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Radio Communications Interdiction Problem under deterministic and probabilistic jamming

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ABSTRACT

The Radio Communications Interdiction Problem (RCIP) seeks to identify the locations of transmitters on the battlefield that will lead to a robust radio communications network by anticipating the effects of intentional radio jamming attacks used by an adversary during electronic warfare. RCIP is a sequential game defined between two opponents that target each other's military units in a conventional warfare. First, a defender locates a limited number of transmitters on the defender's side of the battlefield to optimize the relay of information among its units. After observing the locations of radio transmitters, an attacker locates a limited number of radio jammers on the attacker's side to disrupt the communication network of the defender. We formulate RCIP as a binary bilevel (max–min) programming problem, present the equivalent single level formulation, and propose an exact solution method using a decomposition scheme. We enhance the performance of the algorithm by utilizing dominance relations, preprocessing, and initial starting heuristics. To reflect a more realistic jamming representation, we also introduce the probabilistic version of RCIP where a jamming probability is associated at each receiver site as a function of the prevalent jamming to signal ratios leading to an expected coverage of receivers as an objective function. We approximate the nonlinearity in the jamming probability function using a piecewise linear convex function and solve this version by adapting the decomposition algorithm constructed for RCIP. Our extensive computational results on realistic scenarios show the efficacy of the solution approaches and provide valuable tactical insights.

1. Introduction

Military communications (Beidel et al., 2011; Ryan and Frater, 2002) is a high-value target for intentional electronic attacks aiming to disrupt command and control. Radio jamming through deliberate radiation of electromagnetic energy is a commonly used form of electronic attack. Although several techniques and strategies can be used in jamming, in its basic form, an interfering jamming signal is added into the opponent's receiver to override any other communication signal at the receiver (Adamy, 2001). Vadlamani et al. (2016), Prasad and Thuente (2011), and Mpitziopoulos et al. (2009) present detailed overviews of various types of jammers and commonly used jamming techniques and strategies.

Any military communication system can be analyzed in terms of communication links between radiation sources such as transmitters and jammers, and receiving devices (Adamy, 2001). The source's antenna gain increases the power level of the signal prior to leaving the source. As the signal propagates to the receiver, its power attenuates with distance due to various factors. This power fall is commonly modeled by the path loss exponent rate, which is a function of reflectors, scatters and obstructions in the environment. Aragon-Zavala (2008) states that the value of the path loss exponent rate ranges from 2 to 5 (where 2 is for propagation in free space and 5 is for relatively rough and mountainous areas). Upon arrival at the receiver, the power of the residual signal is increased by the receiver's antenna gain. Finally, communication takes place on this link only if the resulting received power level is greater than the receiver sensitivity threshold value, which denotes the smallest signal power needed for proper reception (Adamy, 2001).

Signal corps is the military branch solely responsible for planning a tactical communication system that provides continuous

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communication service to widely dispersed mobile receivers operating at extended distances. To avoid the adverse effects of electronic attacks, particularly radio jamming, the planners must design a robust communication network with respect to electronic protection measures. Vital decisions in such designs are the locations of transmitters since they regulate the power of electromagnetic transmission and signal level on each receiver in the communication network. Whitaker and Hurley (2004) and Chapman et al. (1999) emphasize that building an effective and efficient radio communication network that can maintain the minimum level of desired signal on each receiver depends mainly on the locations of the transmitters.

In order to design a robust battlefield communication network for a tactical military unit involved in a battle, we propose a game theoretic approach for locating a given number of transmitters that aims to mitigate the adverse effects of radio jamming. Specifically, we solve the Radio Communications Interdiction Problem (RCIP), which determines the locations of a given set of transmitters in order to maximize the worst case coverage of receivers by anticipating the disruptive effects of radio jamming that may be imposed by an intelligent adversary. We formulate RCIP as a binary bilevel programing problem, present the equivalent single level formulation, and solve the problem by a decomposition-based exact solution method. Though similar defender–attacker and interdiction models have been studied in the literature as will be detailed in Section 2, our problem has characteristics pertinent to the military scope that distinguish it from those in the literature. RCIP considers the deterministic case where given the jamming to signal ratio, the knowledge of a receiver being jammed is known with certainty. To provide a more realistic framework, we formulate probabilistic RCIP (P-RCIP) where we associate jamming probabilities as functions of jamming to signal ratios prevalent at each receiver site. In this fashion, the deviation of the electromagnetic signal that may be caused by obstacles affecting the wave propagation is incorporated and the objective function turns into an expected coverage one. Based on real-world scenarios, we discuss the insights and the implications of these models.

The rest of the paper is organized as follows. Section 2 reviews the RCIP related literature and highlights the contributions of the current paper. In Section 3, RCIP is formally defined. Sections 4 and 5 provide the mathematical models and proposed solution methods for RCIP and P-RCIP, respectively. In Section 6, we present our computational results and our tactical insights. Finally, Section 7 concludes the paper.

2. Related literature

A wide variety of research has been carried out on effectively locating transmitters in communication network designs. These studies consider various objectives such as (i) maximizing the total coverage or demand weighted coverage (Ahmed et al., 2012; Akella et al., 2005; Lee and Murray, 2010), (ii) minimizing the path loss of the signal (Ji et al., 2002; Kouhbor et al., 2006; Sherali et al., 1996), and (iii) a hybrid of multiple objectives involving the interference, the number of transmitters, and the energy consumption (Lakashminarasimman et al., 2010; Mathar and Niessen, 2000; Nebro et al., 2007; Whitaker and Hurley, 2004; Zimmermann et al., 2003). Alternatively, numerous optimization problems have been identified to increase the efficiency of radio jamming and hence disable the opponent's communication capability. Among these, Commander et al. (2008, 2007) and determine the minimum number of jammers and their locations to obtain the desired effect. Feng et al. (2014) consider the location of jammers that will partition the communication network into disconnected components. Vadlamani et al. (2014) find out not only the locations but also the channel hopping strategies in order to minimize the expected throughput of the opponent's communication network. Additionally, Vadlamani et al. (2018) deal with the location of jammers under flow-jamming attacks.

The above studies handle the problem unilaterally, either from the perspective of the communication network designer or the adversary that aims to disable the communication network. Shankar's study (Shankar, 2008) is the first attempt to formulate and solve a bilevel optimization problem in order to assess the attack and defense strategies of wireless mesh networks bilaterally. In the first stage, the attacker intentionally locates a limited number of jammers to disrupt the network in the worst possible way. The defender in the second stage, investigates the best strategy to optimize the flow of information after observing the location strategy of the attacker by solving the Simultaneous Routing and Resource Allocation (SRRA) problem of Xiao et al. (2004). Shankar solves moderately sized problem instances by enumerating all possible attacker strategies and devises several jammer location heuristics for larger instances. Different from Shankar's study, we design the transmitter locations and thus the communication network, maximize the number of receivers that are able to communicate rather than improve the flow of information in the given network and we incorporate the Jamming to Signal Ratio metric into our model rather than using the metric in the SRRA problem. Also, we manage to solve considerably larger instances to optimality within reasonable times. Medal (2016) also applies a game theoretic approach to identify the locations of a set of jammers that will induce the largest degradation in a given wireless network and determines the most effective strategies such as channel hopping to mitigate these jamming attacks. This study is one of the pioneering works that optimize network throughput under radio wave interference between transmitters. In our study, we ignore the radio interference effect since we assume that receivers belonging to different units communicate with transmitters by using different frequencies, which prevents the occurrence of interference. Additionally, we optimize and design the locations of the transmitters.

Nicholas and Alderson (2015) are the first to apply the tri-level game theoretic optimization framework to design wireless mesh network topologies that are robust to jamming. In this problem, the network designer as the defender locates the access points in the first stage; after observing the locations of the access points an intelligent adversary as an attacker identifies the jammer locations in the second stage; and finally at the third stage designer as the operator optimizes the value of the network by using the SRRA and Coverage problem (Nicholas and Alderson, 2012), in order to quantify the value of a particular wireless mesh network. This study is also the first to devise a solution algorithm that makes use of DIviding RECTangles sampling algorithm (Jones et al., 1993) to design an electromagnetic interference robust wireless mesh networks. The authors extend Shankar's work (Shankar, 2008) by considering a continuous space for jammer locations. In contrast, with our work we intend to cover non-uniformly distributed receivers by depending on deterministic and probabilistic Jamming to Signal Ratio criteria rather than covering the maximum terrain.

With its features pertinent to military context only, our study is a distinctive example of a defender–attacker type of problem that optimizes military radio communication systems on the battlefield under jamming attacks. We incorporate Jamming to Signal Ratio into a bilevel formulation to identify the location of transmitters that will yield a jamming robust radio communication network. We assume that the transmitters are connected to each other via a backbone network, possibly having a mesh topology. Since directional antennas with very large gains are used between fixed transmitters, this backbone network is robust against jamming and thus the jamming effect in this backbone network is ignored in this paper. Different from the previous works, we do not deal with the flow of information but the coverage of the receivers since, as argued above, the flow of information is enabled whenever the receivers are covered.

Contrary to the mentioned works that consider locating facilities under deterministic conditions, Daskin (1983) and Batta et al. (1989) maximize the expected coverage by considering the probability that a facility may not be able to serve a demand point. Similarly, Patel et al. (2005) determines locations of sensors over a time horizon to maximize the expected coverage of data by considering the probability of a link failure. In a similar fashion, to bring more realism to our problem, we consider the probability that a receiver is not able to communicate due to the deviation in the received signal power because of fading caused by different variables such as geometric spread, atmospheric absorption, radio frequency, and geographical obstacles on the battlefield. We define the Probabilistic Jamming to Signal Ratio which incorporates the randomness in the jamming to signal ratio and introduce and formulate the probabilistic version of RCIP, namely P-RCIP that maximizes the expected coverage of receivers. After approximating the jamming probability function as a piecewise linear convex function, we manage to adapt the decomposition approach for RCIP to solve P-RCIP efficiently.

Many researchers apply game theoretic mathematical modeling approaches to identify system vulnerabilities and/or to create protection plans for critical infrastructure such as radio communications against intentional attacks of an intelligent adversary (e.g. Brown et al., 2006; Morton et al., 2007; Scaparra and Church, 2008; Starita and Scaparra, 2016). Wood (2011) states that such a model, as being a sequential game of non-cooperative players, can be formulated as a type of Stackelberg game: a two-person, zero-sum, sequential-play game with two stages (Simaan and Cruz, 1973; Stackelberg, 1952). Stackelberg games are modeled as bilevel programming problems (BPPs) (Dempe, 2002), which are generally difficult to solve with even the linear form being NP-Hard (Ben-Ayed, 1993). Detailed information for existing solution methods for BPPs can be found in surveys by Labbé and Violin (2013), Colson et al. (2007), Dempe (2003) and in textbooks by Dempe (2002) and Bard (2013). BPPs having integer variables only in the first stage or having a totally unimodular constraint matrix in the second stage problem are generally solved by taking the dual of the second stage problem and solving the resulting single level formulation (Brown et al., 2011, 2005; Medal, 2016; Wood, 1993). Difficulties encountered while solving BPPs with integer decision variables enforce researchers to introduce solution methods that are tailored to the specific bilevel structure of their problems. The general trend is to reformulate the bilevel model as a single level model and solve with appropriate methods typically involving decomposition (Alekseeva et al., 2010; Brown et al., 2009; Church and Scaparra, 2007; Nandi et al., 2016; Pessoa et al., 2013; Roboredo and Pessoa, 2013). Some researchers enhance the decomposition method by adding super valid inequalities to the master problems (Israeli and Wood, 2002; OHanley and Church, 2011; Starita and Scaparra, 2016). Implicit enumeration methods that make use of some problem specific observations are also common (Aksen et al., 2010; Bard and Moore, 1992; Mahmutogullari and Kara, 2016; Moore and Bard, 1990; Scaparra and Church, 2008). In addition to exact approaches, heuristic methods are also frequently used to find quick solutions to BPPs with integer decision variables (Berman et al., 2009; Iellamo et al., 2015; Konak et al., 2015).

We adapt the decomposition algorithms used in Alekseeva et al. (2010) and Israeli and Wood (2002) in different contexts to solve deterministic and probabilistic versions of RCIP in an iterative manner. The bilevel programming problem is decomposed into an upper level master problem and a lower level subproblem. At each iteration, (i) we solve the master problem that yields a transmitter location strategy, (ii) solve the subproblem to identify the best jamming strategy as a response, and (iii) update the master problem by adding valid inequalities that are generated from the solution of the subproblem. The algorithm terminates when upper and lower bounds become equal. To speed up the solution process, we propose three types of enhancements. The efficacy of the solution approach is tested on large scale instances spanning different scenarios that reflect the possible situations of a military operation. Furthermore, sensitivity analyses for different problem parameters are conducted based on these instances. Finally, we analyze the optimal transmitter locations to provide tactical insights from a commander's perspective.

