Multiple Access Communications

EEE 538, WEEK 11

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Multiple Access

- Satellite systems, radio networks (WLAN), ethernet segment
- The received signal is the sum of attenuated transmitted signals from a set of other nodes, corrupted by distortion, delay and noise
- The multiaccess medium is allocated among the various nodes by the MAC (medium access control) sublayer
- We can view multiaccess communication in queueing terms: Each node has a queue of packets to be transmitted and the multiaccess channel is a common server.

In Queueing Terms

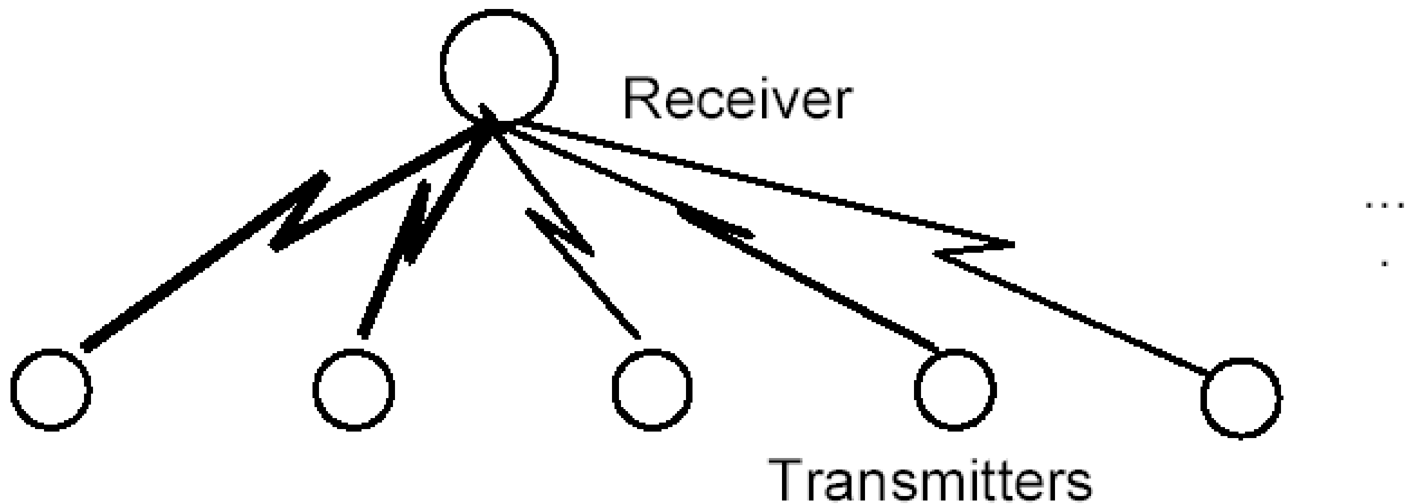
- Ideally, the server should view all the waiting packets as one combined queue to be served by an appropriate queueing discipline.
- Problem: The server doesn't know which nodes contain packets.
- Problem: The nodes are unaware of packets at other nodes.
- Knowledge about the state of the queue is distributed.

Approaches to multiple Access

- Fixed Assignment (TDMA, FDMA, CDMA)
  - Each node is allocated a fixed fraction of bandwidth
  - Equivalent to circuit switching
  - Very inefficient for low duty factor traffic
- Contention systems
  - Polling
  - Reservations and Scheduling
  - Random Access

Ex. Satellite Channel

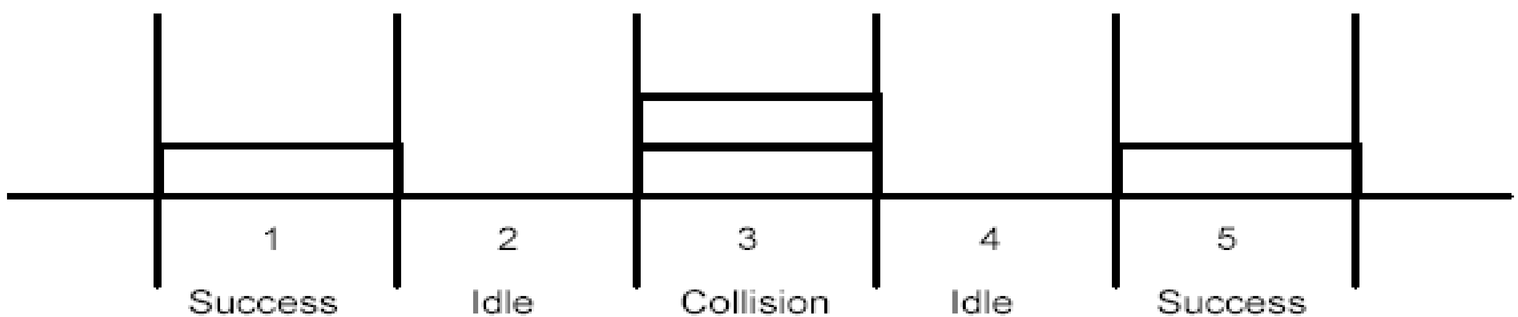
Single receiver, many transmitters



E.g., Satellite system, wireless

Slotted Aloha

- Time is divided into "slots" of one packet duration
  - E.g., fixed size packets
- When a node has a packet to send, it waits until the start of the next slot to send it
  - Requires synchronization
- If no other nodes attempt transmission during that slot, the transmission is successful
  - Otherwise "collision"
  - Collided packet are retransmitted after a random



Slotted Aloha Model

- Slotted system: Same length of all packets, each packet needs one time unit (a slot) for transmission. All transmitters are synchronized, transmission starts at an integer time and ends before next integer time.
- Poisson arrivals: Packets arrive at each of the m transmitting nodes according to independent Poisson processes. Let λ be the overall arrival rate, and λ/m the arrival rate at each transmitting node.
- Collision or perfect reception: If two or mode nodes sends in a time slot there is a *collision* and the receiver obtains no information about the content or sources of the transmitted packages. If just one node sends the packet is correctly received.

