EE 424 – Discrete-time Signal Processing

Midterm Exam

Fall Semester 2007

Duration: 100 minutes

Name: ....SOLUTION........

Student ID: ....MANUAL........

Section: ....

Q1 (25 pts)..................

Q2 (25 pts)..................

Q3 (25 pts)..................

Q4 (25 pts)..................

Attempt all questions and **show all your work**.

1// Find the a) DTFT $H(e^{j\omega})$ and b) 32-point DFT of $x[n] = \left\{ \frac{1}{4}, \underset{\underset{n=0}{\uparrow}}{\frac{1}{2}}, \frac{1}{4} \right\}$

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a) $X(e^{j\omega}) = \sum_{n=-1}^{1} x[n] e^{-j\omega n} = \frac{1}{4} e^{j\omega} + \frac{1}{2} + \frac{1}{4} e^{-j\omega} = \frac{1}{2} + \frac{1}{2} \cos\omega$

b) zero-phase DFT vector: $x[n] = \left\{ \underset{\underset{n=0}{\uparrow}}{\frac{1}{2}}, \frac{1}{4}, 0, 0 \ldots 0, \underset{\underset{n=31}{\uparrow}}{\frac{1}{4}} \right\}$

$$X[k] = \sum_{n=0}^{31} x[n] e^{-j\frac{2\pi}{32}kn}$$

$$X[k] = \frac{1}{2} + \frac{1}{4} e^{-j\frac{2\pi}{32}k} + \frac{1}{4} e^{-j\frac{2\pi}{32}31} = \frac{1}{2} + \frac{1}{4} e^{-j\frac{2\pi}{32}k} + \frac{1}{4} e^{j\frac{2\pi}{32}k}$$

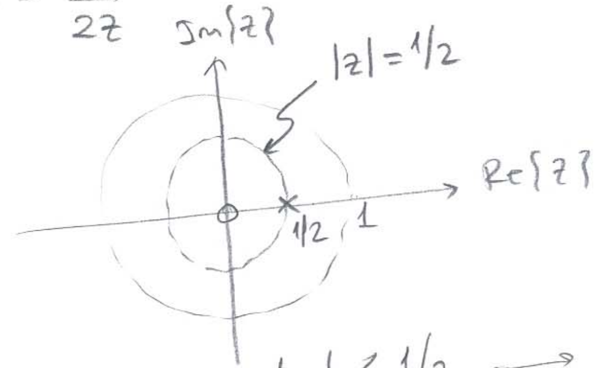
$$X[k] = \frac{1}{2} + \frac{1}{2} \cos\left(\frac{2\pi}{32}k\right), \quad k = 0, 1, \ldots, 31$$

2) Given $H(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}}$

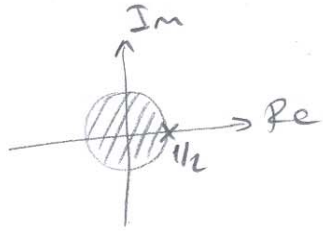
a) Find the time domain impulse responses corresponding to $H(z)$.

b) Indicate if they are stable or not.

a) $H(z) = \dfrac{1}{1-\frac{1}{2z}} = \dfrac{2z}{2z-1}$ → zero at $z=0$, → pole at $z=1/2$

