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Is proportional fair scheduling suitable for age-sensitive traffic?

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A R T I C L E I N F O

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A B S T R A C T

Proportional Fair (PF) scheduling with successful deployments in various cellular wireless networks and wireless LANs, aims at maximizing the sum of the logarithms of user throughputs. PF scheduling is known to strike an appropriate balance between fairness and throughput, for conventional data traffic. On the other hand, there has recently been a surge of interest in status update networks carrying age-sensitive traffic for which information freshness is crucial and therefore network performance metrics driven by Age of Information (AoI) are instrumental, as opposed to conventional performance metrics such as delay, loss, or throughput, used for conventional data traffic. This paper studies the scheduling problem for the downlink of a cellular wireless network with a transmitter sending age-sensitive status update packets from multiple information sources to users with the goal of keeping the information as fresh as possible for the users. For this purpose, under the generate-at-will scenario, an age-agnostic model-free scheduler is proposed with the goal of minimizing the weighted sum peak AoI of the network, which is the performance metric used in this paper for quantifying information freshness. With numerical examples, the proposed scheduler is compared and contrasted with weighted PF scheduling in terms of implementation and performance, in both non-opportunistic and opportunistic scenarios.

1. Introduction

Proportional Fair (PF) network resource allocation provides a balance between fairness and efficiency through the maximization of the sum of the logarithms of user throughputs for a fixed population of users [1]. In particular, proportional fairness has been successfully used for link scheduling in wireless networks in opportunistic scenarios where the available user transmission rates are different for each user due to communication distance, fading, etc., and these rates are a-priori known at scheduling instants. On the other hand, a non-opportunistic scenario refers to one for which the transmission rates are not known in advance at a scheduling instant. The Ref. [2] proposed a PF cellular wireless scheduler for which the Base Station (BS) chooses to serve the user which has the largest ratio of available transmission rate to its exponentially smoothed average throughput. Variations of the PF wireless scheduler emerged following the original proposal of [2]. In Temporal Fair (TF) scheduling, the cell throughput is maximized under the constraint that users receive the same share of air-time resources [3]. It was shown in [3] that the optimum TF scheduler chooses to serve the user which has the largest sum of available transmission rate and another user-dependent term that can be obtained using an on-line learning algorithm. The authors of [4] show that PF and TF allocations are equivalent for wireless LANs and ad-hoc network scenarios, and subsequently propose distributed air-time allocation algorithms for achieving proportional fairness. Weighted Proportional Fairness (WPF) can also be defined as in [1,5] by considering the minimization of the weighted sum of logarithms of user throughputs where the weighting factors are used to give relative importance to certain user throughputs. Similarly, Weighted Temporal Fairness (WTF) amounts to the situation for which the sources receive a weighted share of air-time resources [6,7].

Recently, there has been a surge of interest on timely status updates in networked control and monitoring systems. To quantify the timeliness of information freshness in status update systems, the authors of [8] introduced the Age of Information (AoI) concept; see the surveys on [9,10] and the references therein for recent AoI-related research. In the general AoI end-to-end scenario outlined in [11], information sources, e.g., sensors, sample a random process and generate packets carrying information on the sample values and sampling times. Usually, the sensors send the packets immediately to a server (or also called a transmitter) which then forwards the information packets from the sources towards one or more destinations typically over a wireless channel. In the so-called random arrival scenario, the sampling is done by the sources according to a random process without the involvement of the server which employs a variety of techniques such as preemption, queuing, buffer management, scheduling, etc. so as to keep the information as fresh as possible at the destination(s). On the

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other hand, in the generate-at-will scenario, the server decides when to sample and forward an information packet for transmission. In this regard, the generate-at-will scenario is equivalent to the random arrival scenario where each source has its own queue with a single packet buffer holding the most recent, i.e., freshest, packet from that source at the heavy traffic regime, i.e., as the arrival rates approach infinity. For each of the two scenarios described above, the AoI process for a certain information source–destination pair keeps track of the time elapsed at the destination since the generation of the last successfully received update packet from the source. Sample paths of the AoI process increase in time with unit slope except at information packet reception instances when the AoI process abruptly drops to a value that is equal to the age of the received packet, i.e., generation time of the packet subtracted from the current time. The Peak AoI (PAoI) process for the same pair is obtained by sampling the AoI process just before packet reception instances. The mean AoI or mean peak AoI values have generally been used to quantify the information freshness at the destination. On the other hand, the system-level freshness is described by the weighted sum AoI or weighted sum peak AoI, where weighted averaging is done across all the information source–destination pairs in the system with the weights being used to capture the relative urgency of the underlying streams.

In this paper, we study the problem of scheduling the transmission of time-sensitive information packets generated by N information sources from a transmitter (base station) to N users (subscriber units), for the generate-at-will scenario. The service times of information packets are assumed to be generally distributed and heterogeneous, which is indicative of different channel gains and hence different modulation and coding schemes between the BS and the users. We note that the proposed schedulers can also be used for the cellular uplink where the information sources may transmit their time-sensitive data towards the base station but the focus of the current paper is the downlink. A continuous-time setting is envisioned which is in line with the majority of the existing literature on AoI and the server is in charge of scheduling one of the information sources to generate an information packet and send it over the wireless channel to the intended user. The goal of the scheduler is to minimize the weighted sum peak AoI of the system that is used to quantify the information freshness of the system. Both non-opportunistic and opportunistic scenarios are considered.

