Elec 201
Midterm 2

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Duration: 90 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NAME: ____

SIGNATURE: ____________________________________

Problem	Points
1	
2	
3	
TOTAL	

You may or may not need the following formulas and attached Table:

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\cos^2\theta = \tfrac{1}{2}\left(1 + \cos 2\theta\right)$$

Summary of Fourier Relations

Time	Frequency	Formulae
Continuous Periodic	Aperiodic Discrete	$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0=\frac{2\pi}{T_0}$ $a_k=\frac{1}{T_0}\int_{<T_0>} x(t)e^{-jkw_0 t}dt$ $X(w)=\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega-k\omega_0)$
Continuous Aperiodic	Aperiodic Continuous	$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{jwt}dw$ $X(w)=\int_{-\infty}^{\infty} x(t)e^{-jwt}dt$
Discrete Periodic	Periodic Discrete	$x[n]=\sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$ $a_k=\frac{1}{N}\sum_{n=<N>} x[n]e^{-j\frac{2\pi}{N}kn}$ $X(\theta)=\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta-\frac{2\pi}{N}k); a_{k+N}=a_k$
Discrete Aperiodic	Periodic Continuous	$x[n]=\frac{1}{2\pi}\int_{<2\pi>} X(\theta)e^{j\theta n}d\theta$ $X(\theta)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\theta n}$

$$\mathrm{sinc}(t)=\frac{\sin(\pi t)}{\pi t}$$

$$x(t)=\frac{\sin(\omega_0 t)}{\pi t} \quad \leftrightarrow \quad X(\omega)=\begin{cases}1 & |\omega|<\omega_0\\ 0 & \text{otherwise}\end{cases}$$

$$x(t)=\begin{cases}1 & |t|<T_1\\ 0 & \text{otherwise}\end{cases} \quad \leftrightarrow \quad X(\omega)=\frac{2\sin(\omega T_1)}{\omega}$$

$$x[n]=\begin{cases}1 & |n|\le N_1\\ 0 & \text{otherwise}\end{cases} \quad \leftrightarrow \quad X(\Omega)=\frac{\sin(\Omega(N_1+1/2))}{\sin(\Omega/2)}$$

$$x[n]=\frac{\sin(a\pi n)}{\pi n} \quad \leftrightarrow \quad X(\Omega)=\begin{cases}1 & |\Omega|<a\pi\\ 0 & a\pi<|\Omega|<\pi\end{cases}$$

PROBLEM 1: (40 Points)

1.

Find Fourier Transform of the following signals:

a. $x[n] = \cos(\frac{\pi}{3}n)\sin(\frac{\pi}{5}n)\left(\frac{1}{2}\right)^{|n|}$ (8 Points)

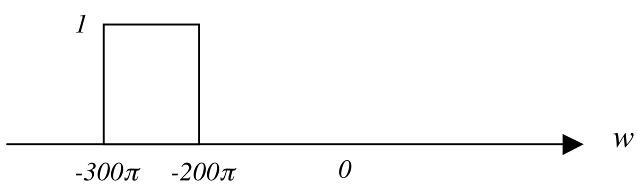
b. $x(t) = \sum_{n=-\infty}^{\infty}\delta(t-3n)$ (8 Points)

2.

Find inverse Fourier Transform of the following:

a. $X(e^{j\Omega}) = \sum_{k=-\infty}^{\infty}(-1)^k\,\delta(\Omega - \frac{\pi}{2}k)$ (8 Points)

b. $X(w)$ is the following (8 Points)



c. $X(\omega) = \dfrac{j}{(j\omega + a)^2}$ (8 Points)

PROBLEM 2: (30 Points)

When the input to an LTI system is given by:

$$\frac{6}{6}y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n-1]$$

a. What is the output of this system if the input is $x[n]=\sin(5\pi n/2-\pi)$? (5 Points)

b. What is the frequency response, $H(\Omega)$, of this system? (5 Points)

c. What is the output of this system if the input is $x[n]=(-1)^n$? (5 Points)

d. What is the impulse response h[n] of this system? (15 Points)

PROBLEM 3: (30 Points)

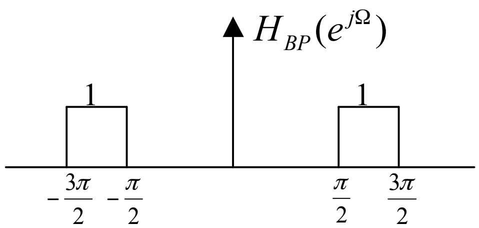
1. Let $x(t) = e^{-|10t|}$

a. Find the corresponding continuous time Fourier Transform of x(t)? (7 Points)

b. If we desire to sample this signal with $w_s = 200\pi$, how should we process x(t) before sampling to avoid aliasing? (8 Points)

2.

a) Write the impulse response h[n] of the following bandpass digital filter, given between $[-\pi, \pi]$: (7 Points)



b) What is the output of this system if $x[n] = \cos(\pi n) + \cos(\pi n / 4)$? (8 Points)