# A Game Theoretic Approach to Channel Switching in the Presence of Jamming

Berk Bozkurt, Ahmet Dundar Sezer, Sinan Gezici, and Tolga Girici

***Abstract*—In this letter, a channel switching problem is investigated in the presence of jamming based on a game theoretic approach. First, a convex formulation of the optimal channel switching problem is proposed for a given jamming strategy. Then, considering a fixed channel switching strategy, an explicit solution of the optimal jammer power allocation problem is obtained. Consequently, a game theoretic formulation is proposed and the existence of a pure-strategy Nash equilibrium is shown for the proposed channel switching game between the transmitter and the jammer.**

***Index Terms*—Channel switching, jamming, Nash equilibrium, capacity, time-sharing, power allocation.**

## I. Introduction

In the availability of multiple communication channels, a transmitter and a receiver can perform channel switching (i.e., time-sharing) among different channels by communicating over only one channel at a given time [1]–[3]. As motivated in [3], channel switching is applied in various scenarios such as in cognitive radio networks, where secondary users can utilize frequency bands of primary users when they are available.

Via channel switching, performance improvements can be achieved in terms of various performance metrics such as the average probability of error, throughput, and channel capacity. In [1] and [2], the aim is to perform optimal channel switching for minimizing the average probability of error. For example, [1] focuses on an average power constrained binary communication system and shows that the average probability of error is minimized by either communicating over one channel exclusively, or switching between two channels with a certain time-sharing factor. The work in [4]–[6] considers channel switching in the context of opportunistic spectrum access in cognitive radio networks. In the presence of multiple frequency bands and channel switching constraints, throughput performance of various bandwidth allocation strategies is investigated in [6]. To maximize the average Shannon capacity between a transmitter and a receiver, optimal channel switching problems are proposed in [3], [7], [8], considering Gaussian channels and the presence of average and peak power constraints. It is shown in [3] that an optimal channel switching strategy can be realized by utilizing at most two different channels. Extensions of the optimal channel switching problem in [3] are performed by considering channel switching delays in [7] and the presence of multiple users in [8].

Although the optimal channel switching problem for average Shannon capacity maximization is investigated in [3], the presence and effects of jamming have not been considered. In this letter, we focus on the channel switching problem in the presence of a jammer, and propose a game theoretic approach by deriving the optimal strategies for both the channel switching problem in the presence of jamming and the jamming problem for a given channel switching strategy. In the literature, there exist various game theoretic investigations of anti-jamming problems in the presence of multiple channels, which employ Stackelberg equilibrium, Markov decision process (MDP), and Nash equilibrium concepts [9]–[11]. However, they do not employ the average capacity metric in the utility functions and the channel switching approach adopted in this manuscript. In addition, based on a hierarchical approach, Stackelberg game formulations between users and jammers are proposed in [12]–[14]. For example, a hierarchical power control algorithm is developed in [12] to obtain the Stackelberg equilibrium by modeling the user as the leader and the jammer as the follower. The studies in [12]–[14] differ from our manuscript since they employ the Stackelberg equilibrium concept and different utility functions in the absence of channel switching. Without any hierarchy or commitment assumptions, we employ the Nash equilibrium concept in this work, where players announce their strategies simultaneously (also see Footnote 2). The main contributions of this letter are as follows:

- The optimal channel switching problem in the presence of a jammer is formulated under average and peak power constraints, and an equivalent convex optimization problem is obtained (Proposition 1) for the first time in the literature.
- For a given channel switching strategy, the optimal power allocation problem for the jammer is formulated as a convex optimization problem and its solution is characterized explicitly (Proposition 2).
- A channel switching game is formulated between the transmitter and the jammer, and it is shown that the game admits at least one pure-strategy Nash equilibrium (Proposition 3). Also, how to calculate the Nash equilibrium is discussed.

## II. System Model

Consider a communication system in which a transmitter and a receiver communicate with each other via channel switching among $K$ different channels (frequency bands). In particular, the transmitter and the receiver can switch among $K$ channels to enhance the average capacity of the communication system as in [3]. In this channel switching approach, only one channel is utilized for the communication between the transmitter and the receiver at any given time, and the transmitter informs the receiver about the occupied channel in order for the receiver to be synchronized during communication [3]. The channels are modeled as flat-fading additive Gaussian noise channels with various bandwidths and constant power spectral density levels. Besides the transmitter and the receiver in the communication system, there exists a jammer which tries to reduce the average channel capacity between the transmitter and the receiver. It is assumed that the jammer transmits zero-mean Gaussian noise with constant power spectral density levels over the considered channel bandwidths [15], [16] for distorting the communication between the transmitter and the receiver, and that the jammer

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can transmit over multiple channels in parallel at a given time In other words, although the transmitter and the receiver communicate with each other by employing time-sharing among channels, the jammer concurrently transmits noise over the channels that are considered in the jamming strategy during the whole communication duration.