The contributions of the paper are listed as follows:

- Since most cellular networks currently deploy PF schedulers or their variants for the downlink, a natural choice for scheduling is to ignore the age-sensitive nature of the incoming traffic and continue to use the WPF or WTF scheduler as if the network is carrying conventional data traffic, while attempting to maximize the system utility that is defined as the weighted sum of the logarithms of user throughputs. As the first contribution of this paper, for the non-opportunistic scenario, we identity regimes of operation for which the WTF scheduler turns out to be relatively ineffective in terms of peak AoI.
- As the second main contribution of this paper, we propose an age-agnostic, model-free, learning-based scheduler that minimizes the weighted sum peak AoI of the system while slightly modifying the implementation of the underlying WTF scheduler. In commercial cellular wireless networks, the characteristics of the system, e.g., per-source service times, may not be known in advance, or may even be time-varying, and the scheduler needs to learn the relevant characteristics in operation, to be of practical use which is the main motivation for the development of a learning-based scheduler. Moreover, it is shown that the proposed scheduler can be tuned to reduce the weighted sum AoI without having to sacrifice from the weighted sum peak AoI performance.
- The majority of the existing schedulers for AoI focus on the non-opportunistic scenario. However, cellular networks with access to channel state information give rise to the possibility of opportunistic scheduling. As our final contribution, we propose an extension of the proposed scheduler so that it can also be effectively used in opportunistic scenarios.

The paper is organized as follows: Section 2 briefs the related work. In Section 3, a detailed description of the AoI and peak AoI processes is presented. The system model is given in Section 4 and the proposed schedulers are presented in Section 5. Numerical examples are provided in Section 6 for comparing weighted temporal fairness and weighted sum peak AoI minimization and also for validating the effectiveness of the proposed scheduler for age-sensitive traffic in terms of both weighted sum peak AoI and weighted sum AoI. Finally, conclusions are given.

2. Related work

Recently, in communication systems literature, there has been growing interest on AoI modeling and optimization since the Ref. [12] first introduced the AoI concept in a single-source, single-server queueing system setting. This model is then extended to multiple sources in [13]. The recent Refs. [9,10] present exhaustive surveys on existing work on AoI models which discuss several variations of AoI models depending on single vs. multiple information sources, random arrival models vs. generate-at-will models, scheduling discipline, buffer management, performance metric of interest, etc.

The analysis of AoI in status update systems with multiple sources and random arrivals has been an active research topic. The mean peak AoI expression for M/G/1 and M/G/1/1 systems with heterogeneous service time requirements are obtained in [14] which makes it possible to optimize system cost in terms of the mean peak AoI. The Ref. [15] obtains the distribution of AoI and peak AoI for each of the sources in a general bufferless M/PH/1/1 status update system with heterogeneous phase-type distributed service times and arbitrary preemption probabilities. The authors of [16] study the multi-source M/M/1 model with FCFS (first come first serve), preemptive LCFS (last come first serve) and non-preemptive LCFS with replacement with a single buffer using Stochastic Hybrid Systems (SHS) and obtain exact expressions for the mean AoI. The authors of [17] consider three source-aware packet management policies in a two-source system for which they obtained the per-source average AoI for each policy using SHS.

Link scheduling for AoI minimization has been one of the main AoI research problems topics studied in the literature. The Ref. [18] considers the problem of minimizing average and peak AoI in wireless networks under general interference constraints and for the generate-at-will scenario, they show that a stationary scheduling policy is peak age optimal. It is also shown in [18] that the proposed scheduling policy achieves average age that is within a factor of two of the optimal average age. The authors of [11] study the link transmission scheduling problem with the objective of minimizing the overall age while proving that the problem is NP-hard in general, and an integer linear programming formulation is provided for performance benchmarking and a steepest age descent algorithm is proposed. The authors of [19] show that ESFS (earliest served first serve) is an effective scheduling policy for system mean AoI minimization for symmetric networks, i.e., source weights and their mean service times are identical. The Ref. [20] considers the joint sampling and scheduling problem for optimizing data freshness in multi-source systems in terms of overall age and shows that the Maximum Age First (MAF) scheduling strategy provides the best age performance. The authors of [21] propose an age-based scheduler that combines age with the interarrival times of incoming packets, in its scheduling decisions, to achieve improved information freshness at the receiver. Although the analytical results are obtained for only heavy-traffic, their numerical results reveal that the proposed algorithm achieves desirable freshness performance for lighter loads as well. The Ref. [22] considers an asymmetric, i.e., heterogeneous weights and service times, discrete-time wireless network with a base station serving multiple traffic streams for the generate-at-will scenario while proposing Max-weight and Whittle index based policies with strong performances whose AoI-optimalities are shown in the more specific symmetric network scenario, i.e., homogeneous weights and service times, and propose nearly optimal age-based schedulers and age-agnostic randomized schedulers for the general case. For the random arrival model, the authors of [23] propose a non-work-conserving stationary randomized policy for the single-buffer case with the policy being independent of the arrival rates. Moreover, they propose a work-conserving age-based Max-Weight scheduler for the same system whose performance is better and is close to the lower bound. Schedulers have also been proposed in the recent literature using reinforcement learning for age-sensitive traffic for the downlink in [24] and the uplink [25] of a cellular wireless network. The Ref. [26] employs belief-based Bayesian reinforcement learning for a learning-based autonomous scheduler for AoI-aware industrial wireless networks.

