SOLUTIONS HW2

Q1) $u(x,y) = \sqrt{xy}$ where $x, y \in \mathbb{C}$

Since $u(0,y) = u(x,0) = 0$ for all x, y $\frac{\partial u}{\partial x}(0,0) = \frac{\partial u}{\partial y}(0,0) = 0$.

Let $f = u + iv$ and let v be identically 0.

Then due to complex-differentiability theorem on page 50:

If u is real-differentiable at $(0,0)$, then $f = u + iv$ is complex-differentiable at $(0,0)$.

Now differentiability of f at z_0 requires that

$\frac{(f(z) - f(z_0))}{z - z_0}$ approach a unique limit as z approaches

z_0 along an arbitrary path. Let this path be $y = x$ line.

$$\Rightarrow f = \sqrt{xy} \Rightarrow \frac{\sqrt{x^2}}{x + ix} = \begin{cases} \frac{1}{1+i} & \text{if } x > 0 \\ \frac{-1}{1+i} & \text{if } x < 0 \end{cases}$$

Since this function is not complex differentiable at the origin, "u" cannot be <u>real differentiable here.</u>

Q2) In order to show that u is harmonic, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

In order to find v, use Cauchy-Riemann

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \quad , \quad \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$$

for i) $v(x,y) = -e^{y}\sin x$

ii) $v(x,y) = -3x^2y + 2y + y^3$

Q4)

$$\left| e^{2x+2iy+i} + e^{i(x+iy)^2} \right|$$

$$= \left| e^{2x} \underbrace{e^{i(2y+1)}}_{A} + e^{-2xy} \cdot \underbrace{e^{i(x^2-y^2)}}_{B} \right| \leq e^{2x} + e^{-2xy}$$

$|A| \leq 1$ and $|B| \leq 1$

Therefore the whole expression should satisfy the inequality