Model Continued

- *(0; 1; e)* Immediate Feedback: At the end of each slot, each node obtains feedback specifying whether 0 packets (idle), 1 packet (successful transmission), or more that 1 packet (collision/error) were transmitted in that slot.
- Retransmission of collisions: Each packet involved in a collision will be retransmitted in some later slot, until successfully transmitted. A node with a packet that must be retransmitted is said to be *backlogged*

Model Continued

- One of the two following is assumed
  - (a) No buffering: If one packet at a node is currently waiting for transmission or colliding with another packet during transmission, new arrivals at that node are discarded and never transmitted.
  - (b) Infinite set of nodes ($m = \infty$): The system has an infinite set of nodes and each newly arriving packet arrives at a new node.

Slotted Aloha

- The Aloha network was developed around 1970 to provide radio communication between central computer and various data terminals at the campuses of University of Hawaii.
- The basic idea is an unbacklogged node transmits a newly arriving packet in the first slot after packet arrival, thus risking occasional collisions but achieving very small delay if collisions are rare.
- When a collision occurs each node sending one of the colliding packets discovers the collision at the end of the slot and becomes backlogged.
- Backlogged nodes wait for some random number of slots before retransmitting.

Preliminary Analysis

- Using infinite node assumption (b)
- New arrivals transmitted in a slot is a Poisson random variable with rate λ
- If retransmissions from backlogged nodes are sufficiently randomized, it is plausible to approximate the total number of retransmissions and new transmissions in a slot as a Poisson random variable with rate $G > \lambda$
- The probability that k nodes will transmit in a slot is then

$$\frac{G^k}{k!}e^{-G}$$

Preliminary Analysis Continued

The probability of a successful transmission is thus Ge^{-G}.

In equilibrium the arrival rate, λ, should be equal to the departure rate, Ge^{-G}

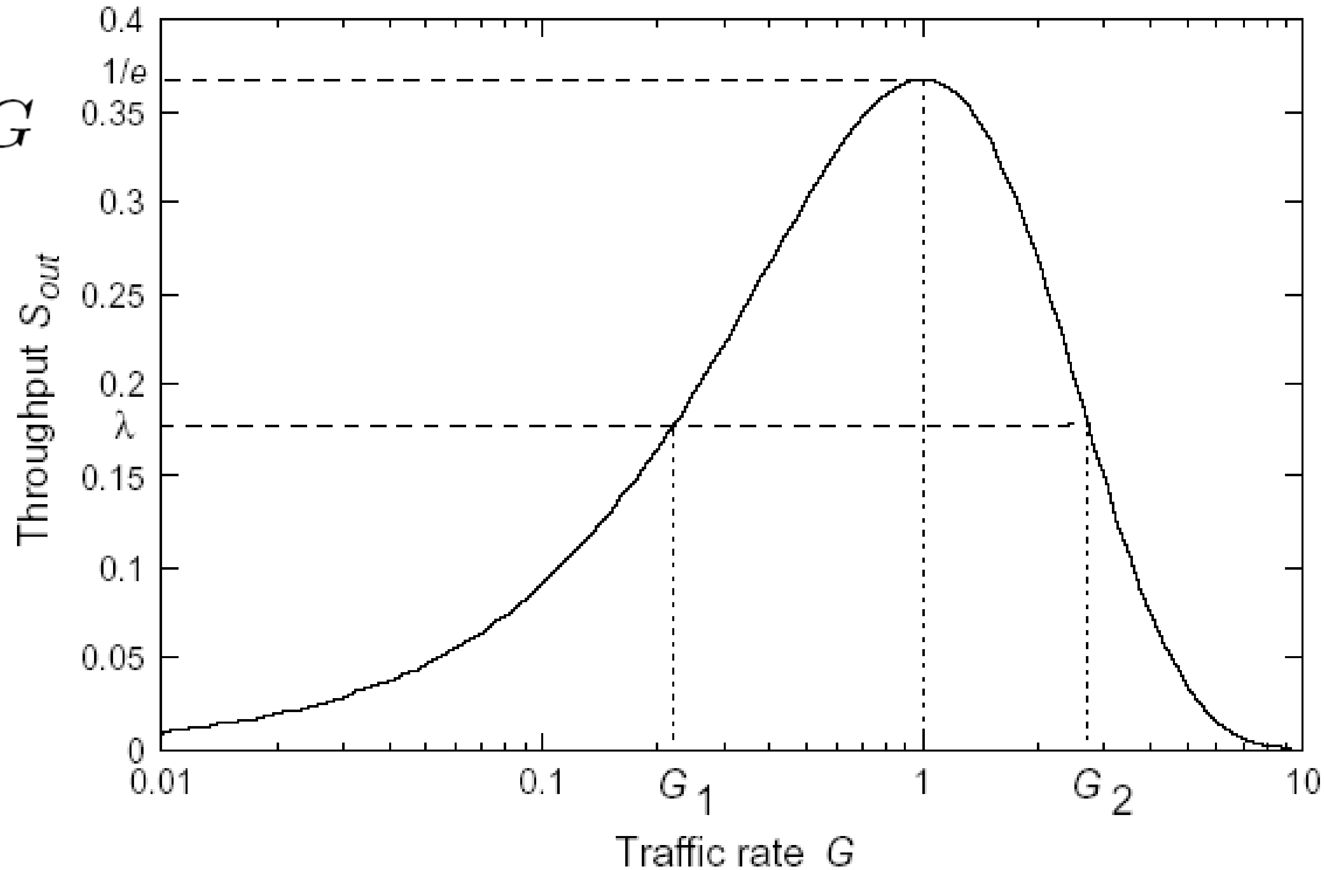
Maximum possible departure rate occurs at $G = 1$ and is $1/e \approx 0.368$

