Discrete-time Signals and Systems
Midterm Exam
Spring Semester 2006

Duration: 110 minutes

Attempt all questions and **show your work. If you don't show your work you get zero credit.**

4) Given the following input/output relation:

$$y[n] = \frac{1}{2}x[n] + \frac{1}{4}x[n-1].$$

a) Is this system linear? Prove your answer

b) Is this system time-invariant? " " "

c) Find the impulse response.

d) Consider the "downsampler by 2":

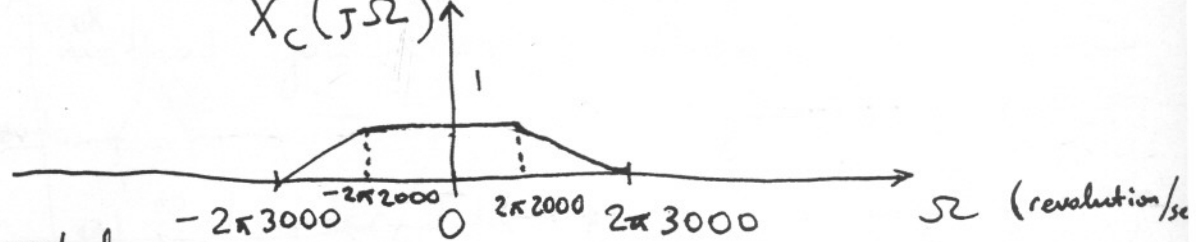
$$x[n] \longrightarrow \boxed{2\downarrow} \longrightarrow v[n] = x[2n]$$

Is it linear? Prove your answer.

e) Is down-sampler time-invariant?

f) Let $x[n] = \delta[n]$. What is $v[n] = ?$

Q2 Let $x_c(t)$ be a continuous-time signal. $X_c(j\Omega)$ is the cont.-time Fourier Transform of $x_c(t)$:



$x_c(t)$ is sampled:

$$x[n] \triangleq x_c\left(n\frac{1}{8000}\right), \quad n = 0, \mp 1, \mp 2, \ldots$$

a) Plot $X(e^{j\omega}) = \mathcal{F}_{DT}\{x[n]\}$.

b) $x[n] \longrightarrow \boxed{2\downarrow} \longrightarrow v[n]$. Plot $V(e^{j\omega})$.

$\frac{N}{2} \log N$

c) Let, $x[n]$, $n = 0, \mp 1, \mp 2, \ldots, \mp 511, +512$. is available.

$X[k]$ is the 1024-point DFT of $[x[-511], x[-510], \ldots, x[512]]$

Approximately plot $|X[k]|$ versus k.

d) $v[n] \longrightarrow \boxed{2\uparrow} \longrightarrow u[n] = \begin{cases} v[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

Plot $U(e^{j\omega})$.

e) What is the computational cost of computing $X[k]$, if the fast Fourier Transform (FFT) is used in part(c

Design a 3rd order FIR low-pass filter with cut-off frequency of $\omega_c = \pi/2$. (i) You can use any method you like. (ii) Make sure to show your work.

b) Obtain a 3rd order FIR high-pass filter from your design in part (a). Cut-off freq. of the high-pass filter is also $\omega_c = \pi/2$.

c) Let $x[n] = \{\ldots\ldots, 1, \underset{\substack{\uparrow \\ n=0}}{1}, 1, 1, 2, 2, 2, 2, \ldots\ldots\}$

Filter $x[n]$ using your low-pass filter.

d) Filter $x[n]$ using your high-pass filter.

e) Comment on filter outputs in parts (c) & (d).

$$\frac{1{,}4 \pm \sqrt{1{,}96 - 2{,}12}}{2} = 0{,}7 \pm j\sqrt{0{,}16} = 0{,}7 \pm 0{,}2j$$

$$H(z) = \frac{z + 0.8}{z^2 - 1.4z + 0.53}$$

take discriminant

3 pts a) Plot the locations of poles and zeros on the complex plane.

5 pts b) How many different LTI filters may have the given $H(z)$? What are their properties? Indicate the associated regions of convergences

3 pts c) If the system is causal, is it also stable?

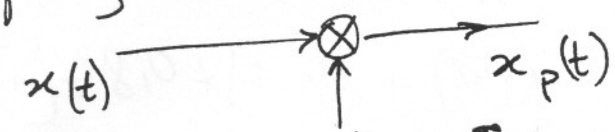
7 pts d) Make a rough sketch of the magnitude response $|H(\omega)|$; What kind of a filter is this?

7 pts e) Give an implementation for the causal system using delay elements, vector adders and scalar (real) multipliers.

5 pts f) Let $H_1(z) = H(z^2)$. Roughly plot $|H_1(\omega)|$ in terms of your plot in part d.

5 pts g) Repeat part e for H_1.

Sampling Theorem

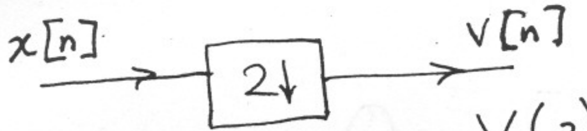


$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$X_p(j\Omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(j\left(\Omega - n\Omega_s\right)\right), \quad \Omega_s = \frac{2\pi}{T_s}$$

Hanning Window: $W_N[k] = \dfrac{1 - \cos\left(2\pi \dfrac{k}{N-1}\right)}{2}$ for $k = 0,1,\ldots,N-1$

Hamming Window: $W_N[k] = \dfrac{54 - 46\cos\left(2\pi \dfrac{k}{N-1}\right)}{100}$ for $k = 0,1,\ldots,N-1$



$$V(z) = \frac{1}{2}\left(X(z^{1/2}) + X(-z^{1/2})\right)$$

- $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$
- $X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}, \quad k = 0, 1, \ldots, N-1$

Table 6.1: Some commonly used z-transform pairs.

Sequence	z-Transform	ROC
$\delta[n]$	1	All values of z
$\mu[n]$	$\dfrac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$\alpha^n \mu[n]$	$\dfrac{1}{1 - \alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n\,\alpha^n \mu[n]$	$\dfrac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$(n+1)\,\alpha^n \mu[n]$	$\dfrac{1}{(1 - \alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$(r^n \cos\omega_o n)\mu[n]$	$\dfrac{1 - (r\cos\omega_o)z^{-1}}{1 - (2r\cos\omega_o)z^{-1} + r^2 z^{-2}}$	$\lvert z\rvert > \lvert r\rvert$
$(r^n \sin\omega_o n)\mu[n]$	$\dfrac{(r\sin\omega_o)z^{-1}}{1 - (2r\cos\omega_o)z^{-1} + r^2 z^{-2}}$	$\lvert z\rvert > \lvert r\rvert$