3. Radio Communications Interdiction Problem: problem definition

RCIP is based on a military conflict between two opposing forces, namely Defender (DF) and Attacker (AT). Both sides are composed of military units that are equipped and deployed on the battlefield according to their respective organizational structure and tactics.

DF aims to establish a reliable tactical radio communications system among all tactical units. These tactical units are assumed to be the smallest maneuver units that have a military radio in their vehicles (e.g. tanks, armored personnel carriers etc.) or the smallest combat support/combat service support units that have a military radio in their organizational structure. The set $\mathcal{R}$ standing for receivers indicates the locations of these radios. All receivers are assumed to be identical with a receiver sensitivity threshold value γ, i.e., the minimum received power for successful reception. DF is assumed to have a limited number of (p) transmitters each radiating a signal with a specific power level and a specific antenna gain. Signal corps determines the possible transmitter location sites by evaluating the geographical characteristics of the area of operation either by making a reconnaissance on the terrain or using a digital or printed map and considering the locations of all tactical units. We refer to this set of potential transmitter locations as $\mathcal{T}$. DF concludes the military decision-making process by selecting the locations of p transmitters from $\mathcal{T}$.

AT, the other side of the military conflict, aims to conduct an interdiction operation in order to interrupt or impede the flow of information and operational tempo (U.S. Joint Chief of Staff, 2016). For this purpose, AT has a limited number of (q) radio jammers with associated power levels and antenna gains. The objective is to locate q radio jammers so as to maximize the number of jammed receivers by conducting intentional jamming attacks. To achieve this objective, AT first identifies possible jammer location sites $\mathcal{J}$, and later, after observing the locations of DF's tactical units and p transmitters, locates q radio jammers among these $\mathcal{J}$ sites.

Whether a receiver is jammed or not is determined by the Jamming to Signal Ratio (JSR), which basically denotes the ratio of the received jamming signal power to the received communications signal power at the receiver. To formally define JSR, let $P_t (P_j)$ in Watt and $G_t (G_j)$ in dB denote the power level and antenna gain of transmitter $t \in \mathcal{T}$ (jammer $j \in \mathcal{J}$), respectively, and let G_r in dB denote the receiver antenna gain for receiver $r \in \mathcal{R}$. Let $\mathcal{T}_p$ be a subset of p transmitters from set $\mathcal{T}$ and $\mathcal{J}_q$ be a subset of q jammers from set $\mathcal{J}$. Schleher (1999) and Shankar (2008) define the jamming to signal ratio for receiver $r \in \mathcal{R}$, say JSR_r, as the ratio of the sum of all individual undesired signal powers to the maximum of desired signal powers. More formally,

$$JSR_r = \frac{\sum_{j \in \mathcal{J}_q} P_j G_j G_r \frac{1}{d_{jr}^{\beta}}}{\max_{t \in \mathcal{T}_p} P_t G_t G_r \frac{1}{d_{tr}^{\alpha}}} \quad (1)$$

where $d_{tr} (d_{jr})$ is the Euclidean distance between the transmitter (jammer) and the receiver in kilometers and $\alpha (\beta)$ is the path loss exponent rate which defines the reduction in signal power attenuation of transmitter's (jammer's) electromagnetic wave as it propagates through space.

Even though the desired signal power at receiver $r \in \mathcal{R}$ of transmitter $t \in \mathcal{T}$ is defined as $P_t G_t G_r \frac{1}{d_{tr}^{\alpha}}$, the received electromagnetic signal power may be affected by the geographical obstacles in the battlefield. To accommodate the random variations in the received signal due to a random number and type of obstructions, a random attenuation term X, in dB scale, is added to the path loss, which is called shadow fading. Variable X typically follows a normal distribution (Rappaport, 2002), i.e., $X \sim N(0, \sigma^2)$. Converting back to the power scale, the received power is multiplied by the random variable $\mathcal{S} = 10^{\frac{X}{10}}$, which has a lognormal distribution. Note that as the received power fluctuates around its average value, $\mathcal{S}$ may not be equal to one. The same shadowing effect also occurs over the channels between the jammers and receivers. However, since the received jammer power is the summation of multiple signals from the jammers, it has a smaller variance due to the central limit theorem. Therefore, we ignore the shadowing that occurs over the jammer received power and define the probabilistic jamming to signal ratio as:

$$PJSR_r = \frac{\sum_{j \in \mathcal{J}_q} P_j G_j G_r \frac{1}{d_{jr}^{\beta}}}{\max_{t \in \mathcal{T}_p} P_t G_t G_r \frac{1}{d_{tr}^{\alpha}} \ \mathcal{S}} \tag{2}$$

To this end, RCIP (P-RCIP) is a sequential game in which DF takes the first step and locates p transmitters. Thereafter, observing the locations of the transmitters, AT locates q radio jammers. The overall purpose of RCIP (P-RCIP) is to determine the optimal locations of DF's transmitters in order to maximize the total (expected) number of receivers that will be able to communicate even after AT's intentional jamming attacks are executed by optimally located radio jammers.

4. RCIP: deterministic approach

We formulate RCIP as a BPP using the following notation.

Sets:

$\mathcal{T} = \{t_1, \ldots, t_T\}$ potential location sites for transmitters
$\mathcal{J} = \{j_1, \ldots, j_J\}$ potential location sites for jammers
$\mathcal{R} = \{1, \ldots, R\}$ location sites of receivers on the battlefield

Parameters:

d_{kr} distance between site $k \in \mathcal{T} \cup \mathcal{J}$ and $r \in \mathcal{R}$ (km)
α path loss exponent for DF's transmitters
β path loss exponent for AT's jammers
P_k transmitting power of transmitter/jammer k located at $\mathcal{T} \cup \mathcal{J}$ (Watt)
G_k antenna gain of transmitter/jammer/receiver k located at $\mathcal{T} \cup \mathcal{J} \cup \mathcal{R}$ (dB)
ε threshold value for JSR (dB)
γ receiver sensitivity (dBm)
p maximum number of transmitters to be located
q maximum number of jammers to be located

Decision variables:

$$x_t = \begin{cases} 1 \text{ if a transmitter is located on transmitter site } & t \in \mathcal{T} \\ 0 \text{ otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 \text{ if a jammer is located on jammer site } & j \in \mathcal{J} \\ 0 \text{ otherwise} \end{cases}$$

$$w_r = \begin{cases} 1 \text{ if the power of desired signal at receiver } & r \in \mathcal{R} \text{ is} \\ \quad \text{at least receiver sensitivity} \\ 0 \text{ otherwise} \end{cases}$$

$$z_r = \begin{cases} 1 & \text{if receiver } r \in \mathcal{R} \text{ is communicating} \\ 0 & \text{otherwise} \end{cases}$$

Without loss of generality, we assume that all transmitters and jammers are identical among themselves and all receivers have omnidirectional antennas with the same antenna gain. Let $\lambda = (P_{j_1} G_{j_1})/(P_{t_1} G_{t_1})$ where j_1 is the first jammer location site and t_1 is the first transmitter location site. Given the location plans $x \in \{0, 1\}^T$ and $y \in \{0, 1\}^J$, $JSR_r(x, y)$ is the jamming to signal ratio at receiver $r \in \mathcal{R}$, and is given as

$$JSR_r(x, y) = \lambda \frac{\sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max_{t \in \mathcal{T}} \ \frac{1}{d_{tr}^{\alpha}} x_t}. \tag{3}$$

For each $r \in \mathcal{R}$, let $\mathcal{T}(r) = \{t \in \mathcal{T} \mid P_t G_t G_r \frac{1}{d_{tr}^{\alpha}} \geq \gamma\}$ denote the potential transmitter locations that can communicate with receiver r.

A receiver $r \in \mathcal{R}$ is assumed to be jammed if $JSR_r(x, y) \geq \varepsilon$; see Iellamo et al. (2015). On the other hand, for a receiver to be deemed communicating, not only $JSR_r(x, y) < \varepsilon$ should hold but there should also exist a transmitter located within its communication range, i.e., $\exists t \in \mathcal{T}(r)$ such that $x_t = 1$.

The mathematical formulation of RCIP then becomes the following.

$$W^* = \max \quad \tau(x) \tag{4}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}} x_t \leq p \tag{5}$$

$$x_t \in \{0, 1\} \qquad t \in \mathcal{T} \tag{6}$$

where

$$\tau(x) = \min \sum_{r \in \mathcal{R}} z_r \tag{7}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}(r)} x_t \leq w_r \ p \qquad r \in \mathcal{R} \tag{8}$$

$$z_r + \frac{\lambda}{\varepsilon} \frac{\sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max_{t \in \mathcal{T}(r)} \ \frac{1}{d_{tr}^{\alpha}} x_t} \geq w_r \qquad r \in \mathcal{R} \tag{9}$$

$$\sum_{j \in \mathcal{J}} y_j \leq q \tag{10}$$

$$y_j \in \{0, 1\} \qquad j \in \mathcal{J} \tag{11}$$

$$z_r, w_r \in \{0, 1\} \qquad r \in \mathcal{R} \tag{12}$$

The above bilevel formulation (4)–(12) is composed of the upper level DF's problem (4)–(6) and the lower level AT's problem (7)–(12). DF locates at most p transmitters (constraints (5) and (6)) so as to maximize the number of receivers that are able to communicate with these transmitters hedging against the best location decisions of AT. For a given set of transmitter locations, AT in turn solves model (7)–(12) and locates at most q jammers (constraints (10) and (11)) in order to minimize the number of communicating receivers of DF (objective (7)). Note that once the x values are fixed, constraints (9) become linear. For a given receiver $r \in \mathcal{R}$, if one of the locations in $\mathcal{T}(r)$ has a transmitter, constraints (8) will force $w_r = 1$. If $w_r = 1$ and $JSR_r(x, y) < \varepsilon$, then constraints (9) will force $z_r = 1$, i.e., if there is a close transmitter and the JSR is low, then receiver r will communicate. On the other hand, if $x_t = 0 \ \forall t \in \mathcal{T}(r)$, then w_r may take value 0 or 1 through constraints (8). However, through constraints (9) and the objective function (7), one can deduce that there exists an optimal solution with $w_r = 0$. In other words, without loss of generality, one may assume that $w_r = \lceil \frac{\sum_{t \in \mathcal{T}(r)} x_t}{p} \rceil$ and these auxiliary w variables simply indicate whether any transmitter in set $\mathcal{T}(r)$ is located or not.

4.1. Solving RCIP using decomposition

To solve RCIP, we present an equivalent single level formulation and propose an exact solution method that decomposes the single level formulation into a master problem and a subproblem. The master problem and the subproblem provide upper and lower bounds, respectively. We solve each problem sequentially until the lower and upper bounds coincide. A similar approach under a different context is used by Alekseeva et al. (2010).

Let $\mathcal{Y} = \{y \in \{0,1\}^J \mid \sum_{j\in\mathcal{J}} y_j \le q\}$ represent all possible AT strategies. For each receiver $r \in \mathcal{R}$, we introduce a new decision variable s_{ry}, which is defined as follows.

$$s_{ry} = \begin{cases} 1 & \text{if receiver } r \in \mathcal{R} \text{ is able to communicate when} \\ & \text{AT's strategy is } y \in \mathcal{Y} \\ 0 & \text{otherwise.} \end{cases}$$

With the addition of an exponential number of such decision variables and an exponential number of constraints, we may reformulate RCIP as the following linear mixed integer programming (MIP) problem, say $MP(\mathcal{Y})$, to stand for the master problem.

$$MP(\mathcal{Y}) \qquad \theta_{MP}(\mathcal{Y}) = \max \quad \omega \tag{13}$$

$$\text{s.t.} \quad \omega \le \sum_{r\in\mathcal{R}} s_{ry} \qquad y \in \mathcal{Y} \tag{14}$$

$$s_{ry} \le \sum_{t\in\mathcal{T}(r,y)} x_t \qquad r \in \mathcal{R},\ y \in \mathcal{Y} \tag{15}$$

$$\sum_{t\in\mathcal{T}} x_t \le p \tag{16}$$

$$x_t \in \{0,1\} \qquad t \in \mathcal{T} \tag{17}$$

$$0 \le s_{ry} \le 1 \qquad r \in \mathcal{R},\ y \in \mathcal{Y} \tag{18}$$

In this model, ω is an auxiliary variable that will correspond to the number of communicating receivers when hedging against all possible AT strategies. Set $\mathcal{T}(r,y)$ represents the transmitter location sites that will enable the communication of receiver $r \in \mathcal{R}$ when AT's strategy is y, i.e., $\mathcal{T}(r,y) = \{t \in \mathcal{T}(r) \mid \lambda\ d_{tr}^{\alpha} / \sum_{j\in\mathcal{J}} d_{jr}^{\beta} y_j < \varepsilon\}$. Constraints (15) enforce one such transmitter to be located when s_{ry} variable takes value one. Through constraints (14), (18) and the objective function, the auxiliary variable ω will be equal to the minimum number of receivers that will be communicating when considering all possible AT strategies. Constraint (16) limits the number of transmitters to be located by p. Constraints (17) are domain restrictions for x_t variables. Note that constraints (18) relax the binary requirements of s_{ry} variables since once the transmitter location variables take integer values, the objective function and constraints (15) imply the integrality of these variables.

