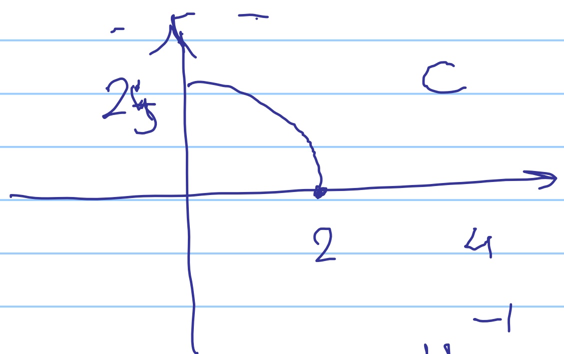
Note Title 3/20/2008

$\boxed{\text{Problem 1}}$

$$\left| \int_C \frac{dz}{z^2-1} \right| \leq \frac{\pi}{3}$$

$|z|=2$

$z^2-1=$



$z=re^{j\theta}$, $r=|z|$

$z^2=r^2e^{j2\theta}$

$M_R^{-1}=|z^2-1| \geq ||z^2|-1| = R^2-1 = 3$

$\frac{2\pi 2}{4} = L = \pi$

$M_R L = \frac{\pi}{3}$ ✓

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% FOR PROBLEM 2
clear all;close all;clc;
% C1 path integral
%leg AO parametrically z=0+iy (0<=y<=1)
%f(z)=y
syms x y;
fintAO=int(i*y^2,1,0);
%leg OB parametrically z=x+i (0<=x<=1)
%f(z)=(1-x-i3x^2)
fintOB=int(-x,0,1);
% C1 contour integral result
fintC1=fintAO+fintOB
% C2 path integral
% C2 denotes segment AB of the line y=x,
%parametric representation z=x+ix
% (0<=x<=1)
fintC2=int(((1-x)^2-x+4*i*x*(1-x))*(1-i),0,1)

ftotal = fintC1 - fintC2
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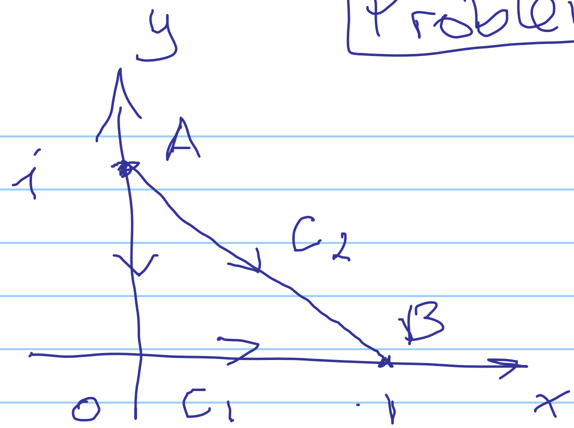
Problem 2



$$I_1 = \int_{C_1} f(z)\,dz = \int_{AO} f(z)\,dz + \int_{OB} f(z)\,dz$$

$$I_2 = \int_{C_2} f(z)\,dz = \int_{AB} f(z)\,dz$$

$$f(z) = y^2 - x + i4xy \quad , \quad z = x+iy$$

$z = 0 + iy$
$z' = i$

$$\int_{AO} y^2 i\,dy = i\int_1^0 y^2\,dy = i\,\frac{y^3}{3}\Big|_1^0 = -\frac{i}{3}$$

$z = x + i0$
$z' = 1$

$$\int_{OB} (-x)\cdot 1\cdot dx = -\frac{x^2}{2}\Big|_0^1 = -\frac{1}{2}$$

$$\rightarrow -\frac{1}{2} - \frac{i}{3}$$

$z = t + (1-t)i$
$z' = 1 - i$

$$(1-i)\int_0^1 \left[(1-t)^2 - t + i4t(1-t)\right] dt$$

$$(1-i)\left[-\frac{(1-t)^3}{3}\Big|_0^1 - \frac{t^2}{2}\Big|_0^1 + 4i\left(\frac{t^2}{2} - \frac{t^3}{3}\right)\Big|_0^1\right]$$

$$(1-i)\left[\frac{1}{3}-\frac{1}{2}+4i\left(\frac{1}{2}-\frac{1}{3}\right)\right]$$

$$\left(-\frac{1}{6}+\frac{2}{3}i\right)(1-i)=-\frac{1}{6}+\frac{2}{3}i+\frac{i}{6}+\frac{2}{3}$$

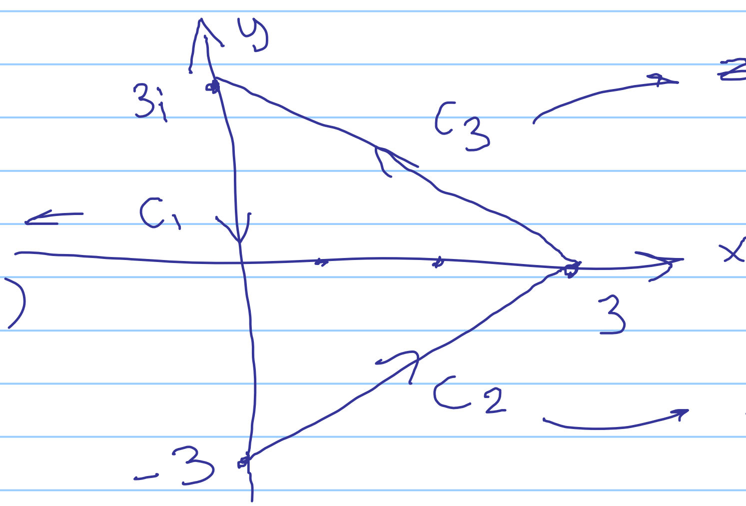
$$\int_{C_1} f(z)\,dz-\int_{C_2} f(z)\,dz=-\frac{1}{2}-\frac{i}{3}-\left(\frac{1}{2}+\frac{5i}{6}\right)$$

$$=-1-\frac{7i}{6}$$

Problem 3

$z=4t$

$(3\to -3)$



$z=3\left(t+(1-t)i\right)$

$(1\to 0)$

$z=3\left(t-(1-t)i\right)$

$(0\to 1)$

parameters for MATLAB & integral limits

$$\frac{1}{2\pi i}\int_C \frac{z^2}{(z-1)^2(z-2)}\,dz = \frac{1}{2\pi i}\Bigg(\underbrace{\int_C \frac{4}{z-2}\,dz}_{4} + \underbrace{\int_C \frac{-3\,dz}{z-1}}_{-3} + \underbrace{\int_C \frac{-1}{(z-1)^2}}_{0}\Bigg)$$

$$= \frac{1}{2\pi i}$$

```
% FOR PROBLEM 3
clear all;close all;clc;
syms t;

a = 1;
b = 2;

% Contour path AB
C1 = double(int((-i*t^2/(i*t-a)^2/(i*t-b)),3,-3))

% Contour path BC
C2 = double(int((3*(1+i)*9*(t-i*(1-t))^2/(3*(t-i*(1-t))-a)^2/(3*(t-i*(1-t))-b)),0,1))

% Contour path CA
C3 = double(int((3*(1-i)*9*(t+i*(1-t))^2/(3*(t+i*(1-t))-a)^2/(3*(t+i*(1-t))-b)),1,0))

%Total Integral
C = C1+C2+C3
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