For a proportional fair scheduling solution, increasing the mean throughput of one user from the optimal solution by $x\%$ should result in a cumulative percentage reduction by larger than $x\%$ of the mean throughput of other users [27]. It is known that achieving proportional fairness is equivalent to the maximization of the sum of the logarithms of the individual throughputs [27]. In the PF scheduling implementation of [28], the BS chooses to serve the user which has the largest ratio of available transmission rate to its exponentially smoothed average throughput [28]. Different variations of the PF algorithm are possible depending on how the scheduler treats empty or short queues and how the average throughput is maintained [29]. In Temporal Fair (TF) scheduling, the cell throughput is maximized under the constraint that users receive the same share of temporal, i.e., air-time, resources [3]. It was shown in [3] that the optimum TF scheduler chooses to serve the user which has the largest sum of available transmission rate and another user-dependent term that can be calculated off-line if the channel models are available or alternatively can be obtained using an online learning algorithm. Under some simplifying assumptions involving channel characteristics of users, proportional fairness is equivalent to air-time fairness in wireless networks [4,30]. Proportional fair scheduling has recently been used in current wireless networking scenarios. The authors of [31] study proportional fair scheduling in vehicular scenarios using the prediction of future throughputs. PF scheduling for the downlink of a mmWave multi-user NOMA system is studied in [32].

3. AoI and peak AoI

We provide a rigorous description of the AoI and peak AoI processes for a generic status update system with N information sources, $s_i, i = 1, \ldots, N$ which generate samples of an associated random process. The description is very general and applies to both generate-at-will and random arrival models. The samples and their time stamps are carried by information packets. These packets are sent to the server with no delay. The transmitter is in charge of sending the information packets from s_i to the intended destination $d_i, i = 1, \ldots, N$ over a shared channel. The notation v_i stands for the source–destination pair (s_i, d_i). We focus on one tagged source–destination pair, say v_ℓ, for which the AoI and PAoI processes will be detailed below. The information packets from the tagged source s_ℓ are called tagged packets. Let t_j and r_j denote the instances at which the jth successful tagged packet is generated by s_ℓ and received by d_ℓ, respectively. Unsuccessful packets are those that are not received by d_ℓ which may stem from channel errors, packet dropping by the server, etc. We also let u_j denote the system time of the jth successful tagged packet which is the sum of the packet's queue wait time and service times, i.e., $u_j = r_j - t_j$. Fig. 1 depicts a sample path of the tagged source AoI process $A_\ell(t)$ which increases with unit slope from the value u_j at r_j until time r_{j+1}. The peak value in the jth cycle is denoted by $P_\ell(j)$ which stands for the Peak AoI process for the tagged source. The buffer management and scheduling decisions made by the server dictate when and which of the information packets will be transmitted and therefore they impact the performance metrics related to the steady-state random variables A_i and P_i of the associated AoI and PAoI processes of source s_i, respectively. In status update systems, the metrics to optimize generally involve the moments or violation probabilities of the related AoI and PAoI processes. In this paper, we consider the problem of minimizing the weighted sum peak AoI denoted by W_P:

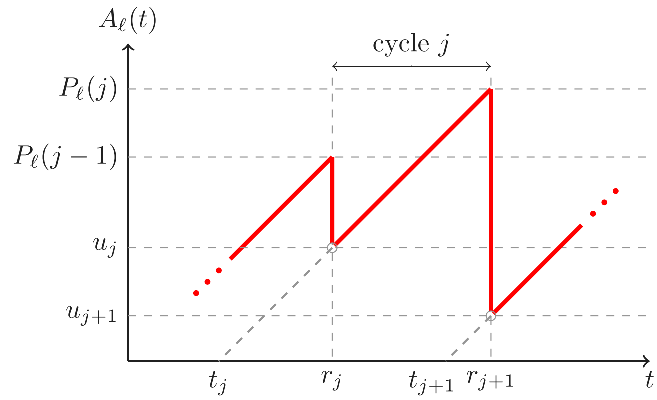

Fig. 1. Sample path of the AoI and PAoI processes for the tagged source ℓ.

$$W_P = \sum_{i=1}^{N} w_i \mathbb{E}[P_i], \tag{1}$$

where w_i is a normalized source weight parameter for source–destination index i used for peak AoI differentiation, i.e., $\sum_{i=1}^{N} w_i = 1$. Minimization of the weighted sum AoI

$$W_A = \sum_{i=1}^{N} w_i \mathbb{E}[A_i], \tag{2}$$

is known to be an open problem for general scenarios and is left for future work. Minimization of weighted sum PAoI and weighted sum AoI have been investigated in many research works; see for example the Refs. [22,33,34].

4. System model

We consider the generate-at-will system given in Fig. 2 with N information sources denoted by $s_i, i = 1, \ldots, N$ co-located with the server. At a scheduling instant, the server (base station) chooses one of the sources, say s_j, to sense and sample the random process associated with the source which then forms an information packet which is subsequently transmitted to the destination (user) d_j. The scheduler is assumed to be age-agnostic, i.e., the instantaneous values of AoI will not be used in the scheduling decisions, for computational efficiency purposes. The completion of a packet transmission initiates a new scheduling instant. The transmission times of packets generated by s_i destined to d_i, denoted by S_i, are generally distributed with mean $\mathbb{E}[S_i] = \mu_i^{-1}$, variance σ_i^2, and squared coefficient of variation $c_i^2 = \sigma_i^2 \mu_i^2$. Packet sizes are random but they have the same mean L bits across all the sources.