For arrival rate $\lambda < 1/e$ there are two values of G for which arrival rate equals departure rate !

We ignore the dynamics of the system, as number of backlogged packets changes the parameter G will change.

Preliminary Analysis Continued

$$Ge^{-G}$$



Detailed Analysis

Assume that each backlogged node retransmits with some fixed probability q_r in each succesive slot until a successful transmission occurs

Number of slots from a collision until a given node involved in the collision retransmits is a geometric random variable, i.e. the probability that the number of slots is $i \geq 1$ is $q_r(1-q_r)^i$

We will now first assume the no-buffering assumption (a)

The behaviour is now described by a discrete time Markov model where the state is the number n of backlogged nodes

Detailed Analysis Continued

Each of the n backlogged nodes will transmit, independently of each other, with probability q_r

Each of the $m - n$ other nodes will transmit if one (or more) packets arrived during the previous slot

Since arrivals are Poisson distributed with rate λ/m, the probability of no arrivals is $e^{-\lambda/m}$; thus the probability that an unbacklogged node will transmit is
$q_a = 1 - e^{-\lambda/m}$

Let $Q_a(i, n)$ be the probability that i unbacklogged nodes transmit when n nodes are backlogged

Let $Q_r(i, n)$ be the probability that i backlogged nodes transmit when n nodes are backlogged

Detailed Analysis Continued

- We get

$$Q_a(i,n) = \binom{m-n}{i}(1-q_a)^{m-n-i}q_a^i$$

$$Q_r(i,n) = \binom{n}{i}(1-q_r)^{n-i}q_r^i$$

- The state n (number of backlogged nodes) increases by the number of new arrivals if we get a collision
- The state decreases by 1 if we get no new arrivals and a succesful transmission (of a retransmitted packet)
- The state is unchanged if we get a new arrival that is successfully transmitted, or if no retransmission occurs, or if retransmission occurs with collision

Detailed Analysis Continued

- If we get $i \geq 2$ new arrivals, collision will occur and the state will increase with i, thus the transition probability $P_{n,n+i} = Q_a(i,n)$ in this case ($2 \leq i \leq m-n$).
- If we get one new arrival and a retransmission, collision will occur and the state will increase with 1,
$P_{n,n+1} = Q_a(1,n)(1 - Q_r(0,n))$
- If we get one new arrival and no retransmission, or no new arrival and no or at least two retransmission, the state will be unchanged
$P_{n,n} = Q_a(1,n)Q_r(0,n) + Q_a(0,n)(1 - Q_r(1,n))$
- If we get no new arrivals and one (i.e. successful) retransmission the state will decrease by 1,
$P_{n,n-1} = Q_a(0,n)Q_r(1,n)$

Detailed Analysis Continued

- This steady state analysis doesn't tell the whole truth!
- We want retransmission probability q_r to be relatively large to avoid large delays after collisions.
- If arrival rate is small, and few packets are involved in collisions, this works well and retransmissions are normally successful
- But, if we get enough backlogged packets n such that $q_r n >> 1$, we get collisions in almost all successive slots and the system remains heavily backlogged for a long time.

Detailed Analysis Cont.

We define the *drift*, D_n, in state n as the expected change in backlog over one slot time, starting in state n

D_n is the expected number of new arrivals accepted into the system $(m-n)q_a$ less the expected number of successful transmissions in the slot, i.e. probability of successful transmission, P_s

Thus $D_n = (m-n)q_a - P_s$ where

$$P_s = Q_a(1,n)Q_r(0,n) + Q_a(0,n)Q_r(1,n)$$

The attempt rate $G(n)$ is the expected number of attempted transmission in a slot when the system is in state n

Detailed analysis Cont.