## III. Formulation and Game Theoretic Analysis

Consider the presence of $K$ channels that are available between the transmitter and the receiver for communication via channel switching, as described in Section II. For channel $i$, let $B_i$ and $N_i/2$ denote, respectively, the bandwidth and the constant power spectral density level of the additive Gaussian noise, where $i \in \{1,\ldots,K\}$. Also, let $h_i^T$ and $h_i^J$ represent the complex channel gains related to channel $i$ from the transmitter to the receiver and from the jammer to the receiver, respectively. Then, the capacity of channel $i$ between the transmitter and the receiver in the presence of the jammer is

$$C_i(P^T,P^J) = B_i \log_2\left(1+\frac{\left|h_i^T\right|^2 P^T}{\left|h_i^J\right|^2 P^J + N_i B_i}\right) \text{ bps} \quad (1)$$

where $P^T$ and $P^J$ represent the average transmit powers of the transmitter and the jammer, respectively.

### *A. Optimal Channel Switching in the Presence of Jammer*

In order to formulate the optimal channel switching problem in the presence of the jammer, time-sharing (channel switching) factors are defined as $\lambda_1,\ldots,\lambda_K$, where $\lambda_i$ represents the fraction of time when channel $i$ is utilized by the transmitter for communication with the receiver. Then, the optimal channel switching problem in the presence of the jammer can be expressed for a given jamming strategy as follows:

$$\max_{\{\lambda_i, P_i^T\}_{i=1}^K} \sum_{i=1}^K \lambda_i\, C_i(P_i^T, P_i^J) \quad (2a)$$

$$\text{subject to } \sum_{i=1}^K \lambda_i\, P_i^T \leq P_{\text{av}}^T\,, \quad (2b)$$

$$P_i^T \in [0, P_{\text{pk}}^T]\,,\ \forall i \in \{1,\ldots,K\} \quad (2c)$$

$$\sum_{i=1}^K \lambda_i = 1,\ \lambda_i \geq 0,\ \forall i \in \{1,\ldots,K\} \quad (2d)$$

where $C_i(P_i^T, P_i^J)$ is as in (1), $P_i^T$ and $P_i^J$ represent the average transmit powers allocated to channel $i$ by the transmitter and the jammer, respectively, $P_{\text{pk}}^T$ denotes the peak power limit of the transmitter, and $P_{\text{av}}^T$ represents the average power limit for the transmitter. The average power constraint at the transmitter can be regarded as the power consumption constraint and/or the battery life constraint at the transmitter. On the other hand, the peak power constraint corresponds to the maximum power level that can be delivered by the transmitter circuitry (i.e., a hardware constraint) [3].

Unlike [3], which considers a similar problem to that in (2) (see [3, eqn. (2)]) and proposes a solution based on the optimality of channel switching between at most two different channels, we transform the problem in (2) into an equivalent problem and prove its convexity in the following proposition.

**Proposition 1:** *For a given jamming strategy denoted by $\{P_i^J\}_{i=1}^K$, the optimization problem in* (2) *can be transformed into the following problem:*

$$\max_{\{\lambda_i, \widetilde{P}_i^T\}_{i=1}^K} \sum_{i=1}^K \lambda_i\, C_i\big(\widetilde{P}_i^T/\lambda_i\,, P_i^J\big) \quad (3a)$$

$$\text{subject to } \sum_{i=1}^K \widetilde{P}_i^T \leq P_{\text{av}}^T\,, \quad (3b)$$

$$\widetilde{P}_i^T \geq 0,\ \widetilde{P}_i^T - \lambda_i P_{\text{pk}}^T \leq 0,\ \forall i \in \{1,\ldots,K\} \quad (3c)$$

$$\sum_{i=1}^K \lambda_i = 1,\ \lambda_i \geq 0,\ \forall i \in \{1,\ldots,K\} \quad (3d)$$

*where $\widetilde{P}_i^T \triangleq \lambda_i P_i^T$ for $i \in \{1,\ldots,K\}$. The problem in* (3) *is a convex optimization problem.*

