, since the prediction is never perfect and we only use the prediction coefficients.

Q.2c (10 pts) Plot a rough flow diagram of the encoder system.

Q.2d (5 pts) Instead of speech, what is transmitted in LPC-10 over the channel?

Solution : Coefficients, error variance, a flag indicating whether the signal part is speech or noise and pitch period.

Q.3

Let $x[n]$ be W.S.S. zero mean white noise with variance σ_x^2. Let $y[n]$ be the output of an ideal low pass filter with cutoff frequency ω_c and input $x[n]$.

Q.3a (5 pts) Determine the autocorrelation sequence $r_x[n]$ of $x[n]$.

Solution : $r_x[n] = \sigma_x^2 \delta[n]$

Q.3b (5 pts) Determine the spectrum $S_x(e^{j\omega})$ of $x[n]$.

Solution : $S_x(e^{j\omega}) = \sigma_x^2$

Q.3c (7 pts) Determine the spectrum $S_y(e^{j\omega})$ of $y[n]$.

Solution : $S_y(e^{j\omega}) = \begin{cases} \sigma_x^2, & |\omega| < \omega_c \\ 0, & |\omega| \in [\omega_c, \pi] \end{cases}$

Q.3d (8 pts) Determine the autocorrelation sequence $r_y[n]$ of $y[n]$.

Solution : $r_y[n] = \text{IDTFT}\{S_y(e^{j\omega})\} = \sigma_x^2 \frac{\sin(\omega_c n)}{\pi n}$.

Q.4

Let $x[n]$ be a sampled signal with sampling period T. The corresponding continuous-time signal shall be denoted by $x_c(t)$, where $X_c(j\Omega)$ is the CTFT of $x_c(t)$. Recall that the DTFT of $x[n]$ is

$$X(e^{j\omega}) = \frac{1}{T}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi r}{T}\right)\right). \tag{1}$$

Consider the downsampled signal $x_d[n] = x[Mn]$. Let the corresponding sampling period of $x_d[n]$ be T'.

Q.4a (3 pts) Express T' in terms of T.

Solution : $T' = MT$.

Q.4b (6 pts) Write down $X_d(e^{j\omega})$ in terms of X_c.

Solution : $X_d(e^{j\omega}) = \frac{1}{T'}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T'} - \frac{2\pi r}{T'}\right)\right)$.

Q.4c (6 pts) Express $X_d(e^{j\omega})$ in terms of T.

Solution : $X_d(e^{j\omega}) = \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right)$.

Q.4d (10 pts) $X_d(e^{j\omega})$ can be expressed in terms of $X(e^{j\omega})$ by

$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X(e^{j(\omega/M - 2\pi i/M)}). \tag{2}$$

Show the way to achieve this result analytically. **Hint**: Use your result from [Q.4c] and substitute the summation index r by $i + kM$, where $i = 0, ..., M-1$ and $-\infty < k < \infty$.

Solution :

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right) \tag{3}$$

$$= \frac{1}{M}\sum_{i=0}^{M-1}\left[\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi k}{T} - \frac{2\pi i}{MT}\right)\right)\right] \tag{4}$$

With $X(e^{j(\omega-2\pi i)/M}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega-2\pi i}{MT} - \frac{2\pi k}{T}\right)\right)$ follows the result.

Formulas

- Discrete-time convolution:

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

- Continous-Time Fourier Transform (CTFT):

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}d\Omega$$

- Discrete-Time Fourier Transform (DTFT):

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{0}^{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Autocorrelation (W.S.S.):

$$r_x[k] = \mathrm{E}[x[n]x[n+k]]$$

Spectrum of W.S.S. random signal $x[n]$:

$$S_x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} r_x[n]e^{-j\omega n}$$

Spectrum of filter output $y[n]$, given W.S.S. random signal input $x[n]$:

$$S_y(e^{j\omega}) = |H(e^{j\omega})|^2 S_x(e^{j\omega})$$