Elec 303
Midterm 1

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Duration: 90 minutes.

Examination is CLOSED-BOOK and CLOSED-NOTES. Do NOT use CALCULATOR.

NO CREDIT will be given for ANSWERS without PROPER JUSTIFICATION.

NAME: ____

ID NUMBER: ___________________________

SIGNATURE: ___________________________________

You may or may not need the following formulas:

$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \qquad \sum_{k=M}^{N}\alpha^k = \frac{\alpha^M - \alpha^{N+1}}{1-\alpha}$$

$$\cos^2\theta = \tfrac{1}{2}\left(1+\cos 2\theta\right) \qquad \sin^2\theta + \cos^2\theta = 1$$

Continuous-Time Fourier Series	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}, \omega_0 = \frac{2\pi}{T_0}$ $a_k = \frac{1}{T_0}\int_{<T_0>} x(t) e^{-jkw_0 t}\, dt$ $X(w) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$
Continuous-Time Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{jwt}\, dw$ $X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt}\, dt$
Discrete-Time Fourier Series	$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn}$ $a_k = \frac{1}{N}\sum_{n=<N>} x[n] e^{-j\frac{2\pi}{N}kn}$ $X(\theta) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\theta - \frac{2\pi}{N}k); a_{k+N} = a_k$
Discrete-Time Fourier Transform	$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(\theta) e^{j\theta n}\, d\theta$ $X(\theta) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\theta n}$

$$x(t) = \frac{\sin(\omega_0 t)}{\pi t} \quad \leftrightarrow \quad X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & \text{otherwise} \end{cases} \quad \leftrightarrow \quad X(\Omega) = \frac{\sin(\Omega(N_1 + 1/2))}{\sin(\Omega/2)}$$

$$x[n] = \frac{\sin(a\pi n)}{\pi n} \quad \leftrightarrow \quad X(\Omega) = \begin{cases} 1 & |\Omega| < a\pi \\ 0 & a\pi < |\Omega| < \pi \end{cases}$$

PROBLEM 1: (30 points) No credit will be given to answers without proper justification.

a) An LTI system is given by the following difference equation:

$$y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2]=x[n-1]$$

i. What is the frequency response $H(e^{j\Omega})$? (3 Points)
ii. Find the corresponding h[n]. (3 Points)
iii. What is the DTFT of $z[n]=n\,h[n]$? (3 Points)
iv. What is the DTFT of $w[n]=\cos(\pi n/2)h[n]$ (3 Points)
v. What is the DTFT of $r[n]=(-1)^n h[n]$ (3 Points)

b)

a. (5 Points) Let $x[n]$ is a periodic signal with period N

$$x[n] \xleftrightarrow{DFT,N} X_1[k]$$
$$x[n] \xleftrightarrow{DFT,4N} X_2[k]$$

What is the relationship between, $X_1[k]$, the N point DFT, and $X_2[k]$, the 4N point DFT?

b. $x[n]=\{0,1,2,3,4\} \xleftrightarrow{DFT,5} X[k]$ (5 Points)
$s[n]=\{0,1,0,0,1\} \xleftrightarrow{DFT,5} S[k]$

Determine y[n], which yields Y[k]=X[k]S[k] where Y[k] is 5 point DFT of y[n].

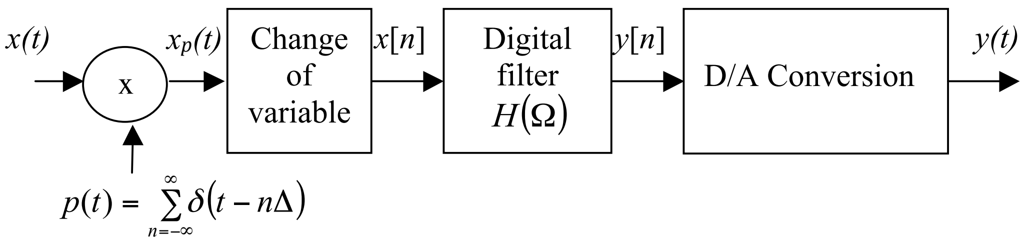
c. (5 Points) Suppose x[n] is a length 8 sequence and X[k] is the length 8 DFT of x[n]. We define a length 24 sequence y[n]

$$y[n]=\begin{cases} x\left[\dfrac{n}{3}\right], n=0,\pm3,\pm6,... \\ 0, \text{ otherwise} \end{cases}$$

What is the relationship between the 8 point DFT X[k] and 24 point DFT Y[k]?

PROBLEM 2: (35 points) No credit will be given to answers without proper justification.

Let $x(t) = \cos(800\pi t) + \cos(500\pi t)$ and Δ=1/600 sec.



a) (15 pts) Suppose we sample $x(t)$ without using an anti-alias filter.

  i. Compute and plot the Fourier transform (CTFT) $X_p(\omega)$ of the sampled signal $x_p(t)$.
  ii. Write an expression for the discrete-time signal $x[n]$ and find the DTFT of $x[n]$.

b) (20 pts) Suppose we now introduce an anti-alias filter before sampling.

  i. Find the cut-off frequency of the anti-alias filter.
  ii. Specify the output $y(t)$ if the impulse response of the digital filter is $h[n] = \left(\frac{1}{2}\right)^{|n|}$

PROBLEM 3: (35 points) No credit will be given to answers without proper justification.

Suppose you are given a sequence x[n] with the corresponding DTFT.

a) Given x[n], you construct y[n] such that (10 Points)

$$y[n] = \begin{cases} x[n], n = 0, \pm 3, \pm 6, ... \\ 0, \text{ otherwise} \end{cases}$$

what is the DTFT of y[n] in terms of DTFT of x[n]?

b) Given x[n], you construct z[n] (10 Points)

$$z[n] = \begin{cases} x\left[\frac{n}{3}\right], n = 0, \pm 3, \pm 6, ... \\ 0, \text{ otherwise} \end{cases}$$

what is the DTFT of z[n] in terms of DTFT of x[n]?

c) Given x[n], you construct (10 Points)

$d[n] = w[n]x[n]$ using the Hanning window:

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n / M), 0 \le n \le M, \\ 0, \text{ otherwise} \end{cases}$$

what is the DTFT of d[n] in terms of DTFT of x[n] for any M?

d) Given x[n], you construct (5 Points)

$e[n] = f[n]x[n]$ using the Bartlett window:

$$f[n] = \begin{cases} 2n / M, 0 \le n \le M / 2, \\ 2 - 2n / M, M / 2 < n \le M \\ 0, \text{ otherwise} \end{cases}$$

what is the DTFT of e[n] in terms of DTFT of x[n] for any M?