*Proof:* Based on the definition $\widetilde{P}_i^T \triangleq \lambda_i P_i^T$, (2) can be transformed into (3). Then, to prove the concavity of the objective function in (3a), we define $\beta_i \triangleq \widetilde{P}_i^T$ for notational simplicity and obtain the following relation for any $\alpha \in (0,1)$, $\{\lambda_i^{(1)}, \beta_i^{(1)}\}_{i=1}^K$, and $\{\lambda_i^{(2)}, \beta_i^{(2)}\}_{i=1}^K$:

$$\sum_{i=1}^K \big(\alpha\lambda_i^{(1)} + (1-\alpha)\lambda_i^{(2)}\big)\, C_i\left(\frac{\alpha\beta_i^{(1)} + (1-\alpha)\beta_i^{(2)}}{\alpha\lambda_i^{(1)} + (1-\alpha)\lambda_i^{(2)}}\,, P_i^J\right)$$
$$\geq \alpha \sum_{i=1}^K \lambda_i^{(1)} C_i\left(\frac{\beta_i^{(1)}}{\lambda_i^{(1)}}\,, P_i^J\right) + (1-\alpha)\sum_{i=1}^K \lambda_i^{(2)} C_i\left(\frac{\beta_i^{(2)}}{\lambda_i^{(2)}}\,, P_i^J\right)$$

where the inequality follows from the concavity of $C_i(x, P_i^J)$ with respect to $x \geq 0$ (see (1)). Hence, the objective function in (3a) is shown to be concave. (The function $\lambda_i\, C_i(x/\lambda_i, P_i^J)$ can be regarded as the *perspective* [17] of $C_i(x, P_i^J)$ and its concavity follows from the concavity of $C_i(x, P_i^J)$.) In addition, it is noted that all the constraints in (3) are linear. Therefore, (3) is a convex optimization problem. ■

Based on Proposition 1, the solution of the optimization problem in (2) can be obtained by solving the optimization problem in (3). Since (3) is a convex optimization problem, it can efficiently be solved by interior-point methods, which are very fast in practice. Once the solution of (3) is obtained, the solution of (2) can be calculated based on the definition $\widetilde{P}_i^T \triangleq \lambda_i P_i^T$ for $i = 1,\ldots,K$.

**Remark 1:** *The formulation of the optimal channel switching problem as the convex problem in Proposition 1 has not been available in the related studies such as [3], [7], which employ different algorithms to obtain the optimal channel switching strategy by utilizing at most two different channels. When Nash equilibria of the channel switching game between the transmitter and the jammer involve channel switching among more than two channels, such equilibria cannot be obtained without the formulation in Proposition 1 (cf. Table I).*

### *B. Optimal Jamming Strategy against Channel Switching*

For a given channel switching strategy between the transmitter and the receiver specified by $\{\lambda_i, \widetilde{P}_i^T\}_{i=1}^K$, the optimal jamming strategy problem can be formulated as follows:

$$\min_{\{P_i^J\}_{i=1}^K} \sum_{i=1}^K \lambda_i\, C_i(\widetilde{P}_i^T/\lambda_i, P_i^J) \quad (4a)$$

$$\text{subject to } \sum_{i=1}^K P_i^J \leq P_{\text{tot}}^J\,, \quad (4b)$$

$$P_i^J \geq 0\,,\ \forall i \in \{1,\ldots,K\} \quad (4c)$$

where $P_i^J$ denotes the average transmit power allocated to channel $i$ by the jammer as in (2), and $P_{\rm tot}^J$ represents the total power limit of the jammer. Since the jammer can emit signals to all the channels simultaneously, a total power constraint is employed in (4) instead of the average power constraint in (2).

The problem in (4) is specified as a convex problem and its solution is presented in the following proposition.

**Proposition 2:** *For a given channel switching strategy denoted by $\{\lambda_i, \widetilde{P}_i^T\}_{i=1}^K$, the problem in (4) is a convex optimization problem with the following solution:*

$$\bar{P}_i^J = \max\left\{\left(\sqrt{\left(\frac{\left|h_i^T\right|^2 \widetilde{P}_i^T}{2\left|h_i^J\right|^2\lambda_i}\right)^2 + \frac{\left|h_i^T\right|^2 B_i \widetilde{P}_i^T}{\ln(2)\left|h_i^J\right|^2\gamma}} - \frac{N_i B_i}{\left|h_i^J\right|^2} - \frac{\left|h_i^T\right|^2 \widetilde{P}_i^T}{2\left|h_i^J\right|^2\lambda_i}\right), 0\right\}, \forall i \in \{1,\dots,K\} \quad (5)$$