5. Proposed schedulers

5.1. Weighted proportional fair scheduler

For the system given in Fig. 2, we first obtain the rules for achieving weighted proportional fairness among the sources. For this purpose, let R_i denote the long-term average bit rate, i.e., throughput, of information packets of s_i scheduled for transmission. Similarly, let τ_i

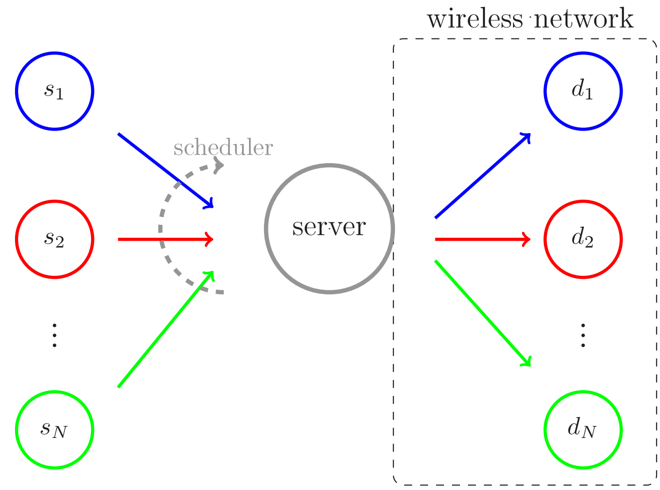

Fig. 2. Wireless network carrying time-sensitive traffic from N information sources.

denote the long-term fraction of time that the link is occupied with the transmission of packets from s_i. The relationship between R_i and τ_i is easy to write:

$$R_i = L\mu_i\tau_i. \tag{3}$$

Subsequently, the optimization problem for the weighted PF scheduler is as follows:

$$\underset{\tau_i}{\text{maximize}} \quad \sum_{i=1}^{N} w_i \log(R_i) = \sum_{i=1}^{N} w_i \log(L\mu_i\tau_i) \tag{4a}$$

$$\text{subject to} \quad \tau_1 + \cdots + \tau_N = 1, \tag{4b}$$

$$\tau_i \geq 0 \tag{4c}$$

which turns out to be a special case of the general formulation in [1] yielding the solution

$$\tau_i = w_i, \quad 1 \leq i \leq N, \tag{5}$$

named as Weighted Temporal Fairness (WTF) which indicates that a scheduling algorithm that achieves (5) by controlling the link occupancy times of each source will be weighted proportional fair, i.e., the weighted sum of the logarithms of user throughputs is maximized. The weighted total throughput W_R in units of packets/s. when WTF is achieved, denoted by W_R^{WTF}, has the following closed-form expression:

$$W_R^{WTF} = \sum_{i=1}^{N} w_i \frac{R_i^{WTF}}{L} = \sum_{i=1}^{N} w_i^2 \mu_i, \tag{6}$$

where R_i^{WTF} is the throughput of the source s_i in units of bits/s. when WTF is achieved according to (5).

When $\tau_i, 1 \leq i \leq N$, is given for a status update system, it is possible to provide a closed-form expression for the weighted sum peak AoI W_P in (1) by revisiting Fig. 1 for the generate-at-will scenario in which case the sample value of P_i is obtained by adding the two service times of successively scheduled s_i packets and the sum of the service times of all the packets scheduled within between, from sources $j \neq i$, denoted by the random variable Y_i. Consequently,

$$\mathbb{E}[P_i] = 2\mathbb{E}[S_i] + \mathbb{E}[Y_i]. \tag{7}$$

From the definition of τ_i and Fig. 1, the following holds:

$$\tau_i = \frac{\mathbb{E}[S_i]}{\mathbb{E}[S_i] + \mathbb{E}[Y_i]}, \tag{8}$$

Algorithm 1 Pseudo-code for WTFS

Require: α, β;
  $B_i \leftarrow 0$, $k \leftarrow 0$;
  while (1) **do**
    $k \leftarrow k + 1$;
    $I \leftarrow \arg\min_i \left((1-\beta)B_i + \beta \frac{S_i^k}{V_i} \right)$;
    Schedule source I packet;
    $U_I \leftarrow U_I + \alpha(S_I^k - U_I)$;
    $V_i \leftarrow V_i + \alpha(S_i^k - V_i)$, $1 \leq i \leq N$;
    $B_I \leftarrow B_I + U_I$;
    $B_i \leftarrow B_i - w_i U_I$, $1 \leq i \leq N$;
  end while

which immediately yields the following closed-form expression for the expected value of P_i in terms of τ_i for the generate-at-will scenario:

$$\mathbb{E}[P_i] = \frac{1}{\mu_i}\left(1 + \frac{1}{\tau_i}\right). \tag{9}$$

Thus, one can write W_P, the weighted sum peak AoI, as a function of $\tau_i, i = 1, \ldots, N$, as follows:

$$W_P = \sum_{i=1}^{N} \frac{w_i}{\mu_i}\left(1 + \frac{1}{\tau_i}\right). \tag{10}$$

The expression for W_P when WTF is achieved, denoted by W_P^{WTF}, can be written as follows from (5):

$$W_P^{WTF} = \sum_{i=1}^{N} \frac{1}{\mu_i}\left(w_i + 1\right). \tag{11}$$

The expressions (6)–(11) are obtained based on the assumption that the mean service times are fixed and a-priori known and also the condition given in (5) is satisfied. For the purpose of achieving (5) when the mean service times are not a-priori known or when they are allowed to vary in time, a model-free WTF-achieving Scheduler (WTFS) is given in Algorithm 1 which schedules a source for sensing and transmission at instant k, i.e., just before transmitting the kth packet from the server. For scheduling purposes, the actual service time of s_i at instant k, denoted by S_i^k, are either known to the scheduler (opportunistic scenario) or not known (non-opportunistic scenario).