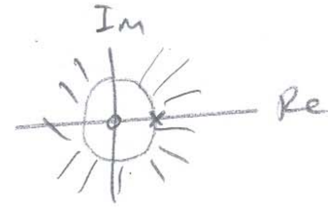


* If RoC is defined on $|z| < 1/2$ →



$h_1[n] = -\left(\frac{1}{2}\right)^n u[-n-1]$

* If RoC is defined on $|z| > 1/2$ →

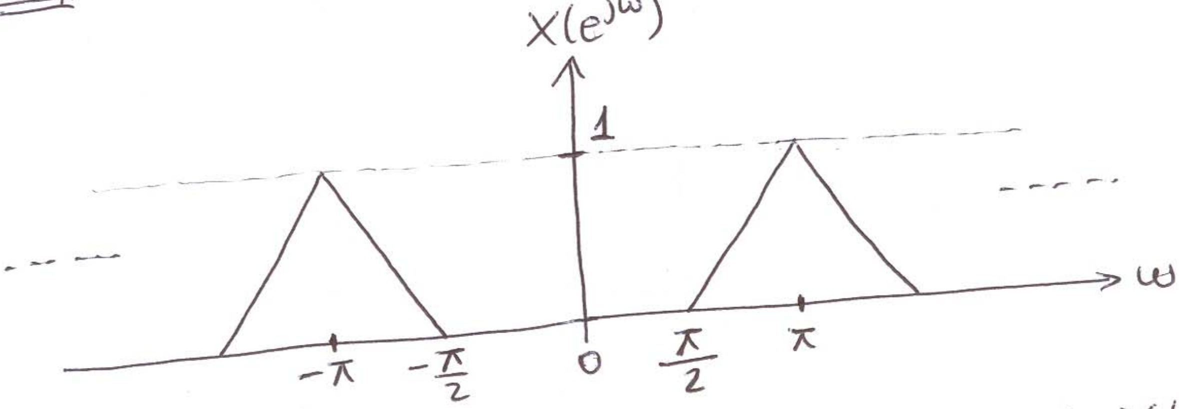


$h_2[n] = \left(\frac{1}{2}\right)^n u[n]$

b) $h_1[n]$ is not stable. RoC does not contain unit circle $|z|=1$

$h_2[n]$ is stable. RoC contains unit circle $|z|=1$.

3) Given $x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$



a) $x[n] \longrightarrow \boxed{2\downarrow} \longrightarrow y[n]$. Plot $Y(e^{j\omega})$.

b) Is it possible to retrieve $x[n]$ from $y[n]$? Explain your answer: No credit will be given to "yes" or "no" answers.

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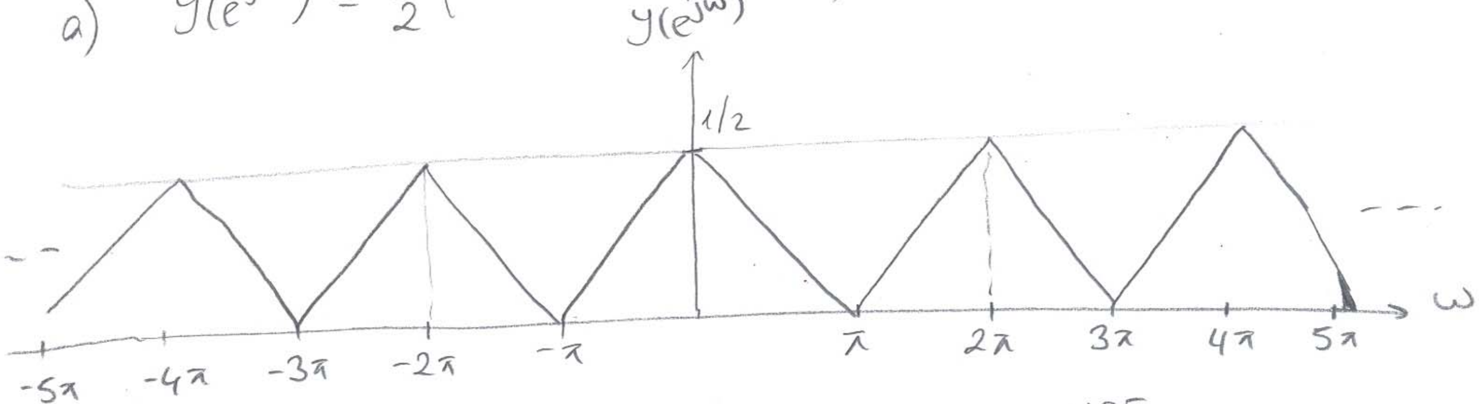
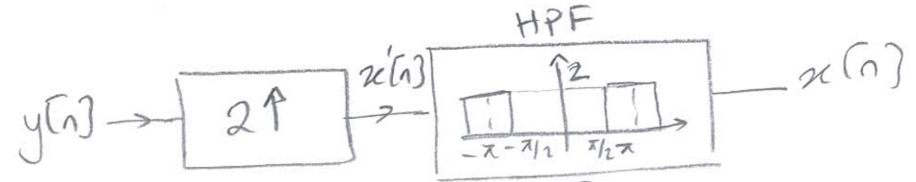
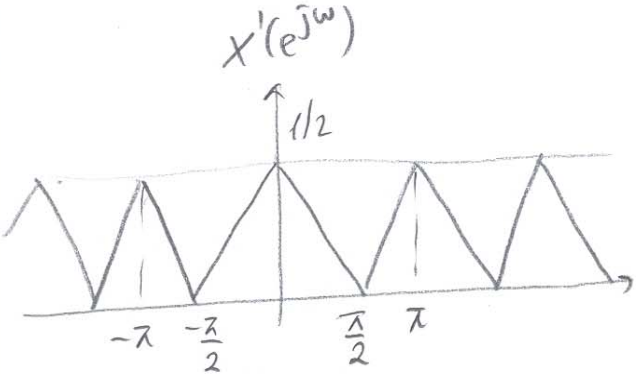
a) $Y(e^{j\omega}) = \frac{1}{2}\left(X(e^{j\frac{\omega}{2}}) + X(e^{j(\frac{\omega}{2}+\pi)})\right)$