*where $\widetilde{P}_i^T/\lambda_i \triangleq 0$ for $\lambda_i = \widetilde{P}_i^T = 0$, and $\gamma$ is a Karush-Kuhn-Tucker (KKT) multiplier, which is calculated by solving*

$$\sum_{i=1}^K \bar{P}_i^J = P_{\rm tot}^J\,. \quad (6)$$

*Proof:* Based on the expression in (1), the second-order derivative of $C_i(x, P_i^J)$ with respect to $P_i^J$ can be shown to be positive for all $P_i^J \geq 0$, $x \geq 0$, and $i \in \{1,\dots,K\}$; hence, $C_i(x, P_i^J)$ is a convex function of $P_i^J$. Since the objective function in (4a) is a nonnegative weighted sum of convex functions, it is also convex [18]. Therefore, together with the linear constraints in (4b) and (4c), the problem in (4) becomes a convex optimization problem. Hence, the KKT conditions present necessary and sufficient conditions for optimality. We first obtain the Lagrangian function for (4) as follows:

$$\mathcal{L}(\boldsymbol{P^J}, \gamma, \boldsymbol{\nu}) = \sum_{i=1}^K \lambda_i C_i\left(\frac{\widetilde{P}_i^T}{\lambda_i}, P_i^J\right) + \gamma\left(\sum_{i=1}^K P_i^J - P_{\rm tot}^J\right) - \sum_{i=1}^K \nu_i P_i^J \quad (7)$$

where $\boldsymbol{P^J} = [P_1^J \cdots P_K^J]^{\mathsf T}$, and $\gamma$ and $\boldsymbol{\nu} = [\nu_1 \cdots \nu_K]^{\mathsf T}$ denote the KKT multipliers related to the constraints in (4b) and (4c), respectively. Among the KKT conditions, the stationarity condition is employed first by setting the partial derivatives of (7) with respect to $P_i^J$ to zero. Based on the expressions in (1) and (7), the stationarity condition leads to the following equalities after some manipulation:

$$\frac{\lambda_i B_i\left|h_i^J\right|^2\left|h_i^T\right|^2 \widetilde{P}_i^T/(\lambda_i \ln(2))}{\left(N_i B_i + \left|h_i^J\right|^2 P_i^J\right)^2 + \frac{1}{\lambda_i}\left(N_i B_i + \left|h_i^J\right|^2 P_i^J\right)\left|h_i^T\right|^2 \widetilde{P}_i^T} = \gamma - \nu_i \quad (8)$$

for $i = 1,\dots,K$. Also, the primal feasibility condition refers to the inequalities in (4b) and (4c), and the dual feasibility condition implies that $\nu_i \geq 0$ for all $i \in \{1,\dots,K\}$ and $\gamma \geq 0$. In addition, the complementary slackness condition can be stated as

$$\gamma\left(\sum_{i=1}^K P_i^J - P_{\rm tot}^J\right) = 0,\ \nu_i P_i^J = 0,\ i \in \{1,\dots,K\} \quad (9)$$

Based on the KKT conditions, it is first concluded that, for the solution of (4) represented by $\{\bar{P}_i^J\}_{i=1}^K$, the total power constraint must be satisfied with equality; that is, $\sum_{i=1}^K \bar{P}_i^T = P_{\rm tot}^J$ must hold. Otherwise, $\gamma$ would be zero due to the first complementary slackness condition in (9) and it would become impossible to satisfy both the stationarity condition in (8) and the dual feasibility condition of $\nu_i \geq 0$ simultaneously. Next, from the second complementary slackness condition in (9), it is concluded that for all positive power levels, i.e., $P_i^J > 0$, the corresponding $\nu_i$'s must be zero. Then, the expressions on the left-hand-side of (8) must be equal for all positive power levels, $P_i^J$. By setting $\nu_i = 0$, (8) becomes a second-order polynomial in terms of $P_i^J$, which can be stated as $\left(N_i B_i + \left|h_i^J\right|^2 P_i^J\right)^2 + \left(N_i B_i + \left|h_i^J\right|^2 P_i^J\right)\left|h_i^T\right|^2 \widetilde{P}_i^T/\lambda_i - \lambda_i B_i\left|h_i^J\right|^2\left|h_i^T\right|^2 \widetilde{P}_i^T/(\gamma\,\lambda_i \ln(2)) = 0$. It can be shown that one root of this second-order polynomial is always negative, hence, not a valid solution due to the primal feasibility condition, and the other root is given by the first expression inside the maximum operator in (5). If the latter root is positive for a given channel index $i$, it is the solution of the optimization problem for that channel. Otherwise, the power level must be zero for that channel to satisfy the stationary condition in (8) with $\nu_i > 0$ (see (9)). Overall, the optimal power levels corresponding to the solution of (4) can be expressed as the maximum of zero and the specified root of the polynomial, as stated in (5). To calculate the KKT multiplier $\gamma$ in (5), the full power utilization property can be used, as stated in (6). ∎

Proposition 2 characterizes the problem in (4) as a convex optimization problem and specifies the optimal jamming strategy for a given channel switching strategy via (5) and (6).