Let us study WTFS operation first for the non-opportunistic scenario in which case the opportunism parameter $\beta, 0 \leq \beta \leq 1$, is set to zero. WTFS maintains a real-valued bucket B_i for each source s_i. The scheduler chooses to transmit the source with the minimum bucket value. When there are more than two sources with the same minimum bucket values, such ties are broken randomly. The bucket of the scheduled source I is incremented by the exponentially smoothed running estimate of the service times of source I, denoted by U_I, obtained by the Robbins–Monro approximation with fixed learning (or smoothing) parameter $\alpha, 0 < \alpha < 1$, [35]. Subsequently, all the bucket values are decremented in proportion with the source weights while preserving the sum of the bucket values. The updates of the bucket values take place after a transmission is over just before the next scheduling instant $k+1$. The choice of a large value for the learning parameter α makes it possible to respond rapidly to changes in the statistics of the service time. However, in this case, a very large transmitted s_i packet service time would give rise to an elongated service interruption for this source which may not be desirable for weighted sum AoI. To see how the algorithm works, assume the mean service time is fixed and known a-priori. In this case, at the bucket update instances,

$$\lim_{t\to\infty} \frac{B_i(t)}{t} = \tau_i - w_i. \tag{12}$$

Since the bucket with the minimum value is chosen by the scheduler, all bucket values will stay bounded. Therefore, Algorithm 1 provides WTF in the long-term, i.e., $\tau_i = w_i$.

The opportunistic scenario is controlled by the opportunism parameter $\beta, 0 < \beta \leq 1$, through which the scheduler is steered to select sources with shorter actual service times relative to their smoothed estimates represented by the variables V_i. However, long-term WTF is achieved for each value of $\beta < 1$. When β is increased, then the algorithm becomes more greedy in selecting sources with short actual service times, i.e., good channels, and the throughput will be higher but at the expense of jeopardizing the short-term WTF. Depending on the time-scale requirements of temporal fairness, a suitable parameter β needs to be chosen. Note that U_i provides an exponentially smoothed estimate for the average service times of scheduled s_i packets. On the other hand, V_i keeps track of both scheduled and un-scheduled packets of s_i and the V_i update step is omitted in the non-opportunistic scenario. The non-zero initial values for the estimates $U_i, V_i, 1 \leq i \leq N$ are arbitrary but can be set using the a-priori information about the service times when the algorithm is first run.

5.2. Weighted Sum Peak AoI Scheduler (WSPS)

In this subsection, we consider the minimization of W_P in (1) for the system of Fig. 2. For this purpose, let us revisit the expression for W_P as a function of the parameters τ_i (10). The minimization of W_P in (10) subject to $\sum_{i=1}^{N} \tau_i = 1$ is a convex optimization problem. To see this, the function $f(x) = \frac{1}{x}$ is a convex function of x for $x > 0$ and a non-negative weighted sum of convex functions is also convex [36]. Therefore, the Karush–Kuhn–Tucker (KKT) conditions [36] applied on the expression for W_P are necessary and sufficient yielding the following rule for $\tau_i, 1 \leq i \leq N$ for the minimization of W_P:

$$\tau_i \propto \sqrt{\frac{w_i}{\mu_i}}, \quad 1 \leq i \leq N, \tag{13}$$

or equivalently,

$$\tau_i = \frac{\sqrt{w_i/\mu_i}}{\sum_{j=1}^{N} \sqrt{w_j/\mu_j}}. \tag{14}$$

Using (14), the weighted total throughput W_R in units of packets/s. when the weighted sum peak AoI W_P is minimized, denoted by W_R^{WSP}, is given by the following analytical expression:

$$W_R^{WSP} = \sum_{i=1}^{N} w_i \frac{R_i^{WSP}}{L} = \frac{\sum_{i=1}^{N} w_i \sqrt{\mu_i w_i}}{\sum_{j=1}^{N} \sqrt{w_j/\mu_j}}, \tag{15}$$

where R_i^{WSP} is the throughput of the source s_i in units of bits/s, when W_P is minimized in line with (14). Above, the term WSP is used to represent the method by which the weighted sum peak AoI is minimized. On the other hand, the minimum value of W_P, denoted by W_P^{WSP}, is given by the following expression by using the Eqs. (14) and (10):

$$W_P^{WSP} = \sum_{i=1}^{N} w_i/\mu_i + \left(\sum_{i=1}^{N} \sqrt{w_i/\mu_i} \right)^2. \tag{16}$$

The above identities are obtained when the mean service times are fixed and a-priori known. In order to obtain a learning algorithm for the case when the mean service times are not known in advance or they may be allowed to vary in time, we propose to use the following additional variable p_i which stands for the long-term fraction of packets scheduled for transmission for source s_i. It is clear that the following relation holds that tie τ_i to $p_j, j = 1, \ldots, N$:

$$\tau_i = \frac{p_i/\mu_i}{\sum_{j=1}^{N} p_j/\mu_j}. \tag{17}$$

Consequently, one can rewrite the expression (13) as

$$\tau_i \propto \frac{w_i}{p_i}, \quad 1 \leq i \leq N. \tag{18}$$

Note the dissimilarity between the optimality condition (5) for WTF and the condition (18) for weighted peak AoI minimization since the additional denominator p_i at the right hand side of the expression in (18) is needed for the latter. A learning-based algorithm for achieving (18) is provided in Algorithm 2, namely Weighted Sum Peak AoI Scheduler (WSPS). The difference of WSPS from WTFS is that we also estimate the quantities p_i by the variables Z_i using again the Robbins–Monro approximation with fixed learning rate σ and properly use it in the bucket updates so as to satisfy (18). Therefore, WSPS is to be implemented by means of a slight modification to the WTFS scheduling algorithm.