We have $G(n) = (m-n)q_a + nq_r$

By inserting the definition of Q_a and Q_r in P_s we get

$$\begin{aligned} P_s &= (m-n)(1-q_a)^{m-n-1}q_a(1-q_r)^n + \\ &+ (1-q_a)^{m-n}n(1-q_r)^{n-1}q_r \\ &= \left((m-n)q_a\frac{1}{1-q_a} + nq_r\frac{1}{1-q_r}\right)(1-q_a)^{m-n}(1-q_r)^n \\ &\approx ((m-n)q_a + nq_r)\, e^{-q_a(m-n)-q_r n} = G(n)e^{-G(n)} \end{aligned}$$

where the approximation is good if q_a and q_r are small

The probability of an idle slot is approximately $e^{-G(n)}$

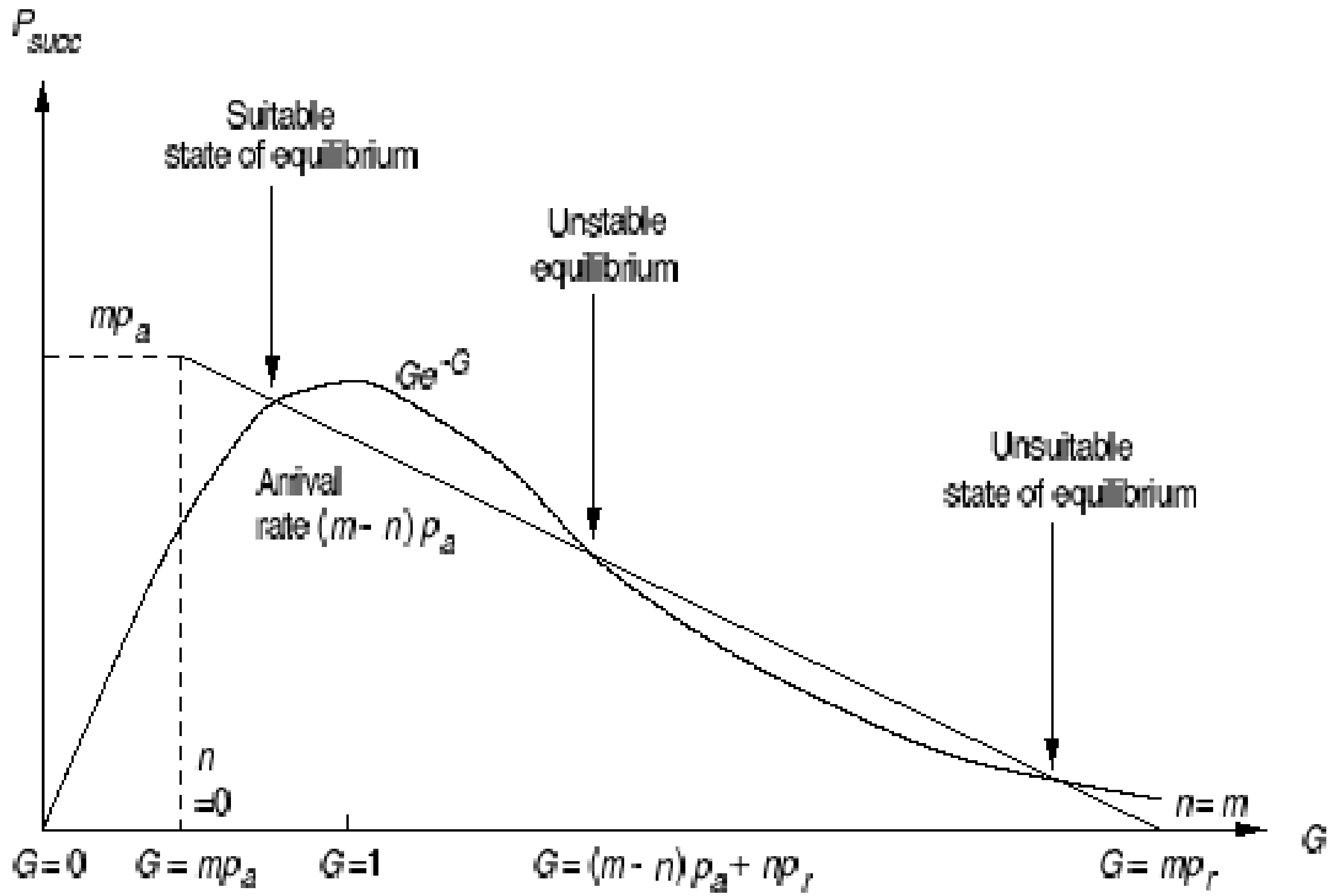
Detailed Analysis Continued

Thus the probability of number of packets in a slot is well approximated by a Poisson process random variable with rate $G(n)$, note that the rate varies with the state.

By plotting $P_s = G(n)e^{-G(n)}$ and the line $(m - n)q_a$ (as function of n) we can see the drift D_n as the difference between the curve and the line