### *C. Channel Switching Game and Nash Equilibrium*

Due to the conflicting aims of the transmitter and the jammer, a game theoretic formulation is well suited for the considered problem. Let $\mathcal{G} = \langle \mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}}\rangle$ denote the channel switching game between the transmitter (i.e., player $T$) and the jammer (i.e., player $J$) in the presence of complete information [19], where $\mathcal{N} = \{T, J\}$ is the index set for the players, $S_i$ is the strategy set for player $i$, and $u_i$ is the utility function of player $i$. For the transmitter, the strategy set $S_T$ is defined as

$$S_T \triangleq \big\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}} \in \mathbb{R}^K |\ \mathbf{1}^{\mathsf T}\widetilde{\boldsymbol{P}}^{\boldsymbol{T}} \leq P_{\rm av}^T \wedge 0 \leq \boldsymbol{e}_i^{\mathsf T}\widetilde{\boldsymbol{P}}^{\boldsymbol{T}} \leq P_{\rm pk}^T\big(\boldsymbol{e}_i^{\mathsf T}\boldsymbol{\lambda}\big), \forall i \in \{1,\dots,K\} \wedge \boldsymbol{e}_i^{\mathsf T}\boldsymbol{\lambda} \geq 0, \forall i \in \{1,\dots,K\} \wedge \mathbf{1}^{\mathsf T}\boldsymbol{\lambda} = 1\big\} \quad (10)$$

where $\boldsymbol{\lambda} = [\lambda_1 \cdots \lambda_K]^{\mathsf T}$, $\widetilde{\boldsymbol{P}}^{\boldsymbol{T}} = \big[\widetilde{P}_1^T \cdots \widetilde{P}_K^T\big]^{\mathsf T}$, $\mathbf{1}$ is the vector of ones, $\boldsymbol{e}_i$ is the unit vector with its $i$th element being one and others being zero, $K$ is the dimension of $\boldsymbol{\lambda}$ and $\widetilde{\boldsymbol{P}}^{\boldsymbol{T}}$, and $P_{\rm av}^T$ and $P_{\rm pk}^T$ are as in (3). Similarly, the strategy set $S_J$ for the jammer node is defined as

$$S_J \triangleq \big\{\boldsymbol{P^J} \in \mathbb{R}^K |\ \mathbf{1}^{\mathsf T}\boldsymbol{P^J} \leq P_{\rm tot}^J \wedge \boldsymbol{e}_i^{\mathsf T}\boldsymbol{P^J} \geq 0, \forall i \in \{1,\dots,K\}\big\} \quad (11)$$

where $\boldsymbol{P^J} = \big[P_1^J\ \cdots\ P_K^J\big]^{\mathsf T}$ and $P_{\rm tot}^J$ are as in (4).

Let $\big\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\big\}$ and $\boldsymbol{P^J}$ denote the strategies of player $T$ and player $J$, respectively. Then, a strategy (action) profile of the game can be denoted by $\big(\big\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\big\}, \boldsymbol{P^J}\big) \in S$, where $\big\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\big\} \in S_T$, $\boldsymbol{P^J} \in S_J$, and $S = S_T \times S_J$. For a given action profile, the utility functions of player $T$ and player $J$ are defined as

$$u_T\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P^J}\big) = \sum_{i=1}^K \lambda_i C_i\left(\frac{\widetilde{P}_i^T}{\lambda_i}, P_i^J\right) = -u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P^J}\big) \quad (12)$$

As $u_T\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ and $u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ satisfy $u_T\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big) + u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big) = 0$, $\forall\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\} \in S_T$ and $\forall \boldsymbol{P}^{\boldsymbol{J}} \in S_J$, it is concluded that the channel switching game between player $T$ and player $J$ corresponds to a two-player zero-sum game [19].

The Nash equilibrium is one of the solution approaches that is commonly used for game theoretic problems [19]. In the game-theoretic notation, a strategy profile of game $\mathcal{G}$, denoted by $\big(\{\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star\}, \boldsymbol{P}^{\boldsymbol{J}}_\star\big)$, is a Nash equilibrium if $u_T\big(\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}_\star\big) \geq u_T\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}_\star\big)$, $\forall\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\} \in S_T$ and $u_J\big(\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}_\star\big) \geq u_J\big(\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}\big)$, $\forall \boldsymbol{P}^{\boldsymbol{J}} \in S_J$. That is, at a Nash equilibrium, no player can improve its utility by changing its strategy unilaterally. Such an equilibrium does not necessarily exist in infinite games. However, the channel switching game $\mathcal{G}$ admits a pure-strategy Nash equilibrium as stated in the following proposition.