Algorithm 2 Pseudo-code for WSPS

Require: α, β, σ;
  $B_i \leftarrow 0$, $k \leftarrow 0$, $Z_i \leftarrow 1/N$;
  while (1) **do**
    $k \leftarrow k+1$;
    $I \leftarrow \arg\min_i \left((1-\beta)B_i + \beta \frac{S_i^k}{V_i} \right)$;
    Schedule source I packet;
    $Z_I \leftarrow Z_I + \sigma(1 - Z_I)$;
    $Z_i \leftarrow (1-\sigma)Z_i \; i \neq I$;
    $U_I \leftarrow U_I + \alpha(S_I^k - U_I)$;
    $V_i \leftarrow V_i + \alpha(S_i^k - V_i), \; 1 \leq i \leq N$;
    $B_I \leftarrow B_I + U_I$;
    $B_i \leftarrow B_i - \frac{w_i/Z_i}{\sum_j w_j/Z_j} U_I, \; 1 \leq i \leq N$;
  end while

6. Numerical examples

In the first subsection, only closed-form expressions will be employed to compare the weighted throughput W_R and weighted sum peak AoI W_P for the two cases (i) WTF is achieved (ii) weighted sum peak AoI is minimized, i.e., termed as WSP. Sections 6.2 and 6.3 evaluate the two scheduling algorithm WTFS and WSPS in terms of W_A and W_P using simulations for the non-opportunistic and opportunistic scenarios, respectively.

6.1. Analytical results

In this example, we fix the number of users N to 100. Five classes of users are assumed where classes 1 to 5 use the modulation schemes BPSK (Binary Phase Shift Keying), QPSK (Quadrature Phase Shift Keying), 16 QAM (Quadrature Amplitude Modulation), 64 QAM, and 256 QAM, respectively [37]. Fixing the time unit to the mean service time of a BPSK user, users belonging to class ℓ, $1 \leq \ell \leq N$, are assumed to have service rates of 1, 2, 4, 6, and 8, respectively. Users are assigned to classes according to a Zipf distribution with exponent parameter $\eta \geq 0$, i.e., the probability that a user belongs to class ℓ is inversely proportional with ℓ^η which reduces to the uniform distribution when $\eta = 0$ whereas when $\eta \rightarrow \infty$, all the users tend to be BPSK users. Therefore, the Zipf exponent η is indicative of how diverse the service times are. Two different scenarios are studied; In scenario A, all the source weights are identical whereas in scenario B, the users from 1 to 50 have a weight 10 times larger than the users from 51 to 100. For a given exponent η, we generate $M = 50\,000$ problem instances for each of which we employ the closed-form expressions (6) and (15) for weighted throughput W_R and the expressions (11) and (16) for the weighted sum peak AoI W_P, when WTF is achieved, and when weighted sum peak AoI is minimized, respectively, and their average values over the M instances are depicted in Fig. 3 as a function of the parameter η. We observe that the weighted throughput is larger with WTF than WPS but on the other hand, the weighted sum peak AoI is significantly lower with WPS than WTF especially when $\eta \rightarrow 0$, i.e., there is heterogeneity in the source service times, and in Scenario B when the source weights are not identical. For both scenarios, W_P values are close to each

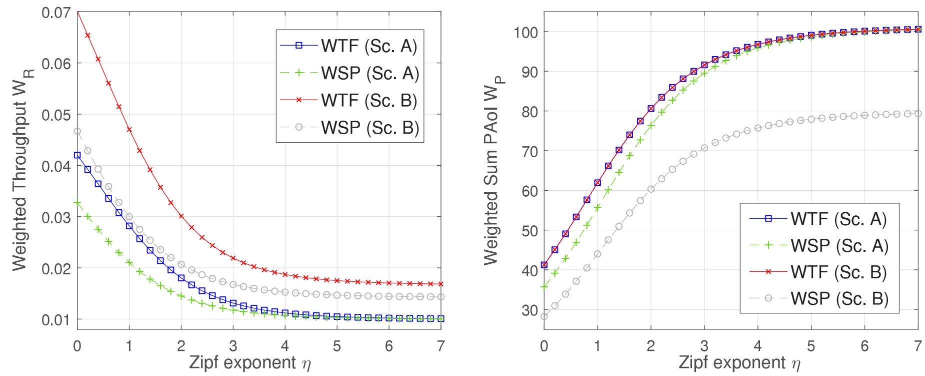

Fig. 3. The performance metrics (a) W_R (b) W_P obtained with analytical expressions for the two weighting scenarios A and B when (i) WTF is achieved (ii) weighted sum peak AoI is minimized.

other with WTF which is evident from the expression (11) since on the average, the number of users in a given class with the low weighting factor is the same as with the high weighting factor. We also note for Scenario A that when $\eta \to \infty$, all the users tend to be of BPSK type and therefore they have the same mean service times giving rise to identical throughput and mean PAoI performances for WTF and WPS in this asymptotic regime.