Set $\mathcal{Y}$ has $\binom{J}{q}$ elements and as such $MP(\mathcal{Y})$ is a huge model to solve directly. To this end, we propose a decomposition approach for its solution. At every iteration, we shall solve this master problem with only a subset of AT strategies, say with $Y \subseteq \mathcal{Y}$. Then, $MP(\mathcal{Y})$ restricted to only the strategies $y \in Y$, i.e. $MP(Y)$, constitutes the relaxed master problem. Its optimal solution will provide an upper bound (UB) for $RCIP$. Let $\hat{x}$ be the optimal solution of the relaxed master problem $MP(Y)$. In order to generate new AT strategies to include in the relaxed master problem, we identify AT's optimal response to $\hat{x}$ by solving model (7)–(12) when $x = \hat{x}$ and the auxiliary w variables are eliminated as discussed. In other words, we solve the following equivalent subproblem $SP(\hat{x})$ where $\hat{R} = \{r \in \mathcal{R} : \sum_{t\in\mathcal{T}(r)} \hat{x}_t > 0\}$ is the set of all receivers having transmitters located within their communication ranges, i.e., set of all potential communicating receivers.

$$SP(\hat{x}) \quad \theta_{SP}(\hat{x}) = \min \sum_{r\in\hat{R}} z_r \tag{19}$$

$$\text{s.t.} \quad \lambda \frac{\sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max_{t\in\mathcal{T}(r)} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \ge \varepsilon(1 - z_r) \qquad r \in \hat{R} \tag{20}$$

$$\sum_{j\in\mathcal{J}} y_j \le q \tag{21}$$

$$y_j \in \{0,1\} \qquad j \in \mathcal{J} \tag{22}$$

$$z_r \in \{0,1\} \qquad r \in \hat{R} \tag{23}$$

Let $\hat{y}$ be the optimal solution to $SP(\hat{x})$. Obviously, $(\hat{x}, \hat{y})$ is a feasible solution of RCIP and $\theta_{SP}(\hat{x})$ is a lower bound (LB) to its optimal objective function value.

Until $LB = UB$, we solve the master and subproblems sequentially in this fashion, each time augmenting the set Y in the relaxed master problem with the optimal solution of the current subproblem. The proposed solution method is formalized with Algorithm 1.

Algorithm 1: Decomposition method to solve RCIP.

```
Data: T, R, J, ε, γ
Result: x*, W*
begin
    LB ⟵ 0, UB ⟵ R, Y ⟵ ∅;
    Select an arbitrary y ∈ Y as an initial solution;
    Y ⟵ Y ∪ {y};
    while LB < UB do
        Solve MP(Y) for x̂;
        if θ_MP(Y) < UB then UB ⟵ θ_MP(Y);
        if LB = UB then
            x* ⟵ x̂, W* ⟵ UB;
            break;
        Solve SP(x̂) for ŷ ;
        if θ_SP(x̂) > LB then LB = θ_SP(x̂);
        if LB = UB then
            x* ⟵ x̂, W* ⟵ LB;
            break;
        Y ⟵ Y ∪ ŷ;
    Print("x* is the optimal strategy for DF that will enable
    W* receivers to communicate")
```

4.2. Enhancements to the decomposition method

We propose three types of enhancements to our decomposition algorithm.

4.2.1. Initial solution

Our preliminary analyses have indicated that the overall computation time is sensitive to the choice of the initial solution y. In order to find an initial solution that will provide a tight upper bound and decrease the overall solution time, we propose a greedy logic for choosing the initial jammer sites. For each potential jammer site, we keep a count of the number of receivers whose closest jammer site is this particular site. We then order the jammer sites in nonincreasing order of their respective count values and simply choose the first q such sites in our initial solution y.

4.2.2. Preprocessing

For a fixed DF solution $\hat{x}$, among the receivers that have the potential to communicate, i.e., those defined by the set $\hat{R}$, some might not be jammable and others will be jammable regardless of AT's location decisions. Such receivers can be identified with the following proposition and the corresponding variables can simply be eliminated from the models.

Proposition. *Let $\hat{x}$ be a given DF solution and consider a particular receiver $r \in \hat{R}$. Assume without loss of generality that $d_{j_1 r} \le d_{j_2 r} \le \ldots \le d_{j_J r}$. Then, the following statements are valid in any optimal solution to* $\mathbf{SP}(\widehat{\mathbf{x}})$.

1. *If* $\left(\lambda \frac{\sum_{1 \le i \le q} \frac{1}{d^{\beta}_{j_i r}}}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t}\right) < \varepsilon$, *then* $z_r = 1$ *(i.e., receiver r is able to communicate).*
2. *If* $\left(\lambda \frac{\sum_{J-q+1 \le i \le J} \frac{1}{d^{\beta}_{j_i r}}}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t}\right) \ge \varepsilon$, *then* $z_r = 0$ *(i.e., receiver r is not able to communicate).*

Proof. The first statement establishes that if the cumulative power of even the closest q jammers to receiver r is not enough to jam for the specific transmitter locations $\hat{x}$, then receiver r will not be jammed in any optimal solution to $\mathbf{SP}(\widehat{\mathbf{x}})$ and the corresponding decision variable can be fixed to 1 in this model. In contrast, the second statement considers the farthest q jammer locations to receiver r. If the jamming to signal ratio is at least the threshold value even when the jammers are located farthest away, then in any optimal solution to $\mathbf{SP}(\widehat{\mathbf{x}})$ it will not be possible to achieve $z_r = 1$ and thus this variable can be fixed to zero in the model without loss of generality. For the specific DF solution $\hat{x}$ and any feasible AT solution y, i.e., $\sum_{j \in \mathcal{J}} y_j \le q$, the above results simply follow from the following relationships:

$$\lambda \frac{\sum_{1 \le i \le q} \frac{1}{d^{\beta}_{j_i r}}}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t} \ge \lambda \frac{\sum_{j \in \mathcal{J}} \frac{1}{d^{\beta}_{jr}} y_j}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t} = JSR_r(\hat{x}, y) \ge \lambda \frac{\sum_{J-q+1 \le i \le J} \frac{1}{d^{\beta}_{j_i r}}}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t}. \quad (24)$$

□

4.2.3. Dominance

Depending on the relative geographical dispersion of two distinct potential jammer location sites j' and j'', one may dominate the other one. More formally, if $d_{j'r} \le d_{j''r}$ $\forall r \in \mathcal{R}$, then site j' dominates site j'' and site j'' cannot be selected unless site j' is selected. In other words, the constraints $y_{j'} \ge y_{j''}$ for each such pair $j', j'' \in \mathcal{J}$ can be incorporated into the subproblem without any loss of generality.

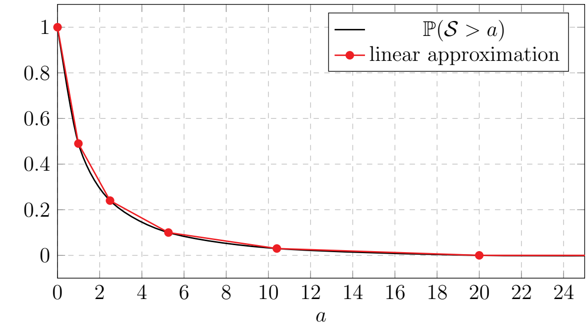
5. RCIP: probabilistic approach (P-RCIP)

Given the location plans $x \in \{0, 1\}^T$ and $y \in \{0, 1\}^J$, $PJSR_r(x, y)$ is the probabilistic jamming to signal ratio at receiver $r \in \mathcal{R}$, which is given as

$$PJSR_r(x, y) = \lambda \frac{\sum_{j \in \mathcal{J}} \frac{1}{d^{\beta}_{jr}} y_j}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} x_t} \frac{1}{\mathcal{S}} \quad (25)$$

where $\mathcal{S}$ is a random variable corresponding to the random fluctuations in the path loss over the channel from the transmitter to the receiver. $\mathcal{S}$ is modeled as a lognormal distributed random variable, i.e., $\log(\mathcal{S})$ has Gaussian distribution (Rappaport, 2002).

Let x_t for $t \in \mathcal{T}$ indicate transmitter locations, y_j for $j \in \mathcal{J}$ indicate jammer locations, and ε be the jamming to signal ratio threshold value, respectively, as in Section 4. The binary decision variables z_r for $r \in \mathcal{R}$ indicating communicating receivers will be replaced with their probabilistic variants called pz_r. In this setting, pz_r corresponds to the probability that receiver r is communicating, i.e., $\mathbb{P}(PJSR_r(x, y) < \varepsilon)$. Letting $a = \lambda \frac{\sum_{j \in \mathcal{J}} \frac{1}{d^{\beta}_{jr}} y_j}{\max_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} x_t} \frac{1}{\varepsilon}$, we denote $\mathbb{P}(PJSR_r(x, y) < \epsilon)$ as $\mathbb{P}(\mathcal{S} > a)$ and the shape of this probability function is depicted in Fig. 1. This nonlinear function can be approximated with a piecewise linear function and after a preliminary computational analysis, we chose to do this approximation using six segments as can be seen in the same figure.



$$\mathbb{P}(\mathcal{S} > a) = \begin{cases} -\frac{51}{99} a + 1 & \text{if } a \le 0.99 \\ -\frac{1}{6} a + 0.655 & \text{if } 0.99 < a \le 2.49 \\ -\frac{14}{277} a + 0.366 & \text{if } 2.49 < a \le 5.26 \\ -\frac{7}{514} a + 0.172 & \text{if } 5.26 < a \le 10.4 \\ -\frac{3}{960} a + 0.0625 & \text{if } 10.4 < a \le 20 \\ 0 & \text{if } a > 20 \end{cases}$$

Fig. 1. Cumulative distribution function and linear approximation of $\mathbb{P}(\mathcal{S} > a)$.

A solution approach similar to that of Section 4.1 can be facilitated for this variation of RCIP. For each receiver $r \in \mathcal{R}$, we introduce a new decision variable ps_{ry}, which is defined as follows.

ps_{ry} = the probability that receiver $r \in R$ is able to communicate when AT's strategy is $y \in \mathcal{Y}$.

To this end, the probabilistic master problem becomes

$$MPp(\mathcal{Y}) \qquad \theta_{MP}(\mathcal{Y}) = \max \quad p\omega \quad (26)$$

$$\text{s.t.} \quad p\omega \le \sum_{r \in \mathcal{R}} ps_{ry} \qquad y \in \mathcal{Y} \quad (27)$$

$$ps_{ry} \le \mathbb{P}(PJSR_r(x, y) < \varepsilon) \qquad r \in \mathcal{R}, \ y \in \mathcal{Y} \quad (28)$$

$$\sum_{t \in \mathcal{T}} x_t \le p \quad (29)$$

$$x_t \in \{0, 1\} \qquad t \in \mathcal{T} \quad (30)$$

$$0 \le ps_{ry} \le 1 \qquad r \in \mathcal{R}, \ y \in \mathcal{Y} \quad (31)$$

where $\mathcal{Y} = \{y \in \{0, 1\}^J \mid \sum_{j \in \mathcal{J}} y_j \le q\}$ and the auxiliary variable $p\omega$ keeps track of the expected number of receivers that are not jammed with respect to all possible AT solutions in $\mathcal{Y}$.

Due to constraints (28), $MPp(\mathcal{Y})$ is a nonlinear MIP model. In order to linearize $MPp(\mathcal{Y})$, we introduce the parameter P_{try}, which denotes the probability that receiver $r \in \mathcal{R}$ is able to communicate when AT's strategy is $y \in \mathcal{Y}$ and a transmitter is located on possible transmitter location site $t \in \mathcal{T}$. The formal definition of P_{try} is

$$P_{try} = \mathbb{P}\left(\lambda \, \frac{\sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\frac{1}{d_{tr}^{\alpha}}} \frac{1}{\mathcal{S}} < \varepsilon \right). \tag{32}$$

Additional variables to linearize $MPp(\mathcal{Y})$ are as follows.

