MATH206 Homework #2
Due 7 March 2008

1.

Let $u(x,y) = \sqrt{|xy|}$, $(x,y) \in \mathbb{C}$. Show that $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ both exist at $(0,0)$, but u is not real-differentiable at $(0,0)$.

2.

In each case show that u is harmonic and find the harmonic conjugate v such that $v(0,0) = 0$.
(i) $u(x,y) = e^y \cos x$;
(ii) $u(x,y) = 2x - x^3 + 3xy^2$.

Use MATLAB for taking derivatives.

3. Sketch the families of level curves of the component functions u and v when

    a. $f(z) = \frac{(z-2)}{(z+2)}$

    b. $f(z) = \frac{1}{z^2}$

and show that these curves are orthogonal to each other using MATLAB.

4. Write $|exp\,(2z + i)|$ and $|exp\,(iz^2)|$ in terms of x and y. Then show that:

$$|ex\,p(2z + i) + exp\,(iz^2)| \leq e^{2x} + e^{-2xy}$$