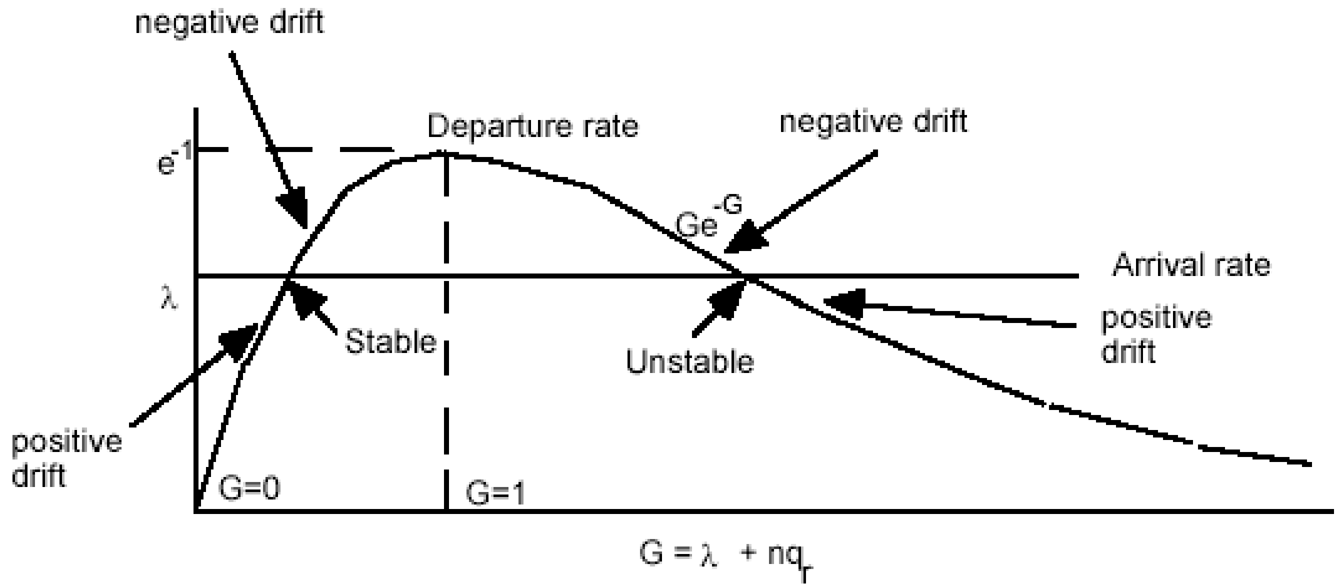
Since the drift is the expected change in in state from one slot to the next, the system tends to move in the direction of the drift although it may fluctuate

Analysis



P_{succ}
Suitable state of equilibrium
Unstable equilibrium
mp_a
Ge^{-G}
Arrival rate $(m-n)p_a$
Unsuitable state of equilibrium
$n=0$
$n=m$
G
$G=0$
$G=mp_a$
$G=1$
$G=(m-n)p_a+np_r$
$G=mp_r$

Infinite Node Assumption



- Undesirable stable point disappears
- If backlog increases beyond unstable point (bad luck) then it tends to increase without limit and the departure rate drops to 0
- Instability of the slotted ALOHA

Stabilization of Slotted ALOHA – Infinite Node

We define a multiaccess system as stable for a given arrival rate if the expected delay per packet is finite

The maximum stable throughput is defined as the least upper bound of arrival rates for which the system is stable

The ordinary slotted Aloha has maximum stable throughput 0

When estimate of backlog is perfect and $G(n) = 1$, idles occur with probability $1/e \approx 0.368$, success occur with probability $1/e$, and collisions occur with probability $1 - 2/e \approx 0.264$, thus the rule for changing q_r should allow fewer collisions than idles

Cont.

If all backlogged nodes use the same retransmission probability the maximum stable throughput is at most $1/e$, since when backlog is large the Poisson approximation becomes more accurate, the success rate is then limited to $1/e$ and the drift is positive for $\lambda > 1/e$

Pseudo-Bayesian algorithm: new arrivals are regarded as backlogged immediately on arrival

Attempt rate $G(n) = nq_r$, probability of successful transmission is $nq_r(1-q_r)^{n-1}$

Each node maintains an estimate $\hat{n}$ of the backlog n at the beginning of each slot

Rivest's Pseudo-Bayesian Algorithm

Each backlogged packed is transmitted with probability $q_r(\hat{n}) = \min\{1, 1/\hat{n}\}$

The estimated backlog $\hat{n}_{k+1}$ at slot $k+1$ is updated from the estimated backlog $\hat{n}_k$ at slot k and feedback for slot k according to

$$\hat{n}_{k+1} = \begin{cases} \max\{\lambda, \hat{n}_k + \lambda - 1\}, & \text{for idle or success} \\ \hat{n}_k + \lambda + (e-2)^{-1}, & \text{for collision} \end{cases}$$

Addition of λ to take new arrivals into account

Subtraction of 1 for successful transmissions to account for succesful departure

Cont.

Subtracting of 1 for idle transmission for decreasing estimate when too many idle slots occur

Adding $(e-2)^{-1}$ on collisions to increase estimate when too many collisions occur.