δ_r = power level of the strongest transmitter signal received by receiver $r \in \mathcal{R}$ (33)

$$u_{tr} = \begin{cases} 1 & \text{if transmitter } t \in \mathcal{T} \text{ transmits the strongest transmitter signal to receiver } r \in \mathcal{R} \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

With these new parameters and variables, the MIP probabilistic master model is formalized as:

$$MPpl(\mathcal{Y}) \quad \theta_{MP}(\mathcal{Y}) = \max \quad p\omega \tag{35}$$

$$\text{s.t.} \quad p\omega \le \sum_{r \in \mathcal{R}} ps_{ry} \qquad y \in \mathcal{Y} \tag{36}$$

$$ps_{ry} \le \sum_{t \in \mathcal{T}} P_{try} u_{tr} \qquad r \in \mathcal{R},\ y \in \mathcal{Y} \tag{37}$$

$$\sum_{t \in \mathcal{T}} u_{tr} = 1 \qquad r \in \mathcal{R} \tag{38}$$

$$u_{tr} \le x_t \qquad r \in \mathcal{R},\ t \in \mathcal{T} \tag{39}$$

$$\delta_r \ge \frac{1}{d_{tr}^{\alpha}} x_t \qquad r \in \mathcal{R},\ t \in \mathcal{T} \tag{40}$$

$$\delta_r \le \frac{1}{d_{tr}^{\alpha}} x_t + M(1 - u_{tr}) \qquad r \in \mathcal{R},\ t \in \mathcal{T} \tag{41}$$

$$\sum_{t \in \mathcal{T}} x_t \le p \tag{42}$$

$$x_t \in \{0, 1\} \qquad t \in \mathcal{T} \tag{43}$$

$$u_{tr} \in \{0, 1\} \qquad r \in \mathcal{R},\ t \in \mathcal{T} \tag{44}$$

$$0 \le ps_{ry} \le 1 \qquad r \in \mathcal{R},\ y \in \mathcal{Y} \tag{45}$$

By constraints (38) and domain restrictions (44), only one u_{tr} variable takes a value of 1 for each receiver and with constraints (39), (40), and (41) $u_{tr} = 1$ only for the transmitter that transmits the strongest transmitter signal to receiver r (M is a large enough number). Note that we no longer use set $\mathcal{T}(r)$ as we did in the deterministic formulation since any transmitter has a positive probability of transmitting to any receiver. By constraints (37), ps_{ry} will be bounded from above with the probability value corresponding to the strongest located transmitter signal and will be equal to this bound value at an optimal solution. The rest of the formulation is the same as that of $MPp(\mathcal{Y})$.

Let $\hat{x}$ be the optimal solution of the relaxed master problem $\mathbf{MP}_{pl}(Y)$ where the set of all AT strategies $\mathcal{Y}$ is replaced with a subset Y and define the constant

$$c_r(\hat{x}) = \frac{\lambda}{\epsilon \max_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \text{ for } r \in \mathcal{R}.$$

The subproblem to be solved for this variant then becomes:

$$SP(\hat{x}) \quad \theta_{SP}(\hat{x}) = \min \sum_{r \in \mathcal{R}} pz_r \tag{46}$$

$$\text{s.t.} \quad pz_r \ge -\frac{51}{99} c_r(\hat{x}) \sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 1 \qquad r \in \mathcal{R} \tag{47}$$

$$pz_r \ge -\frac{1}{6} c_r(\hat{x}) \sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.655 \qquad r \in \mathcal{R} \tag{48}$$

$$pz_r \ge -\frac{14}{277} c_r(\hat{x}) \sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.366 \qquad r \in \mathcal{R} \tag{49}$$

$$pz_r \ge -\frac{7}{514} c_r(\hat{x}) \sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.172 \qquad r \in \mathcal{R} \tag{50}$$

$$pz_r \ge -\frac{3}{960} c_r(\hat{x}) \sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.0625 \qquad r \in \mathcal{R} \tag{51}$$

$$pz_r \ge 0 \qquad r \in \mathcal{R}_{00} \tag{52}$$

$$\sum_{j \in \mathcal{J}} y_j \le q \tag{53}$$

$$y_j \in \{0, 1\} \qquad j \in \mathcal{J} \tag{54}$$

Note that we would like pz_r take the probability value corresponding to the interval where $\mathbb{P}(PJSR_r(\hat{x}, y) < \varepsilon)$ falls, however, due to convexity, by taking the maximum of all these function values as in inequalities (47)–(52) we can guarantee that pz_r will take the correct value.

6. Computational results

In this section, we first investigate the performance of the decomposition method for the deterministic and probabilistic approaches in terms of number of iterations, solution times, and objective function values on different problem instances with varying parameter settings that are defined on a brigade level DF unit with three battalions and test the efficacy of the proposed enhancements. In an attempt to provide tactical insights from the commander's perspective, we test the performance of the decomposition method on larger instances with four battalions by considering different scenarios that reflect not only the initial but also the probable subsequent phases of a military operation. Additionally, two heuristic methods are proposed to assess the value of the exact solution method. Finally, we analyze how parameters like the Jamming to Signal Ratio threshold value (ε) and the path loss exponent rates (α, β) affect the performance of the solution method and the decisions. All experiments are executed on a Lenovo Z580 computer with a 2.2 GHz Intel Core i7-3632QM processor and 6GB RAM by implementing the proposed solution method using Java and CPLEX 12.5.

6.1. Experimental setting

The number of receivers, R, largely depends on the number of battalions. Each battalion is supposed to have three companies and each company is composed of three platoons. Platoon, being the smallest combat unit, consists of four armored personnel carriers and/or tanks and each of them has a military radio mounted on its vehicle. Hence, a company with three platoons has 12 receivers. In addition to these maneuver units, for each company we include one command and control vehicle and two combat support/combat service support vehicles with mounted military radios. In total, the number of receivers in a company sum up to 15 and a battalion with three companies, two command and control vehicles and three combat support/combat service support vehicles has 50 receivers. Finally, with 50 additional receivers regarding the combat support units such as artillery, air defense, corps of engineers and various combat service support units, the value of R is approximately 200 for a brigade with three battalions and 250 for a brigade with four battalions. Nevertheless, the number of receivers in a battalion may be incremented according to the type of operation to be conducted with military units having different capabilities so we let R vary from 200 to 245 and from 250 to 310 for the brigade with three battalions and four battalions, respectively, in our experiments. The number of potential transmitter (T) and jammer location (J) sites are considered to range from 100 to 130 in proportion to the number of receivers.

For the test problems with three battalions, we assume that p ranges from 3 to 6 and for each p value q is assumed to range from 2 to $(p+2)$. Similarly, for the test problems with four battalions, p ranges from 4 to 7 and q ranges from 2 to $(p+2)$. Unless otherwise stated, we use $\alpha = 2$ and $\beta = 2$, i.e., propagation is assumed to take place in free space, $\varepsilon = -3$ dB, i.e., the received signal power should be twice the received jammer power for proper reception, $\gamma = -10$ dBm, i.e., the received signal power should be at least $100\mu W$ for proper reception required by challenging tactical applications, and $\lambda = 1$, i.e., the transmitter power and antenna gain are the same as the jammer's transmitted power and antenna gain, as the values of the parameters used in the computations.

6.2. Experimental results for the brigade with three battalions

The generic scenario is depicted in Fig. 2. The first and the second battalions are located along the frontline and the third battalion is located behind them. The border of transmitter location site surrounds the borders of the battalions and the jammer location site lies approximately 1 km away from the frontline with a depth of 2 km.

In an attempt to evaluate the proposed decomposition method, we solved both the deterministic and the probabilistic RCIP models with this scenario. For each parameter setting provided in Section 6.1, we generated 10 different problem instances by randomly determining the locations of receivers, possible transmitter and jammer location sites depending on the given width, depth and borders of the military unit's deployment on the battlefield. Each row in Table 1 displays the average number of iterations, solution times (in CPU seconds) and the objective function values of 10 randomly generated problem instances. The results for the deterministic and probabilistic approaches are depicted in separate multicolumns. The objective function value of the deterministic RCIP refers to the minimum number of receivers (out of R) that will be covered even under the smartest jamming attack, whereas that of the probabilistic RCIP expresses the expected coverage. We also preface the average percentage coverages these objective values correspond to. The breakdown of solution times into master and subproblems as well as the average number of iterations during the decomposition method are also depicted under columns MP, SP and # iterations in each approach, respectively.

It is readily observed that the coverage improves as the number of transmitters increases and worsens as the number of jammers increases. The results clearly show that both the deterministic and the probabilistic approaches are able to solve all the instances to optimality within reasonable solution times (under five minutes). As expected, solution times increase in both approaches as problem dimensions R, T, and J increase. On the average, 88.3% of the total solution time is spent for solving the subproblems in the deterministic approach, while 97.9% of the total solution time is spent for solving the master problems in the probabilistic approach. This is an expected result as the master problem models for P-RCIP and the subproblem models for RCIP involve extra binary variables when compared with their counterpart variants and hence are computationally more challenging.

6.3. Effects of proposed enhancements

We applied each enhancement proposed in Section 4.2 both individually and collectively, solved RCIP with the same instances presented in Table 1 and observed the results in Table 2. The experiments reveal that starting with the initial AT solution provided by our heuristic reduces the average number of iterations by 14.9%.

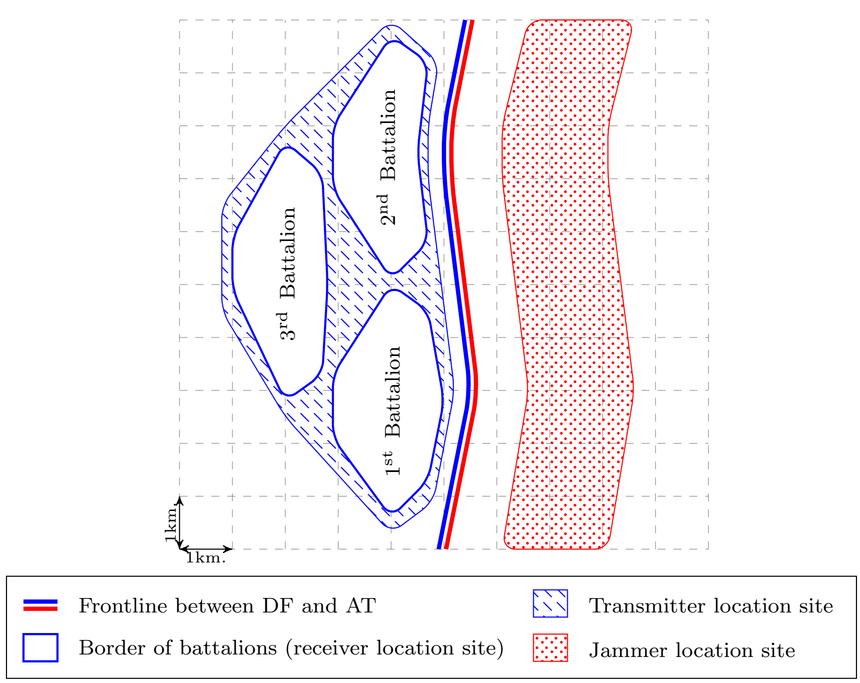


Fig. 2. Sketch of the scenario for a brigade with 3 battalions.

Table 1
Solution statistics of deterministic and probabilistic RCIP for the brigade with 3 battalions.

					Deterministic						Probabilistic					
						Solution times (seconds)			Objective value			Solution times (seconds)			Objective value	
R	T	J	p	q	#iterations	MP	SP	Total	# of receivers covered	Coverage percentage	#iterations	MP	SP	Total	# of receivers covered	Coverage percentage
200	100	100	3	2	6.1	1.4	10.5	11.9	143.2	71.6%	1.7	28.0	0.5	28.5	133.3	66.7%
				3	5.1	0.9	16.8	17.6	119.3	59.7%	2	41.6	0.7	42.3	115.1	57.6%
				4	6.2	1.2	10.1	11.3	106.3	53.2%	2.3	47.5	0.8	48.3	102.7	51.4%
				5	4.5	0.5	3.3	3.8	99.5	49.8%	2	38.6	0.6	39.2	93.8	46.9%
215	110	110	4	2	6.4	3.2	21.7	24.9	169.6	78.9%	2.4	60.3	1.1	61.4	160.6	74.7%
				3	7.3	3.6	78.9	82.5	152.6	70.9%	2.3	60.4	0.9	61.3	142.4	66.2%
				4	9.8	8.8	57.1	65.9	136.5	63.5%	2.6	73.8	1.1	74.9	129.1	60.1%
				5	9.3	7.3	33.9	41.2	127.5	59.3%	2.4	67.8	0.8	68.6	119.2	55.4%
				6	7.1	2.9	21.3	24.2	119.4	55.5%	2.6	100.1	0.9	101.0	111.3	51.8%
230	120	120	5	2	9.6	18.7	35.6	54.3	206.7	89.8%	2.4	98.8	1.0	99.8	181.8	79.1%
				3	8.9	12.9	143.9	156.8	194.8	84.7%	2	68.0	1.0	69.0	164.2	71.4%
				4	10.7	7.7	119.6	127.3	184.0	80.0%	1.9	51.8	0.8	52.6	151.1	65.7%
				5	11.1	13.6	65.5	79.1	155.1	67.4%	2.5	85.9	1.2	87.0	140.4	61.1%
				6	10.2	9.4	46.5	55.9	146.3	63.6%	3	103.7	1.3	105.0	131.6	57.2%
				7	8.2	4.3	44.3	48.6	137.2	59.7%	3.2	122.4	1.4	123.9	124.5	54.1%
245	130	130	6	2	8.5	13.5	32.6	46.1	234.1	95.5%	3.1	208.9	1.7	210.6	203.8	83.2%
				3	10.8	38.4	263.1	301.5	215.9	88.1%	2.8	148.5	1.4	149.9	187.1	76.4%
				4	9.2	14.2	285.2	299.4	199.9	81.6%	1.7	58.5	0.9	59.4	174.3	71.1%
				5	9.2	12.2	209.5	221.7	188.2	76.8%	2.2	78.1	1.2	79.3	163.8	66.9%
				6	8.8	6.6	136.1	142.7	176.8	72.2%	2.6	98.0	1.3	99.3	155.1	63.3%
				7	7.7	3.4	121.3	124.7	165.4	67.5%	2.7	105.5	1.2	106.7	147.3	60.1%
				8	8.7	5.2	82.1	87.3	154.9	63.2%	2.5	121.2	1.6	122.8	140.6	57.4%

Table 2
Effects of proposed enhancements.