**Proposition 3:** *A pure-strategy Nash equilibrium exists in the channel switching game $\mathcal{G}$.*

*Proof:* The channel switching game $\mathcal{G}$ in the strategic form $\langle \mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}}\rangle$ admits at least one pure-strategy Nash equilibrium if the following conditions hold [19]: (**i**) Strategy set $S_i$ is compact and convex for all $i \in \mathcal{N}$, where $\mathcal{N} = \{T, J\}$. (**ii**) $u_i\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ is a continuous function in the profile of strategies $\big(\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\}, \boldsymbol{P}^{\boldsymbol{J}}\big) \in S$ for all $i \in \mathcal{N}$. (**iii**) $u_T(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}})$ and $u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ are quasi-concave functions in $\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\}$ and $\boldsymbol{P}^{\boldsymbol{J}}$, respectively. Since set $S_T$ in (10) and set $S_J$ in (11) are closed and bounded, it can be shown that these sets are compact. Also, the sets in (10) and (11) are convex, as discussed in the proofs of Propositions 1 and 2. Hence, the first condition is satisfied. In addition, $u_T(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}})$ and $u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ in (12) are continuous functions, for which the second condition holds. Regarding the third condition, it is proved in Proposition 1 that $u_T(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}})$ is a concave function of $\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\}$, and it is deduced from the proof of Proposition 2 that $u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ is a concave function of $\boldsymbol{P}^{\boldsymbol{J}}$. Consequently, $u_T(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}})$ and $u_J\big(\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$ are quasi-concave functions in $\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\}$ and $\boldsymbol{P}^{\boldsymbol{J}}$, respectively, as specified in the third condition. Overall, it is concluded that at least one pure-strategy Nash equilibrium exists in the channel switching game $\mathcal{G}$. ■

For analyzing the Nash equilibrium, the best response functions of player $T$ and player $J$ should be specified. For a given strategy of player $J$, denoted by $\boldsymbol{P}^{\boldsymbol{J}}$, the best response function of player $T$ can be stated as $\big\{\boldsymbol{\lambda}_{\text{BR}}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_{\text{BR}}\big\} = \text{BR}_T\big(\boldsymbol{P}^{\boldsymbol{J}}\big) \triangleq \arg\max_{\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\}\in S_T} \sum_{i=1}^{K} \lambda_i\, C_i\big(\widetilde{P}_i^T/\lambda_i\,, P_i^J\big)$. Similarly, for a given strategy of player $T$, the best response function of player $J$ is expressed as $\boldsymbol{P}^{\boldsymbol{J}}_{\text{BR}} = \text{BR}_J\big(\boldsymbol{\lambda}\,, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\big) \triangleq \arg\max_{\boldsymbol{P}^{\boldsymbol{J}}\in S_J} -\sum_{i=1}^{K} \lambda_i\, C_i\big(\widetilde{P}_i^T/\lambda_i\,, P_i^J\big)$. Considering the best response functions together, the following function is defined: $\text{BR}(\boldsymbol{\theta}) = (\text{BR}_T, \text{BR}_J)$ from $S$ to $S$, where $S = S_T \times S_J$ and $\boldsymbol{\theta} \triangleq \big(\boldsymbol{\lambda}\,, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}, \boldsymbol{P}^{\boldsymbol{J}}\big)$. For the Nash equilibrium, denoted by $\boldsymbol{\theta}_\star \triangleq \big(\boldsymbol{\lambda}_\star\,, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}_\star\big)$, the following fixed point equation holds [19]: $\boldsymbol{\theta}_\star = \text{BR}(\boldsymbol{\theta}_\star)$. Since the utility functions in (12) are concave functions of $\big\{\boldsymbol{\lambda}, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\big\}$ and $\boldsymbol{P}^{\boldsymbol{J}}$, respectively, the channel switching game between player $T$ and player $J$ becomes a convex-concave game [18], [20].[1] In such a game, the Nash equilibrium becomes the saddle-point equilibrium; hence, the pure-strategy Nash equilibrium, $\boldsymbol{\theta}_\star = \big(\boldsymbol{\lambda}_\star\,, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}_\star\big)$, of the game $\mathcal{G}$ satisfies the following relation [16], [20]:

$$\begin{aligned} u_T\big(\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}_\star\big) &= -u_J\big(\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}_\star, \boldsymbol{P}^{\boldsymbol{J}}_\star\big) \\ &= \max_{\{\boldsymbol{\lambda}\,, \widetilde{\boldsymbol{P}}^{\boldsymbol{T}}\}\in S_T} \min_{\boldsymbol{P}^{\boldsymbol{J}}\in S_J} \sum_{i=1}^{K} \lambda_i\, C_i\big(\widetilde{P}_i^T/\lambda_i\,, P_i^J\big) \end{aligned} \quad (13)$$

