Name: **ID:** **Section:**

Score: / 100

EEE-424 Digital Signal Processing: Grand Quiz 2010

Duration: 60 minutes
Instructions: No calculators, book or notes allowed. SHOW YOUR WORK! No credit for results without explanations or steps!!

Q.1

Consider the time domain signal $x[n] = 1,\ n = ..., -2, -1, 0, 1, 2, ...$

Q.1a (10 pts) Show that $2\pi\delta(\omega) = \mathrm{DTFT}\{x[n]\}$ is a solution for the DTFT of $x[n]$.

Solution : IDTFT: $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\omega)e^{j\omega n}d\omega = 1\ \forall n.$

Q.1b (15 pts) Determine *all* possible solutions for $\mathrm{DTFT}\{x[n]\}$. Explain why there is more than one solution.

Solution : DTFT is always 2π-periodic:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\omega)e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\omega + 2m\pi)e^{j\omega n}d\omega\ \forall m, n.$$

Q.2

Consider the IIR filter $2y[n] - y[n-1] - y[n-2] + 3x[n] - 5x[n-1] = 0$.

Q.2a (12 pts) Compute the transfer function $H(z)$ of the IIR filter.

Solution : z-Tranf:

$$2Y(z) - z^{-1}Y(z) - z^{-2}Y(z) = -3X(z) + 5z^{-1}X(z) \Rightarrow H(z) = \frac{-3 + 5z^{-1}}{2 - z^{-1} - z^{-2}}.$$

Q.2b (8 pts) Compute the corresponding frequency response $H(e^{j\omega})$.

Q.2c (5 pts) Plot the flow diagram of the filter.

Q.3

Consider the analog filter $H_a(s) = \frac{6\Omega_c^2}{s^2+5s\Omega_c+6\Omega_c^2}$.

Q.3a (6 pts) Determine all poles of this system. Determine the type of this filter, explain your answer.

Solution : $s_1 = -2\Omega_c,\ s_2 = -3\Omega_c$.

Q.3b (12 pts) Digitize this filter by applying the Bilinear transform, where $\omega_c = 0.2\pi$.

Solution : Cut-off freq. conversion to setup analog filter:

$$\Omega_c = \frac{2}{T}\tan(0.2\pi/2) = \frac{2}{T}\tan(0.1\pi).$$

Digital filter:

$$H(z) = \frac{6t^2(z^{-2}+2z^{-1}+1)}{(1-5t+6t^2)z^{-2}+(-2+12t^2)z^{-1}+1+5t+6t^2}, t = \tan(0.1\pi)$$

Q.3c (7 pts) Determine the difference equation (e.g., time domain representation) of the digital filter.

Solution : Digital filter i/o relation:

$$6t^2(x[n-2]+2x[n-1]+x[n]) = (1-5t+6t^2)y[n-2]+(-2+12t^2)y[n-1]+(1+5t+6t^2)y[n]$$

Formulas

- Discrete-time convolution:

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

- Discrete-Time Fourier Transform (DTFT):

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

- z-Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- Bilinear transform:

$$\Omega = \frac{2}{T}\tan(\omega/2),\ \ H(z) = H_a(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$