					Heuristic initial solution				Preprocessing						Dominance relation			All enhancements					
						Solution times (seconds)			# preprocessed z_r variable			Solution times (seconds)			Solution times (seconds)			Iteration		Solution times (seconds)			
R	T	J	p	q	#iter.	MP	SP	Total	$z_r = 0$	$z_r = 1$	Total	MP	SP	Total	MP	SP	Total	#iter.	imp.%	MP	SP	Total	imp.%
200	100	100	3	2	4.3	0.9	7.5	8.3	0.8	108.9	109.6	1.0	1.9	2.9	1.1	1.8	2.9	4.1	33	0.6	0.6	1.2	90
				3	4.1	0.5	12.3	12.9	5.1	93.1	98.2	0.9	11.3	12.2	0.9	4.9	5.7	4.2	18	0.6	3.1	3.7	79
				4	4.9	0.9	5.7	6.6	11.3	89.9	101.2	0.9	4.9	5.8	0.9	2.6	3.5	4.7	24	0.6	0.8	1.5	87
				5	3.3	0.3	1.5	1.8	22.0	84.8	106.9	0.5	2.1	2.7	0.4	1.3	1.6	3.4	24	0.2	0.3	0.5	87
215	110	110	4	2	5.3	2.3	18.8	21.1	0	148.7	148.7	3.2	5.9	9.1	4.4	4.1	8.5	5.5	14	2.6	0.9	3.4	86
				3	6.2	3.2	63.2	66.4	1.1	117.4	118.5	3.9	44.9	48.9	4.4	26.3	30.7	5.8	21	2.3	13.6	15.9	81
				4	9.2	8.1	46.8	54.9	6.4	106.2	112.6	9.3	26.1	35.4	9.3	15.2	24.4	9.6	2	8.8	9.8	18.5	72
				5	8.8	8.3	25.3	33.6	14.9	98.7	113.6	8.1	14.7	22.8	5.7	8.6	14.3	7.9	15	4.1	3.8	7.9	81
				6	5.7	1.6	14.9	16.5	28.9	96.8	125.7	3.3	11.9	15.2	2.8	6.2	9.0	5.7	20	1.3	1.9	3.3	87
230	120	120	5	2	7.5	7.1	29.6	36.6	7.8	164.7	172.5	13.4	4.9	18.3	7.5	4.1	11.7	7.2	25	5.7	1.0	6.7	88
				3	7.3	5.5	102.6	108.1	14.8	132.1	146.9	8.4	50.3	58.7	8.4	39.6	48.0	7.4	17	7.9	1.0	8.8	93
				4	10.4	13.6	129.6	143.2	1.9	122.2	124.1	13.8	59.8	73.7	17.3	44.1	61.4	7.7	28	6.2	21.7	27.9	82
				5	10.3	14.7	58.2	72.8	7.7	155.1	162.8	13.4	32.1	45.5	11.2	22.4	33.6	10	10	11.7	13.4	25.1	68
				6	8.5	5.9	42.2	48.1	18.2	108.5	126.7	6.2	20.2	26.4	6.5	14.6	21.1	8.7	15	6.9	9.2	16.1	71
				7	6.5	2.5	30.4	32.9	26.1	95.5	121.6	3.4	18.7	22.1	4.8	15.1	18.9	7	15	2.8	7.5	10.4	79
245	130	130	6	2	7.9	13.5	32.1	45.6	0	201.3	201.3	14.1	3.6	17.7	14.8	5.4	20.3	8.2	4	15.3	0.9	16.3	65
				3	10.1	30.1	223.5	253.6	0	163.6	163.6	38.9	117.5	156.4	43.1	82.5	125.6	10	7	29.7	40.4	70.1	77
				4	7.4	13.3	151.4	164.6	0.1	154.4	154.5	16.0	157.6	173.6	17.8	98.1	116.1	8.2	11	22.6	33.7	56.3	81
				5	7.3	5.6	108.6	114.1	3.1	142.1	145.2	9.8	78.5	88.3	8.8	43.8	52.7	7.2	22	4.9	19.7	24.7	89
				6	7.5	3.4	89.6	93.1	8.2	131.5	139.7	6.4	68.5	74.9	5.4	40.5	45.9	7.9	10	3.9	21.3	25.1	82
				7	6.7	2.4	82.4	84.8	12.9	121.1	134.0	3.9	63.2	67.1	2.7	36.5	39.3	6.1	21	1.6	17.1	18.6	85
				8	8.6	5.6	79.8	85.4	24.9	113.9	138.8	5.7	41.4	47.1	4.2	23.4	27.7	7.3	16	3.9	13.3	17.2	80

Through preprocessing, 59.4% of the z_r variables are fixed and the average solution time reduces by 49.6%. Finally, identifying dominance relations between possible jammer locations yields an average of 64.9% reduction in solution times. Table 2 also presents the results obtained by applying all enhancements simultaneously, which provides an average of 16.8% reduction in the number of iterations and 81.3% reduction in solution times. Additionally, the results show that after applying all the enhancements, the percentage of the total solution time spent to solve the subproblems reduced from 88.3% to 59.4%.

6.4. Experimental results for the brigade with four battalions

We solved RCIP and P-RCIP for a brigade level military unit with four battalions and tested the performance of the proposed decomposition method on four different probable scenarios. While all enhancements were utilized for RCIP models, the preprocessing enhancement was not available for P-RCIP models.

6.4.1. Scenarios

We designed scenarios to reflect not only the initial but also the probable subsequent phases of a military operation. Sketches of all the scenarios are depicted in Fig. 3. Scenario 1 (Fig. 3(a)) reflects the initial phase of a military operation. We assume that three battalions are positioned along the frontline and the fourth battalion positioned behind serves as a reserve unit. In Scenario 2 (Fig. 3(b)), we assume that the brigade improves its attacks from the north and thereupon the brigade commander deploys the reserve battalion to the north in order to support the improvement or exploit a possible breakthrough. A symmetric scenario can be visualized to represent a southern improvement. To investigate the effects of improvement from the middle of the frontline we provide Scenario 3 (Fig. 3(c)). Finally, we investigate the effects of a withdrawal operation conducted by the brigade in Scenario 4 (Fig. 3(d)). We assume that the battalions of DF, especially the second battalion, strive hard to prevent an AT breakthrough. Hence, the commander is keeping the reserved battalion very close to the second battalion in order to quickly exploit the situation in case of emergency.

6.4.2. Numerical results

Table 3 presents the solution statistics of RCIP and P-RCIP based on the scenarios described above. Each row depicts the average results obtained by solving 10 randomly generated problem instances with the specified parameter choices.

For fixed R, T, J and p values, solution times for RCIP increase rapidly as q increases in the beginning but decrease gradually afterwards. The main reason of this pattern is the number of

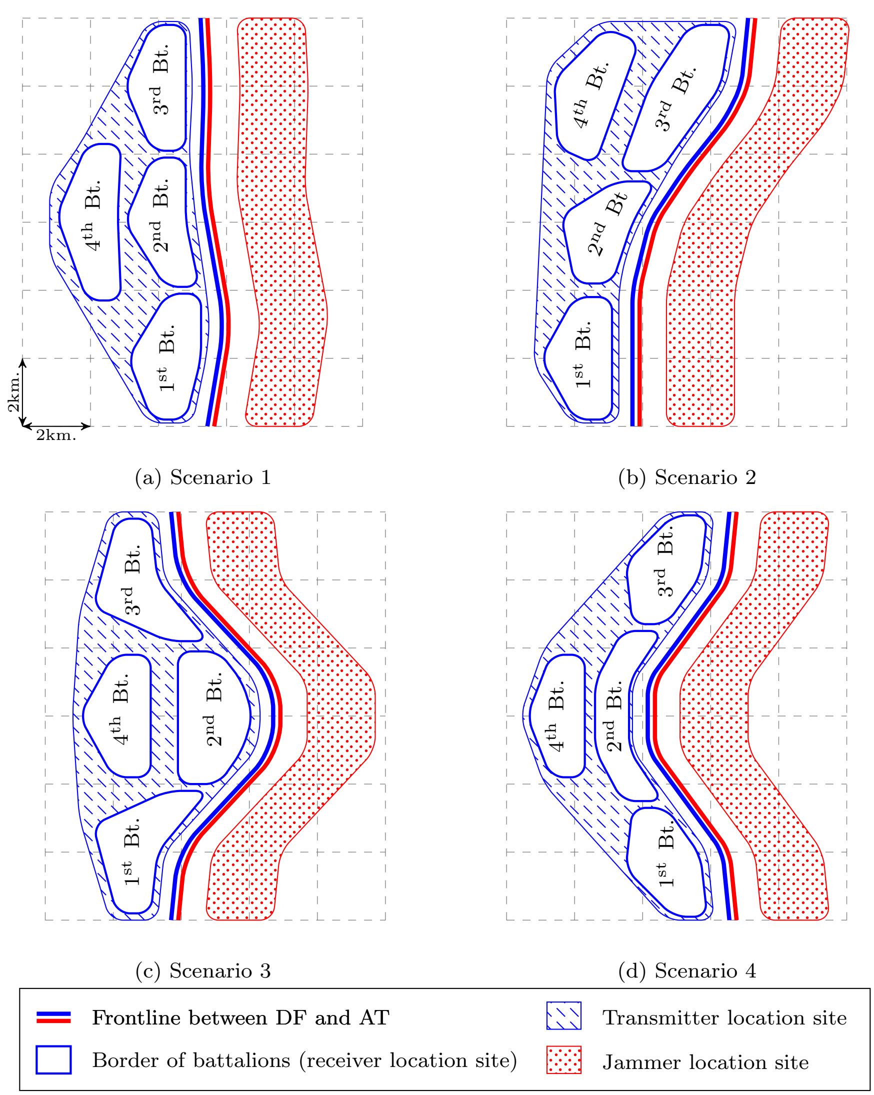


Fig. 3. Scenario sketches.

Table 3
Solution statistics of deterministic and probabilistic approaches on different scenarios.