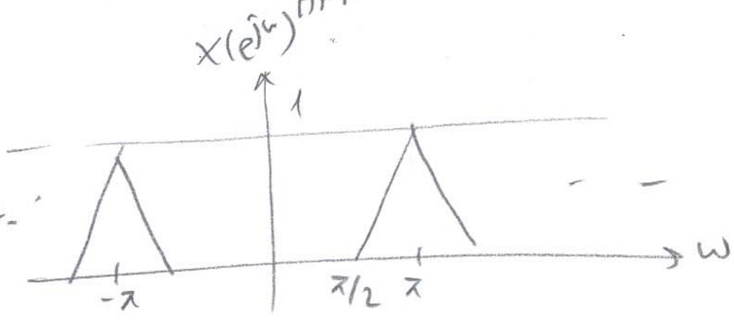


b) Yes. It is possible.

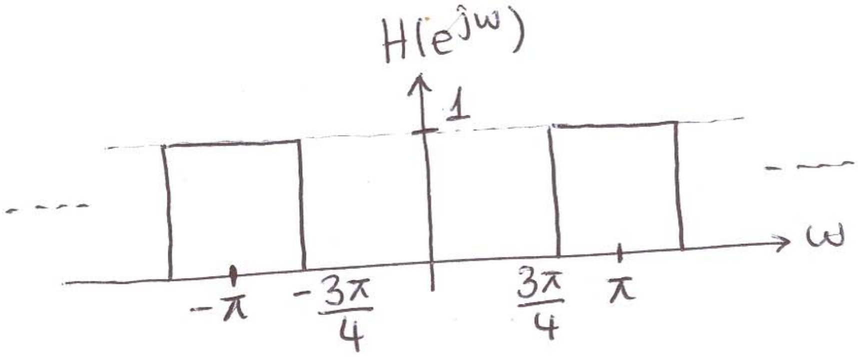




HPF with amplitude 2



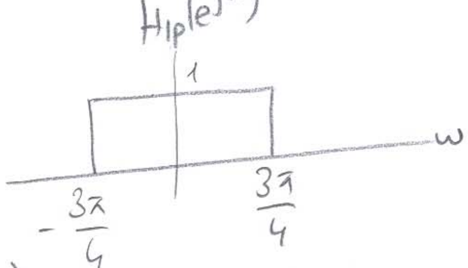
4// Let



Design a third-order FIR high-pass filter whose frequency response is shown above. Use the triangular windowing method.

$h_{hp}[n] = \delta[n] - h_{lp}[n]$

$\downarrow$ low-pass filter with cut-off $\frac{3\pi}{4}$



$$h_{lp}[n] = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} 1.e^{j\omega n}\,d\omega = \frac{\sin\left(\frac{3\pi}{4}n\right)}{\pi n}, \quad -\infty < n < \infty$$

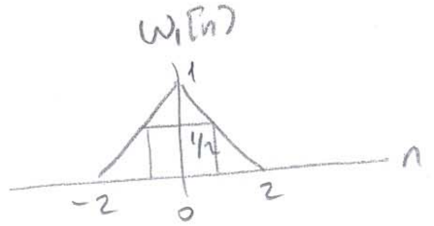
$N = 3 = 2L+1$

$\underline{L=1}$

$$h_{lp}[n] = \left\{ \cdots\cdots, \frac{\sin\left(-\frac{3\pi}{4}\right)}{-\pi}, \underset{\uparrow\, n=0}{\frac{\frac{3\pi}{4}\cos(0)}{\pi}}, \frac{\sin\left(\frac{3\pi}{4}\right)}{\pi}, \cdots \right\}$$

$$h_{lp}[n] = \left\{ \cdots, \frac{\sqrt{2}}{2\pi}, \underset{\uparrow\, n=0}{\frac{3}{4}}, \frac{\sqrt{2}}{2\pi}, \cdots \right\} \rightarrow h_{hp}[n] = \left\{ \cdots, -\frac{\sqrt{2}}{2\pi}, \underset{\uparrow\, n=0}{\frac{1}{4}}, -\frac{\sqrt{2}}{2\pi}, \cdots \right\}$$

$$w_1[n] = \left\{ \frac{1}{2}, \underset{\uparrow\, n=0}{1}, \frac{1}{2} \right\} \longrightarrow$$



$$h_d[n] = h_{hp}[n].w_1[n] = \left\{ -\frac{\sqrt{2}}{4\pi}, \frac{1}{4}, -\frac{\sqrt{2}}{4\pi} \right\}$$

Formulas:

DTFT: $$X\left(e^{jw}\right)=\sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

IDTFT: $$x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{jw}\right)e^{jwn}dw$$

Triangular window: $$W_L[n]=\begin{cases}\dfrac{L+1-|n|}{L+1} &, n=0,\pm1,\pm2,\ldots,\pm L\\ 0 &, o.w.\end{cases}$$