6.2. Simulation results for the non-opportunistic network

In the remaining numerical examples, simulation results for the two scheduling algorithms will be presented. In the non-opportunistic scenario, the information about source transmission rates, or equivalently, packet service times are not available at the scheduling instants. Therefore, the parameter β needs to be set to zero. In the first example, we study the role of the parameter α on the performance of WTFS for an N-source system with equal weights, i.e., $w_i = 1/N, 1 \leq i \leq N$. Linearly-spaced exponentially distributed source service times are assumed such that $\mathbb{E}[S_i] = \mathbb{E}[S_{i-1}] + \delta, 1 < i \leq N, \delta \geq 0$, in a way that the ratio of the largest mean service time to the smallest, i.e., $\frac{\mathbb{E}[S_N]}{\mathbb{E}[S_1]}$, is set to a given ratio r. A unit average of service times across the N sources is assumed, i.e., $\frac{1}{N}\sum_{i=1}^{N}\mathbb{E}[S_i] = 1$. For this purpose, the choice of $\mathbb{E}[S_1] = \frac{2}{r+1}$ and $\delta = \frac{2(1-\mathbb{E}[S_1])}{N-1}$ completes the network construction, which we call the Linearly Spaced Service Times (LSST) traffic model with parameters N and r, the latter parameter being instrumental in determining the dynamic range of the service times used in the network. In this paper, we run simulations with 5×10^7 scheduling instants and U_i values are initialized to 1 for all i for all examples. The performance metrics W_P and W_A are plotted as a function of the smoothing parameter α in Fig. 4 for $r = 100$ and for three values of the number of sources $N \in \{10, 100, 1000\}$. We observe that W_P does not depend on the smoothing parameter α, as can be explained from (11) which yields $W_P^{WTF} = N + 1$ irrespective of the parameter α. However, W_A increases with increased α. Therefore, a sufficiently smoothed estimate of the service times needs to be used in the WTFS algorithm in order not to reduce the performance in terms of W_A. Throughout the paper, the smoothing parameter α will be set to 0.01.

In the second numerical example, we use the same LSST traffic model of the previous example but the ratio parameter r is varied for two values of $N \in \{10, 100\}$ while employing both WTFS and WSPS and the source weights are chosen to satisfy $w_i \propto \mu_i$. We allow the use of an arbitrary squared coefficient of variation for service times as follows. When c_i^2 is the reciprocal of a positive integer j, then the Erlang distribution with order j is used. When $c_i^2 > 1$, then a hyper-exponential distribution with balanced means is used according to the closed-form expressions given in [38]. The case of $c_i^2 = 1$ reduces to the exponential distribution. In the current example, c_i^2 is set to 4 for all the sources. The smoothing parameters α and σ are set to 0.01 for this example. In Fig. 5, the performance metrics W_P and W_A obtained with WTFS and WSPS, are depicted as a function of the ratio parameter r. When the ratio parameter is close to unity, i.e., mean source service times are close to each other, then the performances of WSPS and WTFS are similar for W_P which can be explained by the expression (13) and also for W_A. On the other hand, when r increases, the gap between WSPS and WTFS grows when weighted sum PAoI is taken as the performance metric of interest. We have observed that WSPS outperformed WTFS in terms of W_A for all values of the ratio parameter r and the number of sources N studied in this example. However, the gap between the two schedulers for W_A is not as substantial as in the case of W_P, and moreover, the gap appears to shrink as r increases.

6.3. Simulation results for the opportunistic network

In this subsection, the opportunistic network scenario is considered employing the LSST traffic model with the ratio parameter $r = 100$ and the number of sources $N = 20$. The squared coefficient of variation of all sources are taken to be identical, i.e., $c_i^2 = c^2, 1 \leq i \leq N$. The performance metrics W_P and W_A obtained with WTFS and WSPS, are depicted as a function of the opportunism parameter β for three values of the squared coefficient of variation $c^2 \in \{1/5, 1, 5\}$ in Fig. 6. We have the following observations:

- When $\beta = 1$, the so-called greedy policy, in both WTFS and WSPS, the scheduler chooses to schedule the source with the shortest service time relative to its smoothed estimate. Therefore, the conditions (5) and (18) are not satisfied for WTFS and WSPS, respectively, for the greedy policy.
- In terms of W_P and W_A, WSPS outperforms WTFS for all values of β and the greedy policy with a suitable choice of $\beta < 1$. On the other hand, WTFS is outperformed by the greedy policy for all values of $\beta < 1$ for the two larger values of c^2.
- W_P monotonically decreases with respect to β when $\beta < 1$ for both WTFS and WSPS but the reduction in W_P is more apparent when c^2 is higher which can be explained by the observation that the variability in service times is advantageous for opportunistic scheduling. Note that as $c^2 \to 0$, service times tend to be deterministic and having prior information about service times does not present much value.
- W_A first decreases with increased β for WTFS and then starts to increase once a threshold is exceeded. This behavior is also observed for WSPS when $c^2 = 1/5$ but not for two larger values of c^2 studied in this paper.
- For the minimization of W_P using WSPS, one should use an aggressive opportunism parameter β close to unity but avoiding the greedy policy, i.e., $\beta = 1$. On the other hand, the use of an aggressive β leads to increased W_A for relatively smaller c^2. Therefore, it is crucial to use a proper value of β if one would be interested in controlling W_A as well.