					Solution time (seconds)								Objective function value							
					Scenario 1		Scenario 2		Scenario 3		Scenario 4		Scenario 1		Scenario 2		Scenario 3		Scenario 4	
R	T	J	p	q	det.	prob.	det.	prob.	det.	prob.	det.	prob.	det.	prob.	det.	prob.	det.	prob.	det.	prob.
250	100	100	4	2	7.7	58.2	8.4	55.5	3.4	48.2	19.2	76.1	172.6	156.1	165.1	154.5	167.3	151.9	166.9	151.6
				3	**31.3**	70.7	**117.7**	54.8	**60.5**	56.4	**522.1**	77.1	131.4	131.2	126.5	129.9	125.7	129.7	130.3	128.4
				4	4.7	87.3	55.1	64.5	30.3	56.2	87.8	113.5	115.5	114.4	103.5	113.2	101.4	112.8	102.1	111.7
				5	4.2	**142.5**	12.7	80.6	14.9	**77.8**	27.6	**115.8**	102.6	103.1	91.6	101.1	86.6	101.2	86.9	99.1
				6	1.7	116.5	4.9	**97.1**	5.4	76.1	7.9	90.3	94.5	94.8	82.9	91.8	77.1	92.1	79.3	90.3
270	110	110	5	2	7.7	123.9	33.2	88.9	26.1	57.7	73.6	70.4	193.7	182.1	203.2	184.1	205.5	183.7	201.8	179.8
				3	**117.6**	95.5	**952.3**	81.3	**739.9**	89.8	**952.6**	101.2	159.2	155.5	160.3	157.2	155.8	155.7	159.5	154.9
				4	34.2	129.5	387.6	112.7	513.2	75.3	410.9	122.5	137.9	137.3	131.2	138.5	123.5	139.6	132.8	137.2
				5	17.6	217.7	110.7	124.9	250.4	126.3	166.5	125.3	125.0	124.7	114.8	124.8	113.6	125.9	116.9	123.8
				6	10.4	224.4	34.1	186.5	33.6	106.2	26.8	**148.5**	113.7	115.1	104.1	114.6	102.7	115.3	107.8	114.2
				7	4.1	**326.1**	11.7	**253.4**	21.2	**131.2**	16.6	123.6	107.1	107.5	98.6	106.6	95.7	107.1	101.1	106.7
290	120	120	6	2	24.2	207.6	146.3	134.1	27.3	88.4	171.5	83.8	231.4	206.7	227.6	206.1	235.1	210.8	233.6	209.8
				3	**893.6**	165.1	**2745.9**	150.1	1007.2	**161.5**	**4005.3**	172.4	190.3	179.6	185.3	179.8	200.8	185.4	195.2	183.3
				4	139.4	160.1	1049.8	133.7	**1254.4**	92,6	3729.6	142.9	162.7	161.1	159.0	161.8	162.9	164.9	164.0	163.2
				5	44.1	186.1	359.6	158.5	591.5	150.5	1405.9	156.7	147.4	147.6	142.6	148.3	143.1	150.5	144.9	148.8
				6	22.1	224.9	102.1	286.7	96.7	100.2	335.8	141.1	135.7	137.2	131.6	137.8	132.0	139.4	130.1	137.6
				7	20.6	365.9	45.5	372.8	49.2	147.8	147.6	216.8	126.7	128.9	123.9	129.3	122.8	130.7	121.4	129.1
				8	8.9	**477.5**	23.7	**390.5**	38.7	157.5	52.8	**216.9**	120.3	122.1	117.7	122.3	115.3	123.5	113.2	121.6
310	130	130	7	2	22.1	372.9	104.6	**248.8**	28.8	139.8	117.2	161.7	260.0	230.5	264.9	236.5	265.9	235.6	265.9	231.9
				3	**683.7**	220.8	2787.3	228.7	1463.3	195.8	3107.4	**330.5**	219.0	204.1	228.1	209.1	232.9	208.9	224.8	205.8
				4	266.5	215.2	**4244.6**	170.2	**4583.5**	170.6	**4875.4**	190.7	195.4	185.1	195.4	189.1	196.6	187.7	191.6	186.6
				5	89.6	225.4	1640.7	179.8	689.8	213.4	2466.8	130.8	178.9	170.7	173.4	174.4	169.1	172.5	173.5	171.7
				6	47.3	269.3	498.2	142.9	326.1	168.8	715.8	144.9	165.6	159.5	159.8	162.7	155.1	160.4	157.3	159.7
				7	26.7	418.1	278.6	155.6	158.8	143.2	258.8	179.1	155.2	150.1	147.4	153.2	144.6	150.8	145.9	149.9
				8	15.3	267.4	112.5	190.5	76.5	147.1	238.6	189.3	146.6	142.6	138.3	145.1	135.0	142.4	136.5	141.8
				9	12.3	**435.1**	94.8	215.6	66.4	**299.1**	116.3	202.6	137.6	136.4	126.6	137.9	127.1	135.5	128.6	134.8

preprocessed z_r variables as given in Fig. 4, which directly affects the sizes of subproblems in RCIP. For small q values, DF is at a greater advantage and many receivers are identified as non-jammable. On the other hand, as q increases, AT gains power and the number of receivers that are surely jammed increases. Thus, for small and large values of q, a large number of z_r variables are fixed, reducing the subproblem sizes and thus resulting in smaller CPU times. In general, the overall solution time attains its maximum value (*printed bold in the table*) when the algorithm identifies the least number of preprocessed z_r variables. We do not see this trend in the probabilistic case since the majority of the solution time is spent for tackling the master problem. When compared with the deterministic approach, we observe that 81.2% of the instances in which $q = 3$ or $q = 4$ are solved in shorter times by the probabilistic approach.

The optimal solution values in different scenarios are also presented in Table 3. As expected, coverages in both the deterministic and the probabilistic models decrease as q increases. The marginal loss in the coverage due to the incremental change in the number of available jammers is high for small q values but gradually decreases as q increases. The reason of this gradual decrease stems from the fact that AT is restricted to locate all jammers in a particular area. Hence, as q increases AT wants to jam the communicating receivers that are far behind the frontline, but restriction on the location area causes more overlap on the jammer coverage. Also, we observe that the optimal solution value of the deterministic approach is greater than the optimal solution value of the probabilistic approach in 88% of the instances with $q \leq 3$ and less than the optimal solution value of the probabilistic approach in 90% of the instances with $q \geq 6$. For small q values many of the receivers are counted as communicating ($z_r = 1$) in the deterministic case but only communicating with a high probability ($pz_r \approx 1$) in the probabilistic case. A similar reasoning explains the difference in large q values.

6.5. Heuristic methods for RCIP

RCIP is a fairly large bilevel programming problem with binary variables both in the first and the second stages. Although, we are able to solve large instances in reasonable times, in order to obtain quick solutions for the aforementioned instances and evaluate the exact solution method, we also propose two heuristic solution methods.

6.5.1. Heuristic 1

In this method, we ignore the adversarial effect and the bilevel structure of RCIP and solve the maximum covering location problem (Church and ReVelle, 1974) by the communication range covering criteria.

As in the bilevel formulation of RCIP, we let $\mathcal{T}(r) = \{t \in \mathcal{T} \mid P_t G_t G_r \frac{1}{d_{tr}^{\alpha}} \geq \gamma\}$ denote the potential transmitter locations that can communicate with receiver $r \in \mathcal{R}$ and use the following decision variables.

$$x_t = \begin{cases} 1 & \text{if a transmitter is located on transmitter site } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

$$z_r = \begin{cases} 1 & \text{if receiver } r \in \mathcal{R} \text{ is communicating} \\ 0 & \text{otherwise.} \end{cases}$$

The maximum covering location problem then becomes:

$$\text{Max} \quad \sum_{r \in R} z_r \tag{55}$$

$$\text{s.t.} \quad z_r \leq \sum_{t \in \mathcal{T}(r)} x_t \qquad r \in \mathcal{R} \tag{56}$$

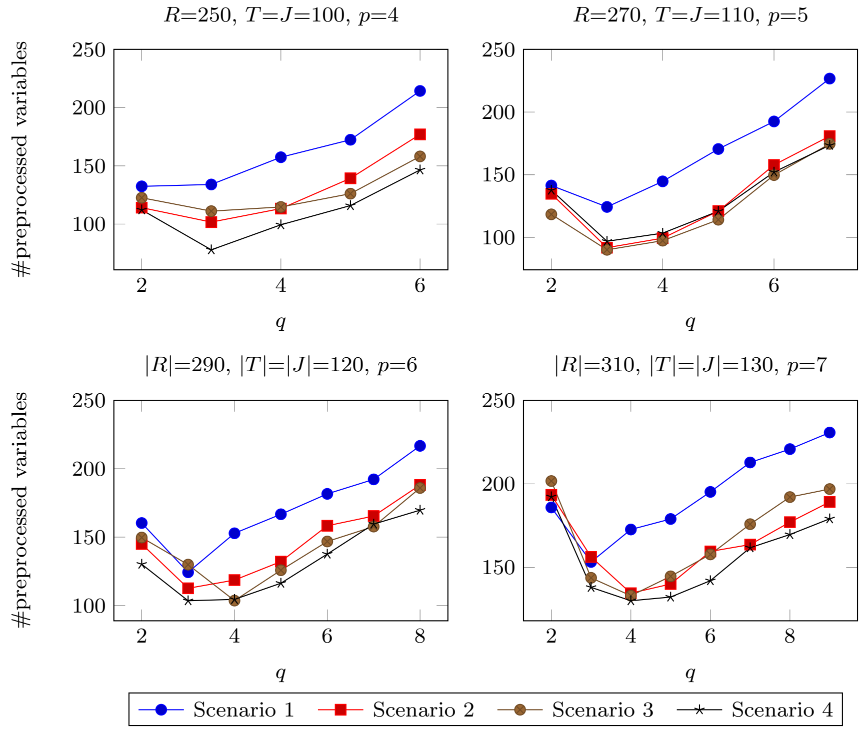


Fig. 4. #preprocessed z_r variables against q in different scenarios for RCIP.

$$\sum_{t\in\mathcal{T}} x_t \le p \qquad r \in R \tag{57}$$

$$x_t \in \{0,1\} \qquad t \in \mathcal{T} \tag{58}$$

$$z_r \in \{0,1\} \qquad r \in \mathcal{R} \tag{59}$$

This mathematical model maximizes the total number of receivers (55) that are determined as covered (56) by locating at most p transmitters (57).

6.5.2. Heuristic 2

Inspecting the optimal transmitter locations as output by our exact solution method, we observed that each battalion has at least one transmitter located to cover the receivers within the battalion site and nearby. This observation is also in sync with the current practices that are used to locate transmitters in the field. Our second heuristic solution method relies on these principles while locating transmitters. For each battalion, a transmitter with the highest cumulative signal power on the receivers of that battalion is chosen. If p is greater than the number of battalions, the remaining transmitters are sequentially located in nonincreasing order of their additional signal power considering all the receivers in the field. Algorithm 2 formalizes our method.

The optimality gaps ($100 \times \frac{RCIP-Heuristic}{RCIP}$) of both of the heuristics for each scenario of Table 3 are presented in Table 4. The optimal and heuristic objective values are also depicted in Fig. 5. Inspecting these results, we observe that Heuristic 2 clearly outperforms Heuristic 1. The main reason behind this difference is the fact that the coverage criteria in Heuristic 1, a simple yes or no value, ignores the level of signal power on receivers, which is utilized in Heuristic 2. Another apparent observation is that for both heuristics, the optimality gaps increase with increasing q values (with the adversary getting stronger) as well as with the dimensions of the instances. In conclusion, even though the heuristic approaches are very efficient in terms of solution times, both of them fail to reflect the adversarial structure of the problem. For some

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Algorithm 2: Heuristic 2.

Result: Transmitter Location Decision
Let $\mathcal{B}$ be the number of battalions and $\mathcal{R}_b$ be the set of receivers of battalion b;
for $t \in \mathcal{T}$ ***and*** $r \in \mathcal{R}$ **do**
  $SP_{tr} = P_t G_t G_r \frac{1}{d_{tr}^{\alpha}}$;
$b \leftarrow 1$;
while $b \le p$ **do**
  if $b \le \mathcal{B}$ **then**
    for $t \in \mathcal{T}$ *and* $r \in \mathcal{R}_b$ **do**
      $total\_SP_t \leftarrow total\_SP_t + SP_{tr}$;
    $\hat{t} = \arg\max_{t\in\mathcal{T}:x_t=0}\{total\_SP_t\}$;
    $x_{\hat{t}} \leftarrow 1$;
    $b \leftarrow b+1$;
  else
    for $r \in \mathcal{R}$ **do**
      $current\_SP_r \leftarrow \max_{t\in\mathcal{T}} SP_{tr} x_t$;
    for $t \in \mathcal{T}$ *and* $r \in \mathcal{R}$ **do**
      $additional\_SP_t \leftarrow$
      $additional\_SP_t + \max\{0, SP_{tr} - current\_SP_r\}$;
    $\hat{t} = \arg\max_{t\in\mathcal{T}:x_t=0}\{additional\_SP_t\}$;
    $x_{\hat{t}} \leftarrow 1$;
    $b \leftarrow b+1$;

Table 4
Optimality gaps of heuristic approaches in each scenario.