For large backlogs, if $\hat{n} = n$ we get attempt rate 1, and idles with probability $1/e$, collision with probability $(e-2)/e$, so decreasing by 1 on idle and increasing by $(e-2)^{-1}$ on collision maintains balance between n and $\hat{n}$ on average

Motivation for the algorithm

Assume that probability distribution of n_k is Poisson with mean $\hat{n}_k \geq 1$, i.e.

$$P(n_k = \nu) = \frac{\hat{n}_k^\nu}{\nu!} e^{-\hat{n}_k}$$

Each packet transmitted with probability $1/\hat{n}_k$

$$\begin{aligned} P(idle) &= \sum_{\nu=0}^{\infty} P(n_k = \nu)(1 - \frac{1}{\hat{n}_k})^\nu = \sum_{\nu=0}^{\infty} \frac{\hat{n}_k^\nu}{\nu!} e^{-\hat{n}_k} (1 - \frac{1}{\hat{n}_k})^\nu \\ &= \sum_{\nu=0}^{\infty} \frac{(\hat{n}_k - 1)^\nu}{\nu!} e^{-\hat{n}_k} = e^{-\hat{n}_k} e^{\hat{n}_k - 1} = e^{-1} \end{aligned}$$

Motivation for the algorithm

The a posteriori probability that there were ν packets in the system given that the slot was idle is

$$P(n_k = \nu | \textit{idle}) = \frac{P(\textit{idle} | n_k = \nu) P(n_k = \nu)}{P(\textit{idle})}$$

$$= \frac{(1 - \frac{1}{\hat{n}_k})^\nu \cdot \frac{\hat{n}_k^\nu}{\nu!} e^{-\hat{n}_k}}{e^{-1}} = \frac{(\hat{n}_k - 1)^\nu}{\nu!} e^{-(\hat{n}_k - 1)}$$

Thus the a posteriori probability is Poisson distributed with mean $\hat{n}_k - 1$

Motivation cont.

Similarly the probability for successful transmission

$$
\begin{aligned}
P(succ) &= \sum_{\nu=0}^{\infty} \frac{\hat{n}_k^{\nu}}{\nu!} e^{-\hat{n}_k} \nu (1 - \frac{1}{\hat{n}_k})^{\nu-1} \frac{1}{\hat{n}_k} \\
&= \sum_{\nu=1}^{\infty} \frac{(\hat{n}_k - 1)^{\nu-1}}{(\nu-1)!} e^{-\hat{n}_k} = e^{-\hat{n}_k} e^{\hat{n}_k - 1} = e^{-1}
\end{aligned}
$$

The a posteriori probability that there were $\nu + 1$ packets in the system given that the slot had a successful transmission is

$$
\begin{aligned}
P(n_k = \nu + 1 | succ) &= \frac{P(succ | n_k = \nu + 1) P(n_k = \nu + 1)}{P(succ)} \\
&= \frac{(\nu + 1)(1 - \frac{1}{\hat{n}_k})^{\nu} \frac{1}{\hat{n}_k} \cdot \frac{\hat{n}_k^{\nu+1}}{(\nu+1)!} e^{-\hat{n}_k}}{e^{-1}} \\
&= \frac{(\hat{n}_k - 1)^{\nu}}{\nu!} e^{-(\hat{n}_k - 1)}
\end{aligned}
$$

Thus the a posteriori probability for the remaining packets is Poisson distributed with mean $\hat{n}_k - 1$

Motivation cont.

- Taking the new arrivals into account we get that given an a priori Poisson probability on n_k with mean $\hat{n}_k \geq 1$, then given an idle or successful slot, the probability distribution of n_{k+1} is Poisson with mean $n_k + \lambda - 1$
- Given a collision, the a posteriori probability is not quite Poisson but may be reasonably approximated by a Poisson with mean $\hat{n}_{k+1} = \hat{n}_k + \lambda + (e-2)^{-1}$
- This is the reason the algorithm is called Pseudo-Bayesian

Motivation cont.

- In applications the arrival rate λ is typically unknown and slowly varying
- One possibility is to estimate λ by time-averaging rate of successful transmissions, however nothing has been proven about stability of the algorithm when using dynamic estimate of λ
- An alternative is to use a fixed value for λ, it can be shown that using $1/e$ will give stability for all actual $\lambda < 1/e$.

Approximate Delay Analysis

Stabilized slotted Aloha with pseudo-Bayesian algorithm

Assuming arrival rate λ is known

Probability of successful transmission $P_s = 1/e$ if backlog $n \geq 2$ and $P_s = 1$ if $n = 1$

Let W_i be the delay from arrival of ith packet until beginning of ith successful transmission

We can assume that the average of W_i over all i is the expected queueing delay W

Let n_i be number of backlogged packets at the instant before i's arrival (not including any packet currently being succesfully transmitted)

Approximate analysis Cont.

- Let R_i be the residual time to beginning of next slot and t_1 the subsequent interval until next successful transmission is completed. Similarly t_j the interval from the end of the $(j-1)$ subsequent success to the end of the jth subsequent success.
- After n_i successful transmissions y_i is the remaining interval until the beginning of next successful transmission, then

$$W_i = R_i + \sum_{j=1}^{n_i} t_j + y_i$$

Approximate analysis Cont.

$$W_i = R_i + \sum_{j=1}^{n_i} t_j + y_i$$

For each interval t_j the backlog is at least 2, thus each slot is successful with probability $1/e$ and the expected value of each t_j is e

Little's theorem gives $E[n_i] = \lambda E[W_i] = \lambda W$

$E[R_i] = 1/2$, and we get

$$W = 1/2 + \lambda W e + E[y]$$

Approximate analysis Cont.

Consider the first slot boundary at which both the $(i-1)$st departure and the ith arrival have occurred

If backlog is 1 then $y_i = 0$

If backlog $n > 1$, then $E[y_i] = e - 1$

Let p_n be steady state probability that backlog is n at a slot boundary

If state is 1 at beginning of a slot we always get a successful transmission, thus p_1 is the fraction of slots in which state is 1 and a packet is successfully transmitted

Approximate analysis Cont.