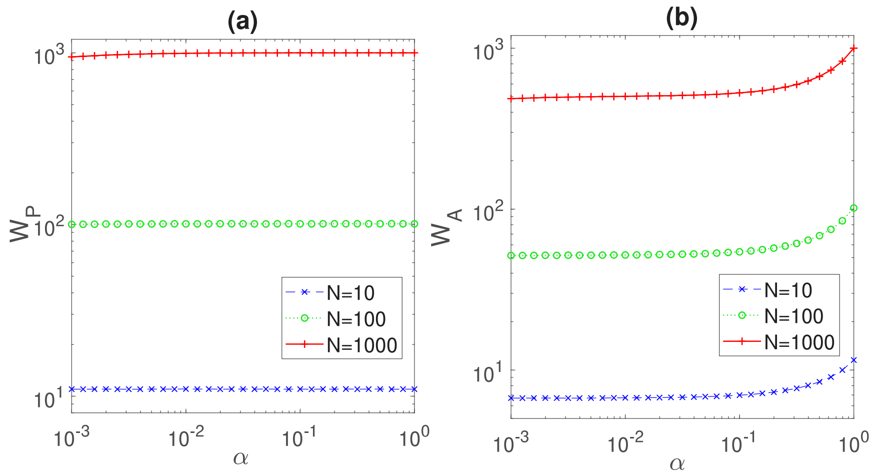

Fig. 4. The performance metrics (a) W_P (b) W_A, obtained with WTFS as a function of the smoothing parameter α for $r = 100$ and for three values of the number of sources N.

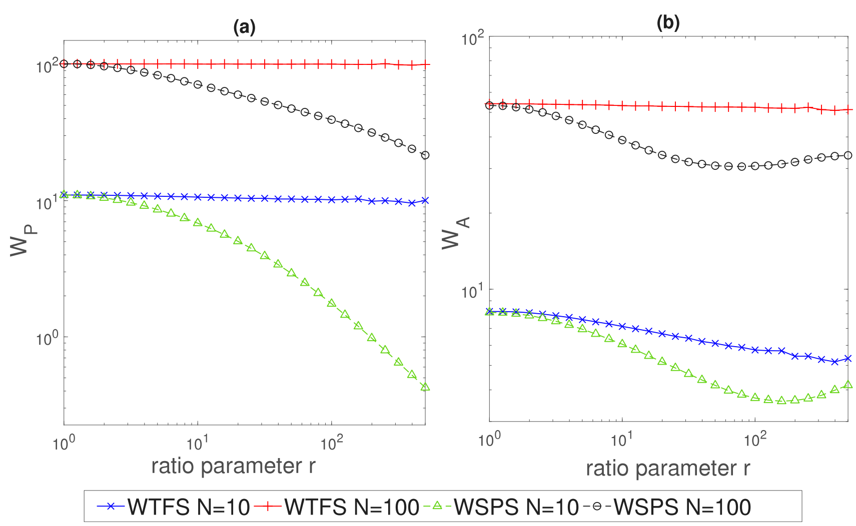

Fig. 5. The performance metrics (a) W_P (b) W_A obtained with WTFS and WSPS, as a function of the ratio parameter r for two values of the number of sources N.

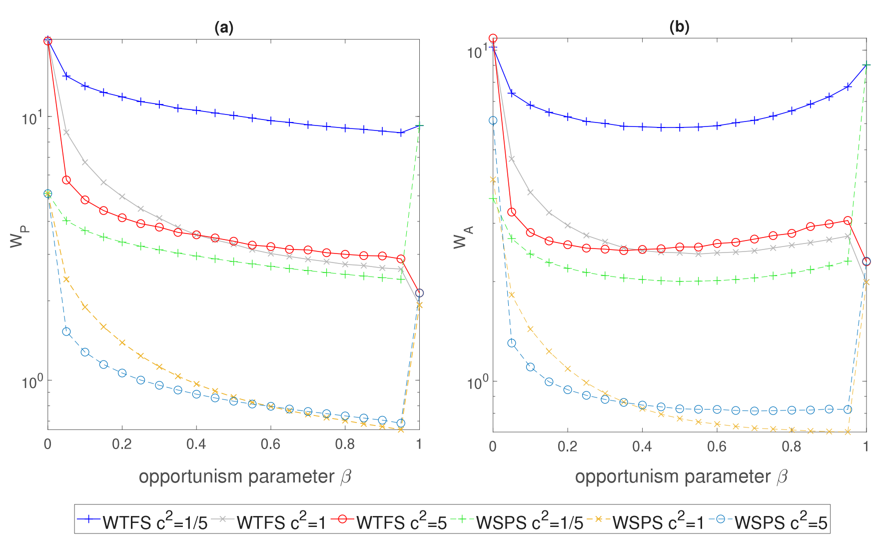

Fig. 6. The performance metrics (a) W_P (b) W_A obtained with WTFS and WSPS, as a function of the opportunism parameter β for three values of the squared coefficient of variation c^2.

7. Conclusions

A model-free learning-based scheduler named WSPS is proposed to minimize the weighted sum peak AoI for age-sensitive traffic for cellular wireless downlinks and it is compared and contrasted with WTFS, whose variations are currently deployed in wireless networks. It is concluded that proportional fair scheduling algorithms such as WTFS are not as effective in fulfilling the AoI and peak AoI requirements of age-sensitive traffic. It is also shown that age-sensitive traffic can effectively be carried in wireless networks using the proposed WSPS algorithm which is very similar to WTFS in terms of implementation and complexity. For future work, model-free learning-based scheduling algorithms are needed for (i) minimizing directly the weighted sum AoI (and not the peak AoI only), (ii) handling random arrivals (in addition to generate-at-will scenario), (iii) coping with scenarios involving a mixture of age-sensitive and conventional data traffic. AoI control in opportunistic networks is another area requiring novel scheduling algorithms.

CRediT authorship contribution statement

Nail Akar: Conceptualization, Design, Analysis, Writing – original draft, Writing – review & editing. **Ezhan Karasan:** Conceptualization, Design, Analysis, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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