					Gap values							
					Scenario 1		Scenario 2		Scenario 3		Scenario 4	
R	*T*	*J*	*p*	*q*	Heuristic 1	Heuristic 2	Heuristic 1	Heuristic 2	Heuristic 1	Heuristic 2	Heuristic 1	Heuristic 2
250	100	100	4	2	26.6	4.5	12.2	2.9	22.2	2.0	17.9	15.4
				3	29.0	10.1	19.2	7.7	28.6	5.5	23.4	18.3
				4	36.1	23.9	27.1	15.2	34.2	12.1	30.7	18.6
				5	40.0	30.9	35.5	25.0	39.7	21.2	38.8	21.6
				6	43.8	38.0	41.4	31.0	44.7	29.2	46.3	28.2
270	110	110	5	2	26.4	7.2	24.8	16.9	25.5	12.9	18.8	18.5
				3	34.5	16.1	34.9	21.5	26.5	16.3	24.0	22.4
				4	39.3	25.5	43.0	25.5	28.6	20.8	30.0	25.0
				5	46.1	34.9	49.7	32.3	38.6	29.9	35.6	28.1
				6	49.5	42.3	54.8	37.1	42.6	35.0	42.5	32.7
				7	53.2	46.5	58.7	44.0	47.9	39.6	48.6	37.8
290	120	120	6	2	30.3	22.2	26.4	23.0	28.4	23.5	21.4	20.0
				3	33.9	33.5	33.9	30.1	36.0	31.7	27.5	24.8
				4	37.8	38.6	40.1	33.8	38.1	32.5	31.8	27.9
				5	43.1	42.7	44.8	35.8	42.8	35.4	37.8	33.6
				6	47.5	46.4	49.0	38.6	47.2	40.1	43.0	38.7
				7	50.9	47.8	51.9	42.4	50.2	43.3	48.0	44.1
				8	55.8	49.8	54.5	45.9	52.1	46.6	51.1	46.9
310	130	130	7	2	28.5	14.5	28.0	17.2	24.0	16.5	26.6	22.0
				3	36.0	20.2	35.5	24.1	32.1	24.5	33.6	24.3
				4	42.4	27.2	38.6	27.4	34.8	29.1	38.0	26.0
				5	47.2	32.3	40.5	31.4	37.5	31.9	43.2	28.5
				6	50.4	36.8	44.2	35.9	39.9	37.6	46.6	30.5
				7	53.2	40.9	46.9	38.3	43.5	41.4	50.4	33.0
				8	55.5	44.0	50.2	40.9	45.3	44.7	53.6	35.2
				9	57.2	45.3	51.5	41.6	47.0	48.2	56.6	36.5

parameter settings, the average gaps can be as large as 50%, which clearly indicate the value of the bilevel solution approach for RCIP.

6.6. Tactical insights

One of the 10 instances where $R = 250$ and $T = J = 100$ is chosen for each scenario in the first multirow of Table 3 and the optimal transmitter and jammer locations for RCIP and P-RCIP corresponding to different choices of p and q values are depicted in Fig. 6.

For Scenario 1, it is observed that if p is equal to the number of battalions, then we have one transmitter located within the borderline of each battalion. With p value exceeding the number of battalions, the surplus transmitters are placed within the borderline starting from the locations that are closer to the frontline since they are exposed to more powerful jamming signals compared to the receivers far from the frontline.

When we investigate the deterministic and probabilistic solutions in Scenario 2, different from Scenario 1 we observe that as q increases, DF locates the surplus transmitter to the 4th battalion that serves as a reserved battalion rather than locating to the 3rd battalion that improved inwards the enemy lines. This result implies that if a battalion accelerates its attacks and moves further forward than the others, it typically becomes more susceptible to jamming.

In Scenario 3, we observe that optimal jammer locations are dispersed on the northern and southern parts of the possible jammer location site and as q increases, jammers are located collectively in order to increase their additive effect. To cope with the situation, the defender locates one transmitter to each battalion when $p = 4$ and generally locates more transmitters to the central region when $p \geq 5$.

In Scenario 4, we realize that optimal jammer locations are gathered in the center of the possible jammer location site since AT has the advantage of controlling the center of the tactical area in this scenario and uses this advantage to jam a larger portion of receivers. This makes the receivers in the center very susceptible to jamming. Therefore, defender locates more transmitters in the central region, especially when $p \geq 5$.

In conclusion, the results indicate that transmitter location decisions are getting complicated for scenarios 2,3, and 4 that reflect the subsequent phases of a military operation. We suggest that rather than using the transmitters homogeneously, commanders must concentrate the effects of available transmitters in the decisive place by allocating minimum essential power to secondary places. To this end, RCIP can provide very useful courses of actions in a very short time, especially for complex situations as in scenarios 2,3, and 4.

One very fruitful observation common to all scenarios is the closeness of location decisions in RCIP and P-RCIP. Depending on the problem parameters, the approach more advantageous in solution time may be utilized to guide the commander.

6.7. Sensitivity analysis on JSR threshold value (ε)

Table 5 presents the solution times and the optimal solution values of the deterministic approach when *JSR* threshold value (ε) varies between −3 dB and −7 dB for specific problem instances in each scenario. The results show that the algorithm attains the maximum solution time (highlighted in bold for each parameter setting) when $\varepsilon = -3$ dB except for two sets of 10 instances in Scenario 4 and decreases dramatically for each 1 dB decrement in ε. This decrease in solution times largely depends on the number of preprocessed z_r variables. As ε decreases, receivers become more susceptible to jamming and therefore the number of receivers that cannot be protected from jamming (i.e. $z_r = 0$) increases and the number of receivers that are not jammed (i.e. $z_r = 1$) decreases. Since the number of receivers that are close to

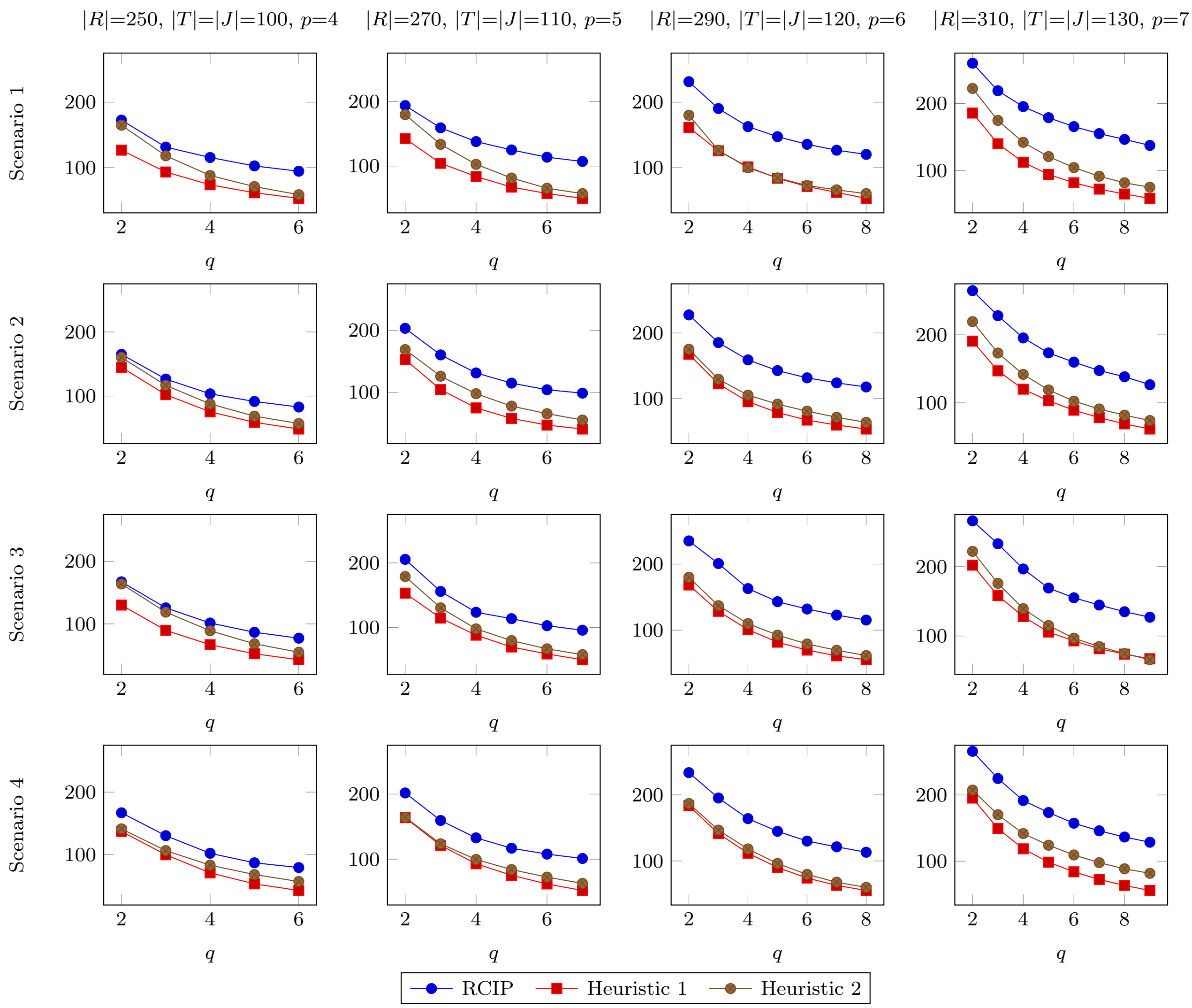

Fig. 5. Comparison of exact and heuristic coverages for different problem instances in each scenario.

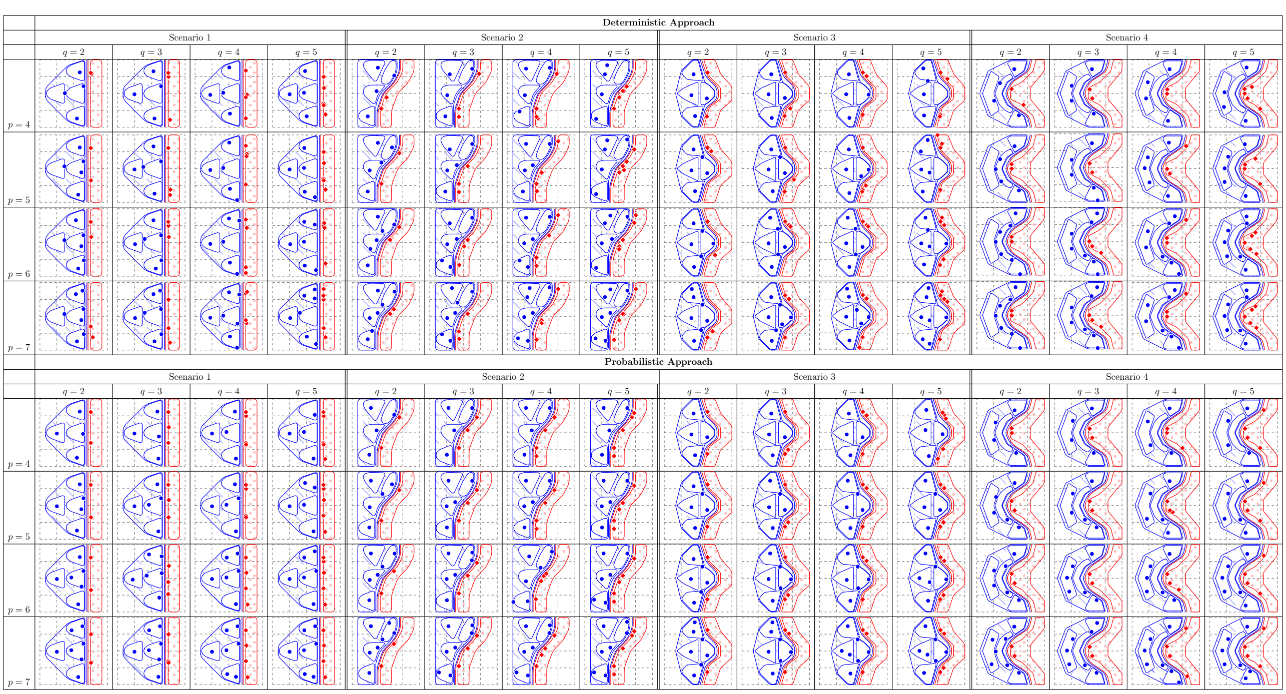

Fig. 6. Optimal transmitter and jammer locations for different p and q values for different scenarios under deterministic and probabilistic case.

Table 5
Sensitivity analysis of *JSR* threshold value (ε).