Total fraction of slots with successful transmission is λ, thus p_1/λ is the fraction of packets transmitted from state 1 and $1 - p_1/\lambda$ is the fraction transmitted from higher numbered states, in total we get

$$E[y] = (e-1)(1 - p_1/\lambda) = \frac{(e-1)(\lambda - p_1)}{\lambda}$$

The rate of packets transmitted from state 1 is p_1

The probability of state higher than 1 is $(1 - p_0 - p_1)$ and successful transmission with probability $1/e$ give rate of packets transmitted from higher states as $(1 - p_0 - p_1)/e$

Thus we get $\lambda = p_1 + (1 - p_0 - p_1)/e$

Approximate analysis Cont.

State 0 entered only if no new arrivals occured in the previous slot and previous state was 0 or 1, thus

$$p_0 = (p_0 + p_1)e^{-\lambda}$$

$$\lambda e = (e-1)p_1 + 1 - p_0$$

$$p_0 = (e-1)p_1 + 1 - \lambda e$$

$$(e-1)p_1 + (1-\lambda e) = ((e-1)p_1 + (1-\lambda e) + p_1)e^{-\lambda}$$

$$(e-1)e^{\lambda}p_1 + (1-\lambda e)e^{\lambda} = (e-1)p_1 + (1-\lambda e) + p_1$$

$$(1-\lambda e)(e^{\lambda}-1) = p_1((e-1)(1-e^{\lambda}) + 1)$$

$$p_1 = \frac{(1-\lambda e)(e^{\lambda}-1)}{1-(e-1)(e^{\lambda}-1)}$$

Approximate analysis Cont.

Now, combining our equations

$$
\begin{aligned}
W &= 1/2 + \lambda W e + E[y] \\
E[y] &= \frac{(e-1)(\lambda - p_1)}{\lambda} \\
p_1 &= \frac{(1-\lambda e)(e^{\lambda}-1)}{1-(e-1)(e^{\lambda}-1)}
\end{aligned}
$$

We get

$$
W = \frac{e-1/2}{1-\lambda e} - \frac{(e-1)(e^{\lambda}-1)}{\lambda(1-(e-1)(e^{\lambda}-1))}
$$

Approximate analysis Cont.

- For comparison, consider the delay in a time division multiplex system with m traffic streams of equal length packets arriving according to a Poisson proccess with rate λ/m each
- Time axis divided into m-slot frames with one time slot dedicated to each traffic stream
- This corresponds to m M/D/1 queueing systems, each with service rate $\mu = 1/m$
- According to M/D/1-formula for queueing delay (3.45) p. 187 the average queueing delay is $W_q = \rho/(2\mu(1-\rho))$ where $\rho = \frac{\lambda/m}{1/m} = \lambda$

Approximate analysis Cont.

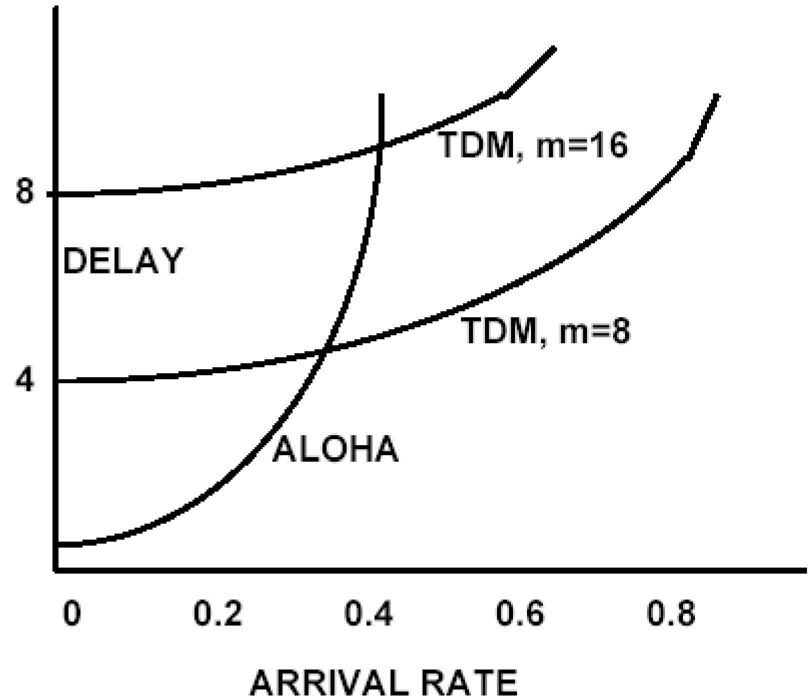
- Thus we get average queueing delay

$$W_q = \frac{m\lambda}{2(1-\lambda)}$$

- In addition to this we have an average delay of $m/2$ waiting for the traffic slot for the traffic stream in question
- Our total average delay from a packet arrival until it begins transmission is

$$W_{\text{TDM}} = \frac{m}{2(1-\lambda)}$$

ALOHA vs TDM



- **Aloha achieves lower delays when arrival rates are low**
- **TDM results in very large delays with large number of users, while Aloha is independent of the number of users**

