

# The Fractional Fourier Transform

## with Applications in Optics and Signal Processing—Supplementary Bibliography

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This is a supplement to the “Bibliography on the Fractional Fourier Transform” found on pages 467–483 of the book *The Fractional Fourier Transform with Applications in Optics and Signal Processing*, by Haldun M. Ozaktas, Zeev Zalevsky, and M. Alper Kutay, John Wiley & Sons, 2000. Wiley Series in Pure and Applied Optics, *xviii* + 513 pages, hardcover, ISBN 0471 96346 1. A small number of supplementary items to the “Other Cited Works” on pages 485–496 of the book have also been included at the end in separate alphabetical sequence.

Taken together, the bibliography in the book and this supplement is meant to be as complete as possible as far as archival publications are concerned. Conference proceedings and other publications are included selectively. It is intended to also include selected works which are very closely related to fractional Fourier transforms, even if they do not directly deal with fractional Fourier transforms.

Presently, items published up to about the beginning of 2002 are included.

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