From (13), a pure-strategy Nash equilibrium of the channel switching game $\mathcal{G}$ can be obtained. Based on the convexity result in Proposition 1, the maximization problem in (13) can be solved via convex optimization tools by calculating the solution of the minimization problem in (13) via the result in Proposition 2. Also, each Nash equilibrium obtained from (13) is Pareto optimal as in all two-player zero-sum games [19].[2]

**Remark 2:** *The results in this section can be applied to a secondary user in a cognitive radio system with the overlay approach [3]. For the underlay approach, a secondary user is affected from interference due to primary users. If that interference is modeled by a Gaussian noise process [7], then the results can still be applied by extending the capacity formula in* (1) *as* $C_i(P^T, P^J) = B_i \log_2\big(1 + \big|h_i^T\big|^2 P^T/\big(\big|h_i^J\big|^2 P^J + 2B_i(0.5N_i + I_i)\big)\big)$ bps, *where* $I_i$ *denotes the spectral density level of the interference in channel* $i$.

**Remark 3:** *The channels between the transmitter and the receiver can be determined based on pilot based channel estimation approaches, and the jammer can learn the channel coefficients by listening to signal exchanges between the transmitter and the receiver and by performing estimations based on the knowledge of geographical locations [21].*

## IV. Numerical Results and Conclusions

In this section, we provide numerical examples to illustrate the theoretical results in Section III by deriving Nash equilibria in various scenarios. We consider $K = 4$ channels with the following parameters: $N_1 = 10^{-12}$ W/Hz, $N_2 = 2 \times 10^{-12}$ W/Hz, $N_3 = 5 \times 10^{-12}$ W/Hz, $N_4 = 10^{-11}$ W/Hz, and $B_1 = B_2 = B_3 = B_4 = 1$ MHz. Also, considering Rayleigh fading channels between the transmitter and the receiver and between the jammer and the receiver, we model $|h_i^T|^2$ and $|h_i^J|^2$ as i.i.d. exponential random variables with a mean parameter of $10^{-4}$ and generate them in MATLAB with seed 1, which results in $\big(|h_1^T|^2, |h_2^T|^2, |h_3^T|^2, |h_4^T|^2\big) = \big(0.87462, 0.32805, 9.0760, 1.1962\big) \times 10^{-4}$ and $\big(|h_1^J|^2, |h_2^J|^2, |h_3^J|^2, |h_4^J|^2\big) = \big(1.919, 2.3823, 1.6806, 1.0626\big) \times 10^{-4}$. In addition, the peak power constraint in (2c) is given by $P^T_{\text{pk}} = 200$ W. In the simulations, Nash equilibria are obtained for the channel switching game between the transmitter and the jammer, and the resulting capacity values (see (12)) are calculated.

In Fig. 1, the capacities achieved at Nash equilibria of the channel switching game are plotted versus $P^T_{\text{av}}$ and $P^J_{\text{tot}}$ by considering three different total jammer powers and average transmit powers, respectively. As expected, lower capacities are achieved when the total jammer power is higher, and the capacity increases with $P^T_{\text{av}}$. It is also noted that while the jammer noise significantly affects the capacity at low values of $P^T_{\text{av}}$, it becomes less significant at high average transmit powers. Moreover, at low total jammer powers, the capacities

[1] In convex-concave games, if there exist multiple Nash equilibria, the value of the game is the same (unique) for each Nash equilibrium.

[2] If the game between the transmitter and the jammer is modeled as a Stackelberg game in which the transmitter is the leader and moves first and the jammer is the follower and moves after the transmitter, it can be solved via backward induction by utilizing the result in Proposition 2.

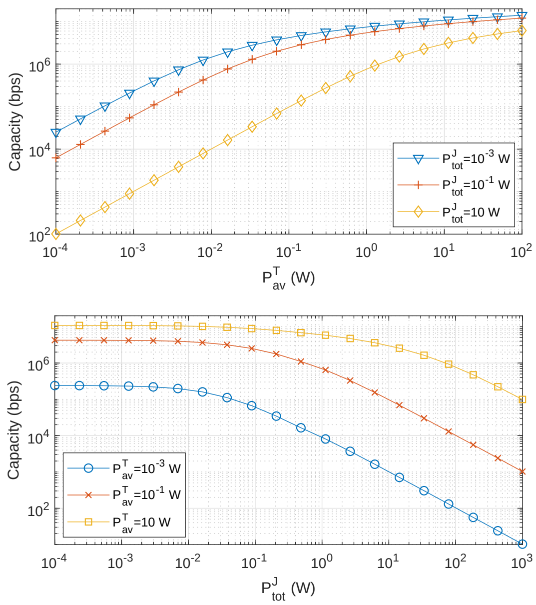


Fig. 1. Capacity versus $P_{\text{av}}^T$ ($P_{\text{tot}}^J$) for Nash equilibria of the channel switching game with various total jammer powers (average transmit powers).

are almost constant since each jammer noise component is insignificant compared to the corresponding channel noise term (the second term in the denominator in (1)) in that region.