Binary Exponential Backoff

- For packet radio networks and some other multiaccess situations the assumption of immediate (0,1,e) feedback is unrealistic
- In some systems a node only receives feedback about its own packets, no feedback in about slots in which it does not transmit
- This limited feedback is insufficient for the backlog estimation of pseudo-Bayesian strategy
- An alternative stabilization strategy is binary exponential backoff used in Ethernet; If a packet has been transmitted unsuccessfully i times the transmission in successive slots is set to

$$q_r = 2^{-i}$$

Unslotted ALOHA

- The original Aloha protocol was unslotted, in this strategy each node, upon receiving a new packet, transmits it immediately rather than waiting for a slot boundary
- We omit the slotted system assumption
- If a packet is involved in a collision, it is retransmitted after a random delay
- We assume that if the transmission times for two packets overlap at all those packets fail and retransmission will be required
- We assume that each node, after a given propagation delay, can determine whether or not its packets were correctly received

Unslotted ALOHA

If one packet starts transmission at time t, and all packets have unit length, any other transmission starting between $t-1$ and $t+1$ will cause a collision

Assume infinite number of nodes

A node is considered backlogged from the time it has determined that its previously transmitted packet was involved in a collision until the time that it attempts retransmission

Number of backlogged nodes is n

Assume that period until attempted retransmission τ is exponentially distributed with probability density $xe^{-x\tau}$, where x is interpreted as retransmission attempt rate

Unslotted ALOHA

With an overall Poisson arrival rate λ, the times of attempted retransmissions is a time-varying Poisson process with rate $G(n) = \lambda + nx$ where n is the backlog at a given time

Let τ_i be the interval between the ith and $(i+1)$th transmission attempt, the ith attempt will be successful if both τ_i and τ_{i-1} exceed 1 (assuming all packets have length 1)

The probability distribution for the interval τ_i is $G(n)e^{-G(n)\tau_i}$ thus the probability that $\tau_i > 1$ is $e^{-G(n)}$

Assuming τ_i and τ_{i-1} independent gives probability of successful transmission $P_s = e^{-2G(n)}$

Unslotted ALOHA

Attempted transmissions occur at rate $G(n)$, the expected number of successful transmissions per unit time, the *throughput* as a function of n is $G(n)e^{-2G(n)}$

The situation is very similar to slotted Aloha, except the maximum throughput is $1/(2e)$ achieved when $G(n) = 1/2$

We have assumed that backlog is same in intervals surrounding a given transmission attempt, but whenever a backlogged packet initiates a transmission the backlog decreases by 1 and whenever a collided packet is detected it increases by 1, for small x this error is relatively small

Splitting

More efficient approach to resolving collisions

- **Simple feedback (0,1,e)**
- **Basic idea: assume only two packets are involved in a collision**

  Suppose all other nodes remain quiet until collision is resolved, and nodes in the collision each transmit with probability 1/2 until one is successful

  On the next slot after this success, the other node transmits

  The expected number of slots for the first success is 2, so the expected number of slots to transmit 2 packets is 3 slots

  Throughput over the 3 slots = 2/3

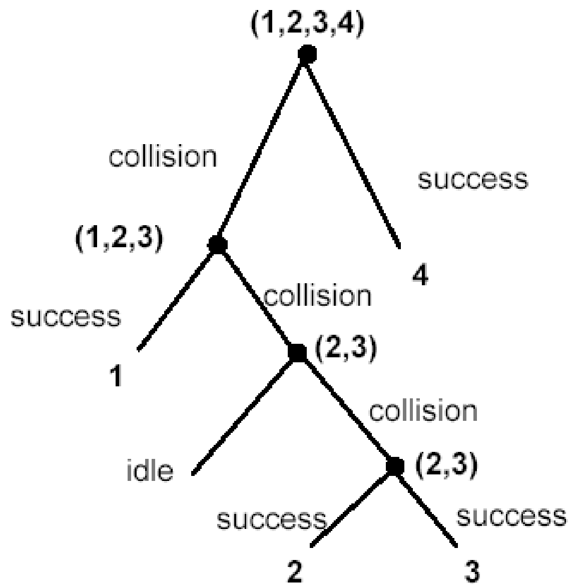
- **In practice above algorithm cannot really work**

  Cannot assume only two users involved in collision

  Practical algorithm must allow for collisions involving unknown number of users

Tree Algorithms

- After a collision, all new arrivals and all backlogged packets not in the collision wait
- Each colliding packet randomly joins either one of two groups (Left and Right groups)
  - Toss of a fair coin
  - Left group transmits during next slot while Right group waits
    - If collision occurs Left group splits again (stack algorithm)
    - Right group waits until Left collision is resolved
  - When Left group is done, right group transmits



Notice that after the idle slot, collision between (2,3) was sure to happen and could have been avoided

Many variations and improvements on the original tree splitting algorithm

Summary

- stabilized pure aloha T = 0.184 = (1/(2e))
- stabilized slotted aloha T = 0.368 = (1/e)
- Basic tree algorithm T = 0.434
- Best known variation on tree algorithm T = 0.4878
- Upper bound on any collision resolution algorithm with (0,1,e)feedback T <= 0.568
- TDM achieves throughputs up to 1 packet per slot, but the delay increases linearly with the number of users