Solution times (seconds)

					Scenario 1					Scenario 2					Scenario 3					Scenario 4				
					ε in dB					ε in dB					ε in dB					ε in dB				
R	*T*	*J*	*p*	*q*	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7
250	100	100	4	3	**140.0**	67.2	33.6	5.6	3.8	**261.8**	190.9	101.5	64.1	22.5	**136.9**	104.3	60.5	26.0	10.0	278.5	435.5	**740.5**	178.6	38.4
				4	**112.1**	10.8	4.4	4.8	3.3	**194.5**	100.1	48.4	10.9	7.9	**101.9**	67.0	30.3	13.8	5.5	**700.5**	566.8	135.8	27.8	10.4
				5	**8.5**	3.0	3.4	2.2	0.8	**111.6**	54.0	15.8	5.4	3.2	**70.9**	32.4	19.7	6.4	2.9	**502.4**	100.5	34.7	8.6	4.5
270	110	110	5	4	**556.1**	107.0	39.2	14.6	6.6	**1476.2**	1296.0	447.4	169.6	37.2	**1423.2**	1036.6	348.0	193.0	48.3	1865.1	**2850.8**	595.6	188.9	35.1
				5	**118.7**	26.6	19.0	7.3	3.7	**1867.5**	449.5	138.0	32.2	11.5	**1165.9**	608.5	261.8	47.0	17.6	**2139.6**	504.3	282.0	38.4	23.0
				6	**28.5**	12.2	16.2	3.2	1.4	**604.0**	212.0	40.0	12.1	11.0	**849.7**	299.1	44.1	21.6	13.1	**750.1**	80.8	36.0	18.2	11.2
290	120	120	6	5	**765.4**	245.7	47.1	30.5	12.7	**5776.7**	1326.1	452.1	123.3	43.6	**2818.7**	2527.5	715.3	116.8	101.3	**8408.7**	7501.0	3145.1	396.8	114.4
				6	**137.6**	60.2	24.9	10.0	5.4	**1194.6**	756.0	109.8	57.1	33.9	**2537.0**	583.2	131.8	73.3	45.1	**9011.7**	2292.3	335.6	158.6	58.9
				7	**85.9**	31.1	21.7	5.7	5.8	**431.8**	155.6	85.5	25.0	30.4	**1019.6**	188.7	67.6	53.5	33.1	**2583.5**	318.5	161.0	64.9	44.3
310	130	130	7	6	**414.6**	90.1	63.9	26.9	16.7	**2524.3**	1780.1	528.4	232.5	191.5	**5761.0**	2340.9	381.9	202.3	79.9	**2930.6**	2614.3	1096.1	138.3	39.9
				7	**132.9**	71.2	29.9	19.4	11.5	**1967.0**	792.6	309.6	221.8	41.8	**2892.7**	342.2	249.7	86.2	70.8	**15,029.7**	3823.0	559.7	264.6	98.2
				8	**58.7**	35.5	17.6	8.3	5.3	**775.0**	325.3	163.7	106.7	35.0	**573.2**	259.1	106.5	69.5	34.7	**4989.5**	615.2	310.9	125.4	85.5

Coverages

					Scenario 1					Scenario 2					Scenario 3					Scenario 4				
					ε in dB					ε in dB					ε in dB					ε in dB				
R	*T*	*J*	*p*	*q*	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7
250	100	100	4	3	175.3	152.6	131.4	118.7	105.7	168.4	147.2	126.5	108.3	93.9	168.6	148.4	125.7	106.1	89.6	171.7	148.2	129.1	106.5	90.6
				4	144.8	127.5	115.5	102.1	91.3	140.6	120.9	103.5	91.1	79.6	142.4	120.6	101.4	86.2	74.6	143.9	120.4	102.1	87.1	77.2
				5	128.1	115.9	102.6	91.8	81.4	120.9	103.5	91.6	80.3	72.6	121.7	101.8	86.6	75.1	68.2	120.5	100.8	86.9	77.7	69.6
270	110	110	5	4	175.8	153.3	137.9	124.6	111.3	182.3	158.3	131.2	114.1	101.7	186.1	159.9	132.9	113.3	100.1	175.4	150.5	132.8	116.1	105.3
				5	153.9	138.3	125.2	114.8	100.7	158.7	131.9	114.8	102.1	93.6	161.6	137.2	113.6	100.4	89.5	151.3	131.3	116.8	105.4	95.1
				6	141.1	127.6	113.7	103.3	93.1	136.2	118.5	104.1	95.8	86.8	137.7	116.9	102.7	92.4	81.7	134.1	119.9	107.8	97.9	86.2
290	120	120	6	5	183.9	163.1	147.4	133.1	120.2	180.7	158.3	142.6	129.1	117.2	192.4	163.5	143.1	126.2	111.8	185.2	162.6	144.9	144.1	114.3
				6	166.7	150.5	135.7	122.5	112.1	164.9	144.7	131.6	119.9	108.7	169.6	146.9	132.4	114.8	101.2	166.6	145.5	130.1	117.8	105.3
				7	155.8	140.7	126.7	116.7	104.4	151.4	135.9	123.9	112.7	100.7	153.4	136.4	122.8	105.9	93.1	153.2	135.6	121.4	108.9	96.6
310	130	130	7	6	199.1	181.9	165.6	149.9	133.5	198.8	178.5	159.8	141.9	126.5	202.8	173.7	155.1	138.8	122.5	200.8	176.5	157.3	144.6	124.1
				7	187.8	171.1	155.2	138.8	123.1	184.7	166.1	147.4	131.5	117.4	182.7	161.2	144.6	128.3	111.7	186.8	164.8	145.9	132.1	117.7
				8	178.5	162.2	146.6	129.4	116.1	172.6	155.5	138.3	122.8	109.8	169.6	151.3	134.9	118.9	103.1	172.3	152.5	136.5	122.4	108.6

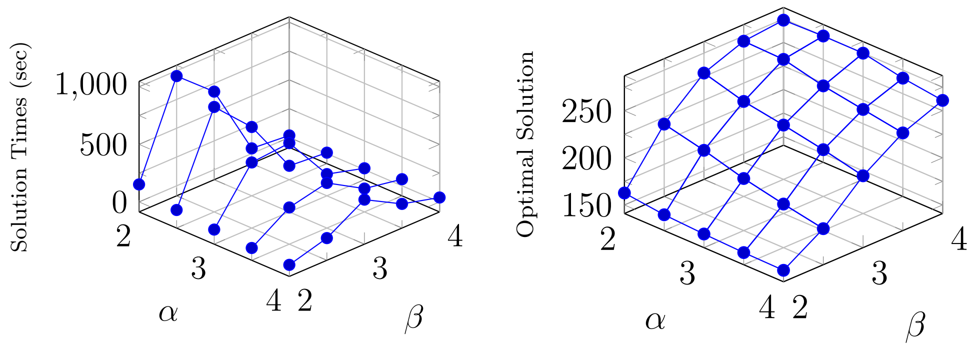

Fig. 7. Sensitivity analysis for the path loss exponent.

the frontline is larger than the number of receivers located at the rear parts of the battlefield, the increment in the number of receivers for which $z_r = 0$ is more than the decrement in the number of receivers for which $z_r = 1$. Consequently, this enables the algorithm to preprocess more variables as ε decreases. Another consequence of this fact is that the optimal solution value uniformly decreases as ε decreases since receivers are more prone to jamming.

6.8. Sensitivity analysis on path loss exponent rates (α, β)

In order to discuss the effects of the path loss exponent rates, we conducted experiments for different values of α and β on a specific problem instance ($R = 290, T = J = 120, P = 6, q = 4$) in Scenario 1. We let α and β vary between 2 and 4 to be able to reflect the situations in which the propagation losses are low and high, respectively. Fig. 7 depicts the solution times and the optimal solution values obtained from these experiments. The results indicate that solution times are larger for intermediate values of β ($\beta = 2.5$ and $\beta = 3$) but considerably lower for other values of β because the proposed solution algorithm can identify more z_r variables to preprocess when $\beta = 2$ (more jamming so rounding down to $z_r = 0$) or when $\beta = 4$ (less jamming so rounding up to $z_r = 1$).

As expected, we obtain the highest coverage when $\alpha = 2, \beta = 4$ and the lowest coverage when $\alpha = 4, \beta = 2$. We also conclude that the optimal value is more sensitive to AT's path loss exponent β rather than DF's path loss exponent α because β becomes more decisive as we add jammer signals in calculating JSR when compared to single transmitter signal effect. In order to find out how path loss exponent rates α and β affect the location decisions of DF and AT, we solved an exemplary instance of Scenario 1 with parameters $R = 250,\ T = J = 100,\ p = 4,$ and $q = 3$ and presented the optimal locations of transmitters and jammers in Fig. 8. The chosen locations indicate that DF locates transmitters very close to the frontline for high β values but prefers the interior of possible transmitter location area for low β values. The location decisions of DF are more sensitive to path loss exponent β and optimal transmitter locations differ only a little for different α values. For a fixed value of α, transmitter locations get closer to the frontline as β increases, i.e., jamming effect decreases. Moreover, we establish that optimal jammer locations of AT are independent from the path loss exponent rate and are always very close to the frontline. The average distance of transmitter locations to the frontline decreases gradually from 2.81 to 0.73 km with a slope of −1.03 as we increase β from 2 to 4. In contrast, the average distance of transmitter locations to the frontline increases from 0.96 to 2.18 km with a slope of 0.61 as we increase α from 2 to 4. When we compare the absolute values of both slopes we conclude that location decisions of transmitters are more sensitive to β than α.

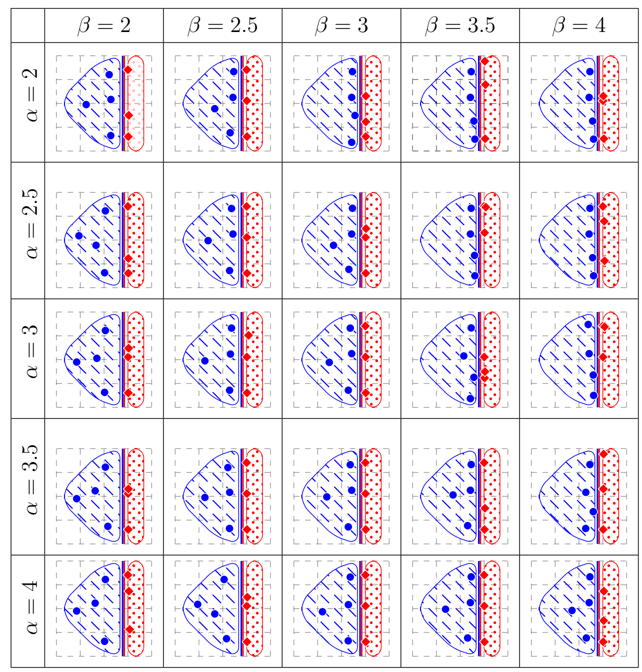

Fig. 8. Optimal transmitter and jammer locations for different values of α and β when $R = 250,\ T = J = 100,\ p = 4,$ and $q = 3$ in Scenario 1.

7. Conclusions

This paper presented results on the Radio Communications Interdiction Problem (RCIP), which identifies the optimal locations of transmitters to construct a robust radio communications network among all military units on the battlefield by anticipating the probable jamming attacks of an intelligent adversary. We incorporated the probabilistic jamming to signal ratio and introduced the probabilistic variant, P-RCIP, in order to include the possible deviation in the received signal power due to geographical obstacles on the battlefield.

Adopting a game theoretic approach, RCIP and P-RCIP were formulated as binary bilevel programming problems and solved by decomposition. In order to improve the solution times, we proposed three enhancements that utilize the dominance relations between possible location sites, preprocessing, and initial starting heuristics. In anticipation of different probable subsequent phases of military operations, we presented four different scenarios and investigated the computational efficacy of the proposed solution methods with different parameters based on these scenarios.

We showed that our treatment of formulating the problem with a bilevel formulation that incorporates the adversarial effect yields considerably better decisions when compared against two fast solution methods, a traditional one in the location literature and one that mimics the decision making process in practice.

We provided some useful tactical insights on transmitter and jammer location decisions by analyzing optimal solutions under varying p and q values in each scenario. The results showed that even though the optimal locations obtained in Scenario 1 are consistent with the expected layout, for other scenarios that reflect the subsequent phases of a military operation, solutions obtained by RCIP outperform the experiential results, highlighting the value of our treatment of RCIP especially in complex military situations. We also presented sensitivity analyses for problem parameters to provide invaluable tactical insights in military communication network design.

Considering that armies are not willing to use a wide variety of transmitters and jammers, we assumed that all the transmitters and jammers are mutually identical in our study. However, as a future research direction, our treatment can be adapted not only to include non-identical transmitters and jammers having different technical and tactical capabilities but also to incorporate sophisticated jammers and transmitters that are far more proficient thanks to new emerging technologies. For instance, rather than using constant jamming, which is energy inefficient, easy to detect but also easy to launch and disruptive, deceptive, random or reactive jammers that can perform advanced jamming techniques may also be considered as a future research direction. The modeling framework will have to be enhanced to consolidate these type of jammers, which are harder to detect and more energy efficient. A challenging future research direction would be to investigate cutting-edge technology function-specific and smart-hybrid jammers, which can either work on a single channel or jam multiple channels and maximize jamming throughput by having frequency and channel hopping capabilities, irrespective of the energy usage (Grover et al., 2014). Finally, integrating transmitters that are capable to use state-of-the-art approaches to avoid jamming attacks such as channel and frequency hopping, jam mapping, spatial retreat, and hybrid techniques may certainly enrich the insights of such a research direction.

Another fruitful research area can be the extension of the static version of RCIP in a setting where receivers are considered as mobile in the direction of the development of the operation. Reflecting the dynamism on the battlefield, not only the location but also the relocation decisions of transmitters and jammers may be included in the analyses.

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