To investigate the strategies of the transmitter and the jammer at Nash equilibria, Table I is presented, where $(\boldsymbol{\lambda}_\star, \widetilde{\boldsymbol{P}}_\star^{\boldsymbol{T}}, \boldsymbol{P}_\star^{\boldsymbol{J}})$ specify the Nash equilibrium in a given setting as in (13). It is observed that when the total jammer power is low; i.e., when $P_{\text{tot}}^J = 10^{-3}$ W, the jammer allocates all of its power to channel 3 (i.e., to the best channel in the absence of jamming) and the transmitter employs channel 3 exclusively since it is still the best one in the presence of jamming. (This single-channel solution is the Nash equilibrium for all $P_{\text{tot}}^J \leq 0.031995$ W.) When the total jammer power increases to $P_{\text{tot}}^J = 10^{-1}$ W, the jammer distributes its power among the best two channels such that they become identical due to jammer noise (and the other two channels remain worse). In response to this strategy, when the transmitter employs these two channels with specific time-sharing factors at the average power limit of $0.1$ W, a Nash equilibrium is formed as specified in Table I. A similar scenario is also observed for $P_{\text{tot}}^J = 10$ W, where all the channels are utilized. When $P_{\text{tot}}^J$ is sufficiently high, the max-min problem in (13) leads to an equalizer solution that is achieved by using all the channels; hence, there exist no Nash equilibria without employing all the channels.

For comparison, the transmitter and the jammer can randomly and independently select which channel to use among the four channels and employ them at the available power limits in each communication interval. Then, the resulting capacities become $5.8286\times10^5$, $5.1794\times10^5$, and $4.4071\times10^5$ bps for $P_{\text{tot}}^J = 10^{-3}$ W, $P_{\text{tot}}^J = 10^{-1}$ W, and $P_{\text{tot}}^J = 10$ W, respectively when $P_{\text{av}}^T = 10^{-1}$ W. Compared to the capacity values in Table I, the random channel switching leads to lower (higher) capacities when the total jammer power is low (high) since it is not an optimal approach for neither the transmitter nor the jammer, and this non-optimality becomes more crucial for the transmitter (jammer) when $P_{\text{tot}}^J$ is low (high) compared to $P_{\text{av}}^T$.

TABLE I
STRATEGIES OF TRANSMITTER AND JAMMER AT NASH EQUILIBRIA.

| | | | | |
|---|---|---|---|---|
| | $P_{\text{av}}^T = 10^{-1}$ W, $P_{\text{tot}}^J = 10^{-3}$ W | | | |
| $\boldsymbol{P}_\star^{\boldsymbol{J}}$ (W) | 0 | 0 | $10^{-3}$ | 0 |
| $\boldsymbol{\lambda}_\star$ | 0 | 0 | 1 | 0 |
| $\widetilde{\boldsymbol{P}}_\star^{\boldsymbol{T}}$ (W) | 0 | 0 | $10^{-1}$ | 0 |
| Capacity | $4.2143 \times 10^6$ bps | | | |
| | $P_{\text{av}}^T = 10^{-1}$ W, $P_{\text{tot}}^J = 10^{-1}$ W | | | |
| $\boldsymbol{P}_\star^{\boldsymbol{J}}$ (W) | 0.0052926 | 0 | 0.094707 | 0 |
| $\boldsymbol{\lambda}_\star$ | 0.077827 | 0 | 0.92217 | 0 |
| $\widetilde{\boldsymbol{P}}_\star^{\boldsymbol{T}}$ (W) | 0.0077827 | 0 | 0.092217 | 0 |
| Capacity | $2.4166 \times 10^6$ bps | | | |
| | $P_{\text{av}}^T = 10^{-1}$ W, $P_{\text{tot}}^J = 10$ W | | | |
| $\boldsymbol{P}_\star^{\boldsymbol{J}}$ (W) | 0.64374 | 0.18768 | 7.6597 | 1.5088 |
| $\boldsymbol{\lambda}_\star$ | 0.064016 | 0.019341 | 0.75852 | 0.15812 |
| $\widetilde{\boldsymbol{P}}_\star^{\boldsymbol{T}}$ (W) | 0.0064016 | 0.0019341 | 0.075852 | 0.015812 |
| Capacity | $9.7923 \times 10^